Combination of Mixed Adjustment Model and Geodetic Lines Method to Transform GPS Coordinates into National Coordinates

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Key words: Cadastre, Geodetic Lines Method, GPS coordinates, local Coordinates, Mixed Adjustment Model, Nord Sahara 1959, Transformation, WGS84.

SUMMARY

The paper describes a simplified transformation between geographical coordinates of two ellipsoidal surfaces (i.e. WGS84 and Clarke 1880), applied in the North of Algeria to express the new GPS coordinates into local coordinates, useful only in small regions and suited for large scale applications such as cadastre. This transformation approach (A. Leick, 90) is based on the mixed adjustment model to estimate the four parameters transformation (2 translations, 1 rotation and scale factor) and on the geodetic lines concept, respectively the called " Inverse " and " Forward " problem of geodesy.

In this context, the program developed was based on methods described above. The necessary data, the steps of processing and the results obtained are presented.

RESUME

L’article décrit une transformation simplifiée entre coordonnées géographiques ellipsoïdiques (ex : WGS84 et Clarke 1880), appliquée dans le nord de l’Algérie afin d’exprimer les nouvelles coordonnées GPS en coordonnées locales (nationales), valide seulement dans des régions de faibles dimensions et recommandée pour les applications à grandes échelles tel que le cadastre. Cette approche de transformation développée par (A. Leick, 90) est basée sur un modèle d’ajustement mixte pour estimer les quatre paramètres de transformations (2 translations, 1 rotation et un facteur d’échelle) et sur le concept des lignes géodésiques, respectivement appelée, le problème " Inverse" et " Direct " de la géodésie.

Dans ce contexte, le programme développé est basé sur les méthodes décrites ci-dessus. Les données nécessaires, les étapes de traitements et les résultats obtenus sont présentés.

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1. INTRODUCTION

Densification of a new cadastral project by GPS techniques require that surveyed GPS coordinates must be transformed into national coordinates using appropriate transformation parameters between both WGS84 and Nord Sahara 1959 datum. To do this, knowledge of some characteristics geographical data (system of common points, homogeneous of gravimetric data) is necessary. In several situations, Algerian surveyor's are confronted to the non-availability of the ellipsoidal heights referred to the local system Nord Sahara 1959 (i.e. the less knowledge of the geoid undulations) and to the imprecision of orthometric heights. To remedy of the lack of local geoidal and orthometric height in small region, we study several approaches of transformation. The simplified transformation described here between geographical coordinates of two ellipsoidal surfaces (i.e. WGS84 and Clarke 1880) is performed using combination of least squares method (mixed adjustment model) and geodetic lines concept, respectively called forward and inverse problem of geodesics. The results are the four parameters transformation (two translations, one rotation and a scale factor).

The transformation approach developed here remains only valid in small area up to (10Km x 10 Km) and must be considered only approximately valid. This special local transformation can provide quite accurate transformed coordinates of new points under some criteria. The required data, the considered criteria, the steps of processing and the mathematical formulas used for this transformation are described in the paper.

2. REQUIRED DATA AND CRITERIA

The transformation approach described in this paper was developed by (Leick, 1990). The aim consist to transform GPS planimetric coordinates $(\lambda, \varphi)_{GPS}$ into local planimetric coordinates $(\lambda\ , \varphi)_{local}$. The required data to perform the transformation are:

- Two sets of geographical coordinates for common reference points in both system (i.e. WGS84 and Nord Sahara 1959), at least three common points are required. We denote by the subscripts "GPS" for the GPS coordinates and by the subscripts 'NS" for the terrestrial coordinates referring to a local system Nord Sahara 1959.
- Sets of geographical coordinates for new GPS points of cadastral project.

The following criteria's must be considered when developing this approach:

- GPS observations provide $(X,Y,Z)_{GPS}$ coordinates as computed from the minimal or inner constraint solution using specific GPS software (i.e. Winprism), these coordinates can be converted into ellipsoidal coordinates $(\varphi, \lambda, h)_{GPS}$ using appropriate formulas. For this conversion ellipsoid parameters (a, e) of geodetic system (i.e. Clarcke 1880) must be used.
- The inter station line between at least two reference points must have similar length as the overall size of the project up to (10 km x 10 km). In fact, this help to control scale and will minimise the effect of coordinates errors, which propagate as scale errors.
- The GPS coordinates of initial point must be computed first in the area of study. An initial point is the centroid of all reference points considered in the project area (see figure 1).
- The control points, at least three, must be well distributed in area of project.
- Area of study must have size up to (10km x 10km).

