Reliability of the Conventional Deformation Analysis Methods for Vertical Networks

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Key words: Deformation analysis, vertical deformation, reliability, homoscedasticity heteroscedasticity.

ABSTRACT

Reliability of the conventional deformation analysis methods is defined with mean success rate. The mean success rate is given as the number of successes divided by the number of experiments. Four different vertical networks are generated by simulation. The observations for two epochs and deformations are also generated. The mean success rates of the methods are computed for certain number of deformed points, for a given interval and for kinds of deformation. Consequently, the reliability of the methods changes depending on number of points, magnitude of deformations, degrees of freedom and number of deformed points. The reliability of the methods increases when the degrees of freedom and the magnitudes of deformations increase. It decreases when the number of points and the number of deformed points increase. As known, the random errors' variances are changing depending on the distance of the levelling lines. This Gauss-Markov model is called as heteroscedastic. If the distances are approximately equal to each other, the random errors have a common variance. So, the model is called as homoscedastic. If the model is homoscedastic, the reliability of the analysis methods increases very rapidly with respect to the ones of the heteroscedastic model.

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1. INTRODUCTION

One of the main aims of geodesy is detection of the deformations imposed on an object or an area which is characterized with points of a geodetic network. Since it is essential to detect deformations for many purposes (monitoring plate tectonics, determination of global datum, taking precautions for a construction which may be under damage, etc.), many considerable efforts and investigations have been performed on deformation analysis (Chen 1983; Chrzanowski et al. 1982; Liu and Chen 1998; Niemeier 1985; Pelzer 1971; Welsch et al. 2000).

Conventionally, for all types of networks (vertical, horizontal or 3D networks), for detecting deformations, same points' estimated coordinates obtained from least-squares adjustment of observations made at different epochs are compared with each other by using the statistical tests. Therefore, this procedure is called as conventional or geometrical analysis which comprises global congruency test and localization steps (Welsch and Heunecke 2001). Although it is also used widely to verify stabilities of some points (to define reference points) or instabilities of others, it is not known exactly that if it gives correct results in all probable cases or which situations may increase its reliability according to imposed deformations. To get information about reliability of the method, true deformations must be known in advance. However, in real cases, it is not entirely possible to estimate what deformation magnitudes are imposed on which points. For that reason, some authors use simulation that creates probable cases in model to find some methods are effectual (Betti et al. 1999; Liu and Chen 1998).

In this study, we investigated how reliability of the conventional deformation analysis methods can be measured and how it changes for the vertical networks. Therefore, we adapted the reliability concept introduced by Hekimoglu and Koch (1999; 2000) to the conventional deformation analysis. The reliability is defined as a mean success rate obtained from number of success divided by the number of experiments to identify outliers. In this concept, we put the deformations instead of outliers. For this purpose, we simulated four vertical networks. In the vertical networks, the random errors' variances are changing depending on the distance of levelling lines. So, the Gauss-Markov model is called as heteroscedastic in robust statistics (Carroll and Ruppert 1982; Hekimoglu and Berber 2001). We know that if the heteroscedasticity is strong, it affects on the reliability of robust estimators badly (Hekimoglu and Berber 2001). Therefore, for the vertical networks' observations, we considered two random error types: heteroscedastic and homoscedastic. Then, simulated deformations for the vertical networks are added to certain number of points' heights at present epoch. They are defined for different kinds and a given interval of deformation. After the deformed points detected separately with using two localization methods (Gauss Elimination and Implicit Hypothesis) are compared with known deformed points, the mean success rates of these methods are obtained for the vertical networks.

2. CONVENTIONAL DEFORMATION ANALYSIS

In the conventional deformation analysis (CDA), the deformations are accepted as significant geometrical differences verified with statistical test, i.e., F-test. So, detection of the deformations basically depends on comparison of geometry of the geodetic networks observed at different epochs, e.g., initial and present epochs (Welsch and Heunecke 2001).