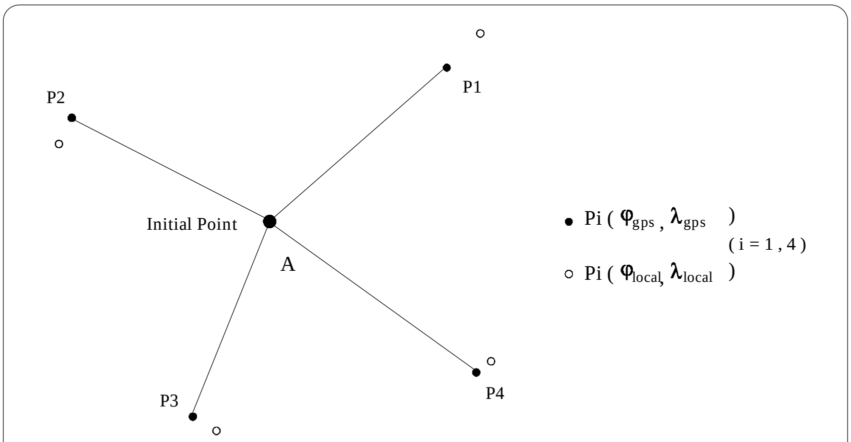
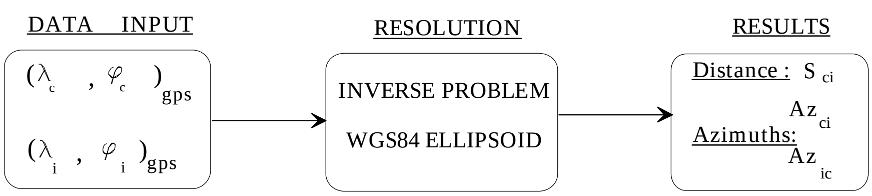


Figure 1: Transformation on Ellipsoidal Surface

3. STEPS OF PROCESSING

The resolution of the inverse and forward problem of geodesics is treated in several books of geodesy. For the inverse problem resolution we need geographical coordinates of initial point and point P_i (i=1, n, n number of reference points), the forward and back azimuth and the length of the geodesic from initial point (centroid) to point P_i are computed as follow:



The inverse solution constitutes the mathematical model for the adjustment on the ellipsoid. The general form is written as:

$$S_{ci} = f(\varphi_c, \lambda_c, \varphi_i, \lambda_i)$$
$$Az_{ci} = f(\varphi_c, \lambda_c, \varphi_i, \lambda_i)$$

The respective coefficients of partial derivatives for the design matrix are show in table 1:

	$d\varphi_c$	$d\lambda_c$	$d\varphi_i$	$d\lambda_i$
dS_{ci}	$- M_c \cos Az_{ci}$	$N_i \cos\varphi_i \sin Az_{ic}$	$- M_i \cos Az_{ic}$	$- N_i \cos\varphi_i \sin Az_{ic}$
dAz_{ci}	$(M_c \sin Az_{ci}) / S_{ci}$	$(N_i \cos\varphi_i \cos Az_{ic}) / S_{ci}$	$(M_i \sin Az_{ic}) / S_{ci}$	$(- N_i \cos\varphi_i \cos Az_{ic}) / S_{ci}$

Table 1: Coefficients of partial derivatives

Where S_{ci} is the length of the geodesic from initial point to station i, Az_{ci} and Az_{ic} designate the azimuth of geodesic from initial point to points $P_i(\lambda,\varphi)_{GPS}$ and from point $P_i(\lambda,\varphi)_{GPS}$ to initial point respectively. The quantities S_{ci} , Az_{ci} and Az_{ic} are computed from the inverse problem resolution. M is the radius of curvature of the ellipsoidal meridian and N is the radius of curvature of the prime vertical section.

The differences in coordinates between GPS and local coordinates (φ_{NS} - φ_{GPS}) and (λ_{NS} - λ_{GPS}) are used as observations to compute the four transformation parameters using a least squares adjustment. Because these differences are small quantities, the coefficients that are traditionally used in the adjustment of two-dimensional networks on the ellipsoid are also appropriate for this transformation (Leick, 1990). Thus, the changes in length and azimuth of the geodesic from initial point (centroid) to any reference point P_i are written as follow:

$$dS_{ci} = -M_i \cos Az_{ic} d\varphi_i - M_c \cos Az_{ci} d\varphi_c - N_i \cos\varphi_i \sin Az_{ic}(d\lambda_i - d\lambda_c) \qquad \text{(a)}$$

$$dAz_{ci} = \frac{M_i}{S_{ci}} \sin Az_{ic} d\varphi_i + \frac{M_c}{S_{ci}} \sin Az_{ci} d\varphi_c - \frac{N_i}{S_{ci}} \cos Az_{ic} \cos\varphi_i (d\lambda_i - d\lambda_c) \qquad \text{(b)}$$

The two equations (a) and (b) can be written for each reference point P_i. It should be emphasised that all geodesics are rotated by the same amount in azimuth and are scaled by the same factor. The following hypotheses are valid:

$$dS_{ci} = \Delta S_{ci} \qquad \text{(c)}$$
$$dAz_{ci} = dAz_c = \text{const} \qquad \text{(d)}$$

Where Δ denote the scale change.

The observations used in the adjustment model are the differences between GPS and local geographical coordinates:

$$d\lambda_i = \lambda_{iNS} - \lambda_{i\,GPS} \qquad \text{(d)}$$
$$d\varphi_i = \varphi_{i\,NS} - \varphi_{i\,GPS} \qquad \text{(e)}$$

Substituting the equation (c) through (e) into the two first equation (a) and (b) gives:

$$\Delta S_{ci} = -M_i \cos Az_{ic}(\varphi_{ins} - \varphi_{iGPS}) - M_c \cos Az_{ci} d\varphi_c - N_i \cos\varphi_i \sin Az_{ic}(\lambda_{ins} - \lambda_{iGPS}) + N_i \cos\varphi_i \sin Az_{ic} d\lambda_c \tag{f}$$

$$dAz_{ci} = \frac{M_c}{S_{ci}} \sin Az_{ci}\, d\varphi_c + \frac{M_i}{S_{ci}} \sin Az_{ic}(\varphi_{ins} - \varphi_{iGPS}) - \frac{N_i}{S_{ci}} \cos Az_{ic} \cos\varphi_i (\lambda_{ins} - \lambda_{iGPS}) + \frac{N_i}{S_{ci}} \cos Az_{ic}\, \varphi_i \cos\varphi_i\, d\lambda_c \tag{g}$$

Note that into the two equations (f) and (g), the parameters and observations already appear in linear form. This is the particularity of the mixed adjustment model where the observations and the parameters are implicitly related. The mixed adjustment model is a more recent tools that applies to research involving unknowns parameters whose level can be controlled by researcher.

The generalized linear mixed model is written as:

$$BV + AX + W = 0$$

Where:

A and B: are matrices of observations and approximate parameters,
W: denotes the value of the mathematical function,
V: is the vector of estimates residuals,
X: is the vector of unknown's parameters;

the estimated results are the four transformation parameters defined as follows:

- The two translations of the initial point (centroid) ($d\lambda_c$, $d\varphi_c$),
- The rotation (dAz_c) of the geodesics at the initial point, and
- The scale factor (1- Δ) of the geodesics.

The respective sub-matrices B and A for each reference point P_i is:

$$B = \begin{pmatrix} M_i \cos Az_{ic} & N_i \cos\varphi_i \sin Az_{ic} & -M_i \cos Az_{ic} & -N_i \cos\varphi_i \sin Az_{ic} \\ -\frac{M_i}{S_{ci}} \sin Az_{ic} & \frac{N_i}{S_{ci}} \cos A_{ic} \cos\varphi_i & \frac{M_i}{S_{ci}} \sin Az_{ic} & -\frac{N_i}{S_{ci}} \cos Az_{ic} \cos\varphi_i \end{pmatrix}$$

$$A = \begin{pmatrix} -M_c \cos Az_{ci} & N_i \cos\varphi_i \sin Az_{ic} & -S_{ci} & 0 \\ \frac{M_i}{S_{ci}} \sin Az_{ci} & \frac{N_i}{S_{ci}} \cos Az_{ic} \cos\varphi_i & 0 & -1 \end{pmatrix}$$