2.1 Free Adjustment of the Vertical Deformation Networks

In order to remove geodetic datum deficiency, heights and their cofactor matrix of each epoch of the vertical network are computed individually by free adjusment method. All of the heights of the network are incorporated to the model as unknowns. So, the Gauss-Markov model can be expressed as (Koch 1999)

$$\mathbf{v}_k = \mathbf{A}_k \mathbf{x}_k - \mathbf{l}_k \,, \tag{1}$$

$$\mathbf{P}_k = \mathbf{Q}_{l_k}^{-1} \,, \tag{2}$$

where k is the number of epochs, $\mathbf{v}_k$ is the $n \times 1$ residual vector, $\mathbf{A}_k$ is the $n \times u$ design matrix, $\mathbf{x}_k$ is the $u \times 1$ unknown vector, $\mathbf{l}_k$ is the $n \times 1$ observation vector, $\mathbf{P}_k$ is the $n \times n$ diagonal weight matrix of observations, $\mathbf{Q}_{l_k}$ is the $n \times n$ weight coefficient matrix of the observations, u is the number of unknowns (equal to number of points in vertical networks), n is the number of observations. When the conditions $\mathbf{v}_k^T \mathbf{P}_k \mathbf{v}_k = \min$, $\mathbf{x}_k^T \mathbf{x}_k = \min$ are satisfied, estimated unknown vector can be formulated as

$$\hat{\mathbf{x}}_k = \mathbf{Q}_{\hat{x}_k \hat{x}_k} \mathbf{A}_k^T \mathbf{P}_k \, \mathbf{l}_k \,, \tag{3}$$

where $\mathbf{Q}_{\hat{x}_k \hat{x}_k}$ is the cofactor matrix of estimated unknowns,

$$\mathbf{Q}_{\hat{x}_k \hat{x}_k} = (\mathbf{A}_k^T \mathbf{P}_k \mathbf{A}_k)^+ \,, \tag{4}$$

where $(\mathbf{A}_k^T \mathbf{P}_k \mathbf{A}_k)^+$ is the pseudo inverse of the normal equation (Koch 1999; Teunissen 1985). The estimated variance factor as fallows:

$$s_{0_k}^2 = \frac{\mathbf{v}_k^T \mathbf{P}_k \mathbf{v}_k}{f_k} \,, \tag{5}$$

where f_k is the degrees of freedom. If the two epochs' estimated variance factors are equal to each other statistically, the pooled variance factor is written by

$$s_0^2 = \frac{\mathbf{v}_1^T \mathbf{P}_1 \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{P}_2 \mathbf{v}_2}{f} \quad , \quad f = f_1 + f_2 \,. \tag{6}$$

2.2 Global Congruency Test

Whether the difference between the estimated unknown vectors for two epochs is the result of the deformations or random errors of the observations is investigated with global congruency test. For this aim, expected values of the estimated unknown vectors are assumed equal to each other by the following null-hypothesis

$$H_0 : E\{\hat{\mathbf{x}}_1\} = E\{\hat{\mathbf{x}}_2\}. \tag{7}$$

The influence of the hypothesis can be expressed as

$$R = \mathbf{d}^T \mathbf{Q}_{dd}^{+} \, \mathbf{d}\,, \tag{8}$$

where **d** is the difference vector of the estimated unknown vectors

$$\mathbf{d} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1\,, \tag{9}$$

$\mathbf{Q}_{dd}$ is the its cofactor matrix,

$$\mathbf{Q}_{dd} = \mathbf{Q}_{\hat{x}_1\hat{x}_1} + \mathbf{Q}_{\hat{x}_2\hat{x}_2}\,. \tag{10}$$

If the null-hypothesis is true, the test quantity T_G follows the central F-distribution,

$$T_G = \frac{R}{h\, s_0^2} \sim F_{h,f}\,, \tag{11}$$

where h is the rank of $\mathbf{Q}_{dd}$. If $T_G \geq F_{h,f,1-\alpha}$ $(1-\alpha$ is confidence level), the null hypothesis is rejected. In other words, the difference is accepted as a result of deformation. Therefore, the next step is localization of deformations.