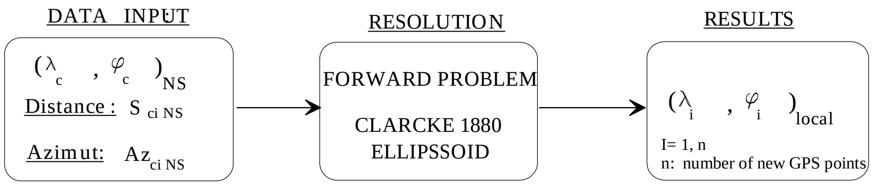
The respective coefficients of W vector are:

$$W = \begin{pmatrix} -M_i \cos Az_{ic}(\varphi_{ins} - \varphi_{iGPS}) - N_i \cos\varphi_i \sin Az_{ic}(\lambda_{ins} - \lambda_{iGPS}) \\ \frac{M_i}{S_{ci}} \sin Az_{ic}(\varphi_{ins} - \varphi_{iGPS}) - \frac{N_i}{S_{ci}} \cos Az_{ic} \cos\varphi_i (\lambda_{ins} - \lambda_{iGPS}) \end{pmatrix}$$

Resolution of this problem by the least squares method while applying the mixed adjustment model, permits to estimate the four transformation parameters ($d\lambda_c$, $d\varphi_c$, dAz_c, Δ) of the adjustment, that will serve to obtain lengths, azimuths of geodesics and planimetric coordinates of the initial point into the local system through the following equations:

$$\begin{aligned} S_{ci,NS} &= S_{ci,GPS} + \Delta S_{ci} \\ Az_{ci,NS} &= Az_{ci,GPS} + dAz_c \end{aligned} \qquad \text{and} \qquad \begin{aligned} \varphi_{c\,NS} &= \varphi_{c\,GPS} + d\varphi_c \\ \lambda_{c\,NS} &= \lambda_{c\,GPS} + d\lambda_c \end{aligned}$$

The final step of processing consist to transform all new GPS surveyed points into local system using forward problem of geodetic lines as follow:



4. APPLICATION

We have used IDL (Interactive Data Language) to develop the respective program of this approach of transformation named **WGS84_NS**. To validate it, we have used two sets of GPS data issued from two different GPS campaigns. The results were compared to other approaches of transformation.

4.1 First Test and Preliminary Results

The first GPS campaign has been carried out in Oran city during May 1995 using two receivers wild system 200. Firstly, GPS coordinates surveyed were transformed by **stepwise** method integrated as module of transformation in SKI (Static Kinematic Software) and by the first version of the program of transformation named **2dtrans** as described in my paper (NABED, GOURINE, 2000). Secondly, the same data are used for this new version of transformation developed in this paper as shown in figure (3).

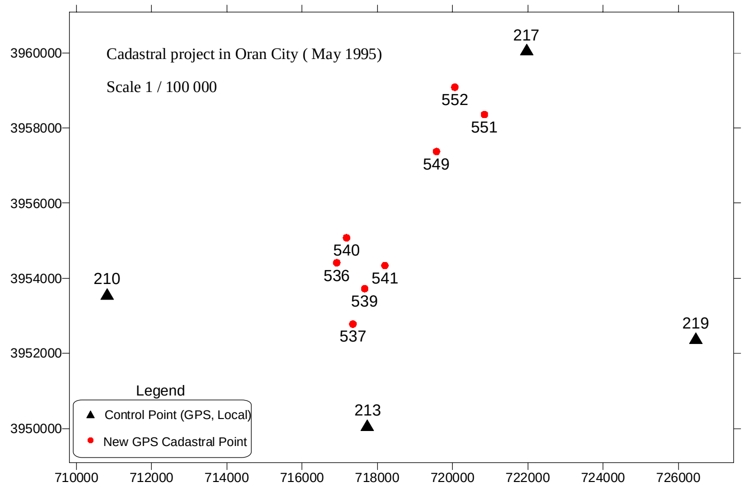


Figure 3:

Extension of Cadastral Network in Oran region

To perform this approach of transformation we have the following data sets:

- A second order Geodetic Network composed of four reference points (217,213,210,219) known in both system (WGS84 and Nord Sahara Datum 1959). The control points were observed by GPS using static mode.
- An extension of the cadastral network composed of eight new points localised inside the area of study. The new points were observed by GPS using rapid static mode.

In general, after two at three iterations the mixed adjustment process is performed and the unknown parameters are estimated. The computed transformation parameters and its residuals are shown in table (2):

Unknown parameters	$d\varphi_{Initial}$ (arc second)	$d\lambda_{initial}$ (arc second)	dAz (arc second)	Δ (scale factor) ppm
Parameters Value	-0.35	3.08	-4.28	-1.48
Residuals	0.0390	0.0117	0.0124	0.08

Table 2: Parameters values and residuals

We applied this transformation parameters to transform all new GPS points into local system. The new transformed coordinates were compared to results obtained by **stepwise** method of SKI software and **2Dtrans** program (using geodesic lines). The residuals in coordinates computed are illustrated in tables (3) and (4) respectively. Comparison with **stepwise** method and **2Dtrans** program gives residuals in coordinates in order of centimeter.

	Minimum (m)	Maximum (m)	Average (m)	EMQ
Residuals in Easting	-0.178	0.051	-0.043	0.074
Residuals in Northing	-0126	0.03	0.089	0.142

Table 3: Comparison with **Stepwise** method

	Minimum (m)	Maximum (m)	Average (m)	EMQ
Residuals in Easting	-0.110	0.362	-0.037	0.052
Residuals in Northing	-0.110	0.362	0.110	0.139

Table 4: Comparison with **2Dtrans**

4.2 Second Test and Results

The second GPS compaign has been carried out in Arzew city during December 1996 using three receivers ASHTECH Z12. We have established an important project for SONATRACH (Industry Petroleum Company in Algeria) named the Micro triangulation network composed of (58) new GPS points. To conceive the microtriangulation, firstly we have observed in static mode with some hours of observation a reference network composed of eight points Where three points belong to the second order geodetic network and five new points belong to the new microtriangulation as shown in the figure (4), the distance between these points varies from 5 to 12 Km. Secondly we have observed all new GPS points of the microtriangulation using fast static method with a minimum of constraints (better precision, easiness of access to points, good distribution of points through all the project (see figure 5)). GPS coordinates were transformed first into national datum (Nord Sahara 1959) using the **2Dtrans** program and **Stepwise** method.

For this new approach of transformation, we used the same sets of data to obtain the four parameters of transformation and finally to transform the new GPS points of microtriangulation in our local system (figure 5). A new comparison is carried out between the developed approach and the two other methods, the **Stepwise** and the **2Dtrans** program (using only geodesic lines) respectively.

After two iterations, the transformation is performed and the unknown parameters are estimated. The computed transformation parameters and its residuals are shown in table (5).

Unknown parameters	$d\varphi_{Initial}$ (arc second)	$d\lambda_{initial}$ (arc second)	dAz (arc second)	Δ (scale factor) ppm
Parameters Value	- 0.35	3.06	6.23	-1.27
Residuals	-0.0018	-0.0097	0.0015	-0.697