2.3 Localization of Deformations

Among the many localization methods presented in Welsch et al. (2000), Gauss elimination and implicit hypothesis methods are reviewed below.

2.3.1 Localization with Gauss elimination

The difference vector **d** and $\mathbf{Q}_{dd}^{+}$ are divided into sub-vectors and sub-matrices as follows (Niemeier, 1985; Welsch et al. 2000)

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_B \end{bmatrix} \quad , \quad \mathbf{Q}_{dd}^{+} = \begin{bmatrix} \mathbf{P}_{FF} & \mathbf{P}_{FB} \\ \mathbf{P}_{BF} & \mathbf{P}_{BB} \end{bmatrix}, \tag{12}$$

where index B describes assumed deformed point P_B, index F is for the other points. To distinguish independently the effect of the point P_B on R in (8), the following equation is obtained by using Gauss elimination method

$$\bar{\mathbf{d}}_B = \mathbf{d}_B + \mathbf{P}_{BB}^{-1}\,\mathbf{P}_{BF}\,\mathbf{d}_F\,. \tag{13}$$

So, the partial effect of P_B on quadratic form R is given by

$$R_B = \overline{\mathbf{d}}_B^T \, \mathbf{P}_{BB} \, \overline{\mathbf{d}}_B \,. \tag{14}$$

Sequentially, each point is considered as deformed point. The point which gives maximum R_B is accepted as deformed point and it defines the current datum. To test whether remaining points are deformed, $\mathbf{d}$ and $\mathbf{Q}_{dd}$ is transformed into the current datum with using S–transformation (Teunissen 1985)

$$\mathbf{d}_i = \mathbf{S}_i \, \mathbf{d} \quad , \quad \mathbf{Q}_{d_i d_i} = \mathbf{S}_i \, \mathbf{Q}_{dd} \, \mathbf{S}_i^T \quad , \ (i = 1, 2, \ldots u) \,. \tag{15}$$

Using elements related to the remaining points in (15), the new global congruency test is applied. The same localization procedure is performed until the test can not verify existence of deformation any more. As a result of the localization, deformed points are detected.

2.3.2 Localization with implicit hypothesis

Some points' estimated unknowns are considered stable for two epochs by the null-hypothesis (Welsch et al. 2000)

$$H_0 : E\{\hat{\mathbf{x}}_{1F}\} = E\{\hat{\mathbf{x}}_{2F}\} = \hat{\mathbf{x}}_F \,. \tag{16}$$

The hypothesis is included into the following observation equation implicitly

$$\mathbf{v}_H = \begin{bmatrix} \mathbf{A}_{1F} & \mathbf{A}_{1B} & \mathbf{o} \\ \mathbf{A}_{2F} & \mathbf{o} & \mathbf{A}_{2B} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_F \\ \hat{\mathbf{x}}_{1B} \\ \hat{\mathbf{x}}_{2B} \end{bmatrix} - \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} , \tag{17}$$

where $\hat{\mathbf{x}}_{1B}$ and $\hat{\mathbf{x}}_{2B}$ are unknown vectors related to assumed as the deformed point. The influence of the hypothesis can be expressed as

$$R_I = \mathbf{v}_H^T \, \mathbf{P} \, \mathbf{v}_H - (\mathbf{v}_1^T \mathbf{P}_1 \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{P}_2 \mathbf{v}_2) \,, \tag{18}$$

where $\mathbf{P} = \mathrm{diag}(\mathbf{P}_1, \mathbf{P}_2)$. The point i that gives minimum R_I is defined as the deformed point. For the new global test, the minimum value of R_I and degrees of freedom $(h_{min} = h - 1)$ are used. If the test quantity is greater than $F_{h_{min}, f, 1-\alpha}$, the observation equation (17) is augmented as fallows:

$$\mathbf{v}_H = \begin{bmatrix} \mathbf{A}_{1F} & \mathbf{A}_{1B} & \mathbf{o} & \mathbf{A}_{1i} & \mathbf{o} \\ \mathbf{A}_{2F} & \mathbf{o} & \mathbf{A}_{2B} & \mathbf{o} & \mathbf{A}_{2i} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_F \\ \hat{\mathbf{x}}_{1B} \\ \hat{\mathbf{x}}_{2B} \\ \hat{\mathbf{x}}_{1i} \\ \hat{\mathbf{x}}_{2i} \end{bmatrix} - \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} , \tag{19}$$

where $\hat{\mathbf{x}}_{1i}$ and $\hat{\mathbf{x}}_{2i}$ are unknown vectors of point i obtained as deformed from previous localization. Its location in the model is not changed until the end of the procedure.