Table 5: Parameters values and residuals

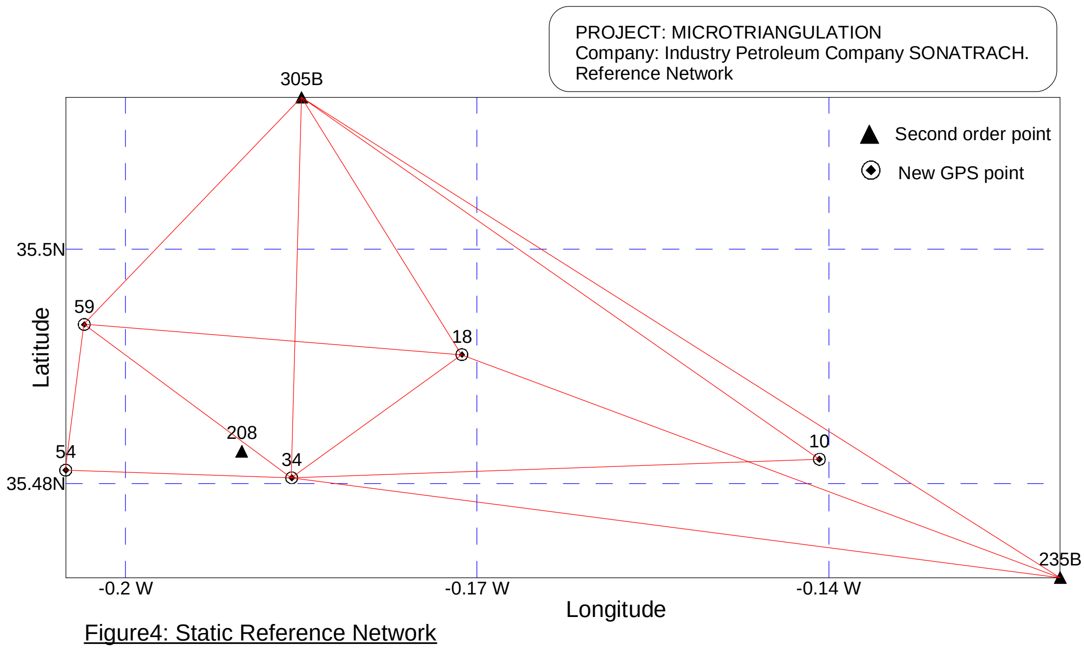


Figure4: Static Reference Network

The transformed coordinates were compared to results obtained by **Stepwise** method of SKI software and **2Dtrans**. The residuals in coordinates are illustrated in table (6) and (7).

	Minimum (m)	Maximum (m)	Average (m)	EMQ
Residuals in Easting	-0.012	0.267	0.086	0.087
Residuals in Northing	-0.229	0.027	-0.117	0.063

Table 6: Comparison with **Stepwise** method

	Minimum (m)	Maximum (m)	Average (m)	EMQ
Residuals in Easting	-0.043	0.173	0.083	0.055
Residuals in Northing	-0256	0.200	-0.099	0.131

Table 7: Comparison with **2Dtrans** program

The comparison with **Stepwise** method and **2Dtrans** program gives residuals in order of centimeter. In conclusion, results obtained by combination of mixed adjustment model and geodetic lines method gives quite accurate transformed coordinates, but this method remains only valid in small sized network.