3. THE RELIALIBITY CONCEPT: THE MEAN SUCCESS RATE

The reliability concept is defined as the mean success rate which is obtained by number of success divided by the number of experiments to identify outliers (Hekimoglu and Koch 1999). We adapt this concept to the CDA with the aim of obtaining its reliability.

Vertical deformations occurs as uplift or subsidence. In other words, the signs of the deformations are plus or minus. Therefore, random and influential deformations are used and called as kind of deformations. Random deformations (RD) have signs selected randomly, while influential deformations (ID) have the same signs, i.e., only all plus or all minus.

If a point is translated as $\tilde{d}_i$ $(i=1,2,\ldots u)$ in vertical direction between initial and present epochs, u×1 vertical deformation vector $\tilde{\mathbf{d}}$ and the set of deformed points M can be expressed as

$$\tilde{\mathbf{d}} = [\,0\ \ 0 \ldots \tilde{d}_i \ldots 0\ \ 0\,]^T , \tag{20}$$

$$M = \{\,0{,}0, \ldots i \ldots 0{,}0\,\} . \tag{21}$$

It is considered that the magnitute of $\tilde{d}_i$ lies in the interval (int) as fallows:

$$\text{int} = a\sigma < \left|\tilde{d}_i\right| < b\sigma \quad , \ a \ge 3, a \ne b \text{ and } b > a \ , \tag{22}$$

where σ^2 is the variance of unit weight. So the initial and present epoch's observation vectors are written as

$$\mathbf{l}_1 = \mathbf{h} + \mathbf{e}_1 - \mathbf{A}_1\, \mathbf{H}_0 , \tag{23}$$

$$\mathbf{l}_2 = \mathbf{h} + \mathbf{e}_2 + \mathbf{A}_2\, \tilde{\mathbf{d}} - \mathbf{A}_2\, \mathbf{H}_0 , \tag{24}$$

where $\mathbf{h}$ is the true height differences vectors, $\mathbf{e}_1$ and $\mathbf{e}_2$ are n×1 normal distributed random error vectors and $\mathbf{H}_0$ is the vector of approximate heights of points. Both observation vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ constitute a working sample.

If the set M is equal to the set of detected points M_L from one of the localization methods that use the estimated unknown vectors and cofactor matrices from $\mathbf{l}_1$ and $\mathbf{l}_2$, the method was considered as successful.

According to the generating different random error vectors $(\mathbf{e}_1, \mathbf{e}_2)$ and different deformation vector $\tilde{\mathbf{d}}$, we can produce many working samples. When the deformation vector $\tilde{\mathbf{d}}$ contains n_d $(1 \le n_d < u)$ number of deformation of any kinds with any magnitudes in the given interval, we can define the mean success rate as fallows (Hekimoglu and Koch 2000)

$$\gamma^r_{mean}(L, \mathbf{l}_1, \mathbf{l}_2, \text{int}, n_d, n, u) = \sum_{s=1}^{N} \frac{q}{N} , \tag{25}$$

where r is the deformation kind, L is the localization method, q is the number of success, s denotes a certain working sample, N is the number of working samples. Thus, the different mean success rates for the different deformation kinds can be computed. The smallest one of them is accepted here as the reliability of the CDA for a certain number of deformation and for a given interval (int):

$$\gamma(L, \mathbf{l}_1, \mathbf{l}_2, \text{int}, n_d, n, u) = \text{minimum}\left\{\gamma^r(L, \mathbf{l}_1, \mathbf{l}_2, \text{int}, n_d, n, u), r = 1, 2\right\}. \tag{26}$$

4. SIMULATION

In order to obtain the reliability of the CDA for vertical application areas, we simulated four vertical networks as shown in Figure 1a-d. Number of observations, number of unknown parameters and degrees of fredoom for free networks are given in Table 1.

Table 1. Number of observations, number of unknown parameters, degrees of freedom for vertical networks

Network	I	II	III	IV
Number of observations (n)	36	27	19	15
Number of unknown parameters (u)	16	16	8	8
Degrees of freedom (f)	21	12	12	8

4.1 Generating of Simulated Deformations

a) Random deformations (RD): The magnitude of $\tilde{d}_i$ of one deformation is generated by the uniform distribution for a given interval as fallows:

$$\text{int} = a\sigma < \tilde{d}_i < b\sigma, \tag{27}$$

$$\tilde{d}_i = \text{sign}(t_{1i})\,\tilde{d}_{0i} \quad, (i = 1, 2, \ldots u), \tag{28}$$

$$\text{sign}(t_{1i}) = \begin{Bmatrix} + & t_{1i} > 0.5 \\ - & t_{1i} \leq 0.5 \end{Bmatrix}, \ 0 < t_{1i} \leq 1, \tag{29}$$

$$\tilde{d}_{0i} = a\sigma + t_{1i}(b\sigma - a\sigma), \ i = u\, t_{2i} \ , \ 0 < t_{2i} < 1 \ , \ a \geq 3 \ , \ b > a, \tag{30}$$

where t_{1i} and t_{2i} are distributed uniformly (Hekimoglu and Koch 1999; 2000). This algorithm has been computed 2500 times for each working sample in (23) and (24).

The magnitudes $\tilde{d}_i$, $\tilde{d}_j$ of two deformations, and etc. are generated correspondingly by the uniform distribution.

We used the three intervals for the magnitutes of deformations as follows: 3σ–6σ, 3σ–10σ, 10σ–50σ.

b) Influential deformations (ID): ID's magnitudes are generated also by the uniformed distributions for a given interval as done for the RD. Only they all have the same sign, i.e., all plus or all minus.

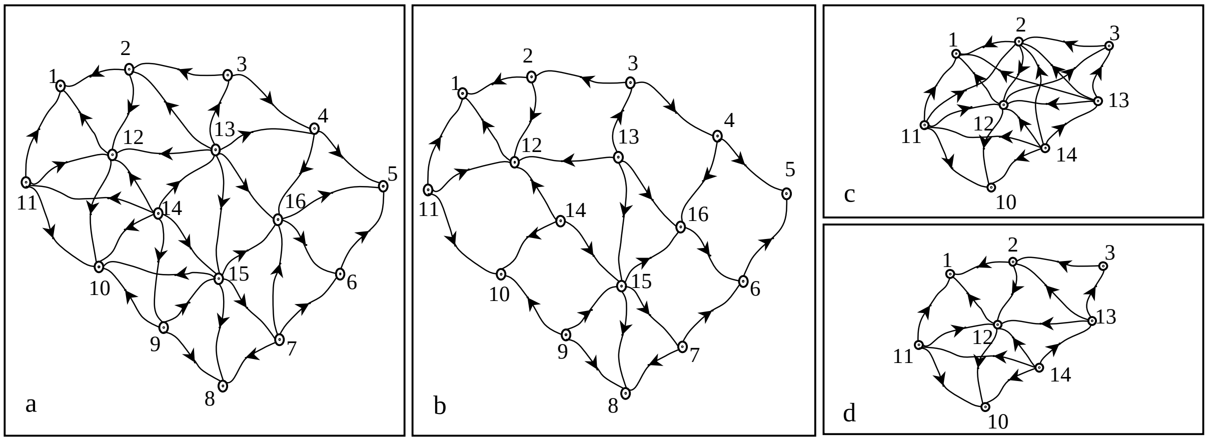

Fig. 1a-d. Configurations of simulated networks: **a** Network