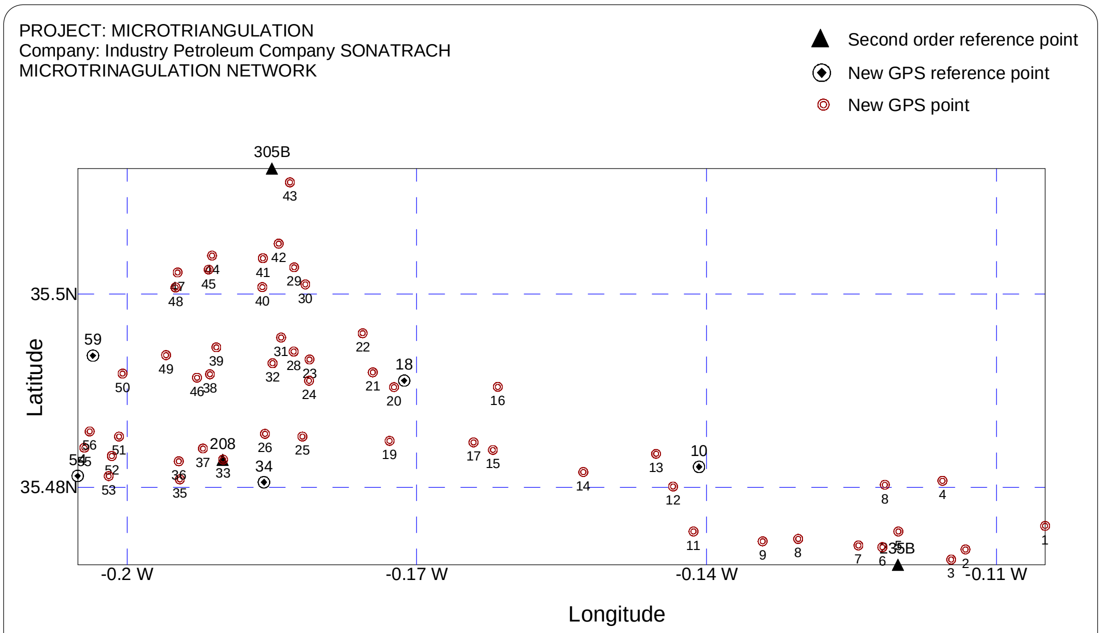


Figure 5: Static Reference Network

5. CONCLUSION

The developed approach using combination of mixed adjustment model and geodetic lines method provide a mean precision of transformed coordinates in order of a centimeter as described in comparison to other approaches of transformation, which allow it's utilisation for large scale application such as cadastre. The mixed adjustment model is a more recent tools that applies to research involving unknowns parameters whose level can be controlled by researcher. This new approach permits to check the measurements and to adjust the cadastral networks in an independent way and not require height data in reference points nor in new GPS points. This method works well for small sized network with low elevations and for areas where geoidal heights do not change significantly. If the size of area project is very large, in this case we must devide our area project into several regions and assure to have the junction between these regions.

It is emphasised that the obtained coordinates are affected by some errors, which depends on GPS localisation and on the existence of local system distortions. Even if local distortions could adequately modeled with this technique of transformation, the accuracy of transformed coordinates still be of low quality (Leick, 1990) for the principal reason that the national network adjustment used less and low observations than the GPS solution. For this reason it's very important to have access to the (original) observations and to perform a new regional adjustment.

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BIOGRAPHICAL NOTES

Mr. Abdelkader Nadir NABED works as researcher in the geodetic laboratory of National Center of Spatial techniques (CNTS –Arzew –Algeria), has his post-graduate Magister diploma in Geodetic Sciences since 1996, he have presented his research about GPS Techniques and Cadastre. The main tasks of his actual research is articulated on positioning by GPS techniques for large scale applications (cadastre, road survey, auscultation , etc…) on methods of observations (Static, Rapid Static, Kinematic, OTF technique, …), on treatments of GPS data, and on problem of transformation from GPS coordinates into national coordinates. He teaches GPS courses for engineer and Magister students since 1996.

Mr. Bachir GOURINE, graduated as a geodetic engineer from the National Center of Spatial Techniques (CNTS), Arzew, Algeria, in 1994. He teaches geodesy and micro-triangulation since 1995, at the CNTS. He is working as geodesist in the laboratory of Geodesy, CNTS. The main subjects of his research concern the combination of the terrestrial and spatial (GPS) observations for geodetic applications, the transformation methods of GPS coordinates to national coordinates, the modelling and evaluation of the network deformations by the Monte-Carlo method and finite element method (FEM).

Mr. Sid Ahmed BENAHMED DAHO is researcher in the Geodetic Laboratory of National Center of Spatiales techniques (CNTS –Arzew –Algeria) since 1996. His research is related on: Gravity field modelling in particular Geoid determination from gravity and GPS data in Algeria, and satellite altimetry and applications in geodesy and oceanography.

Mr. Boualem GHEZALI, researcher in the Laboratory of Geodesy of the National Center of the Spatial Techniques (CNTS), Arzew / Algeria. Domains of intervention, concerning geodesy, articulate around themes of research relative to the precise positioning by spatial techniques and to the geodesic data treatments. The achieved works are about the technical GPS and its applications in geodesy, in geodynamics and works of large scale, on the development of tools related to the geodesic astronomy and about the technical VLBI for the geodesic applications. In parallel of research activities he teach courses about "Geodesic Astronomy" and "Reference Frames" for the engineer students and Magister.

Mr. Ali ZEGGAI, has his Magister diploma in geodesy since 1997. He works at the National Centre of Spatial Techniques (CNTS - Arzew- Algeria) as researcher to the laboratory of geodesy. Except the teaching that he dispenses in the CNTS since 1978, his main activity is based on GPS works (Levelling by GPS, transformation of co-ordinates, maritime navigation by differential GPS).

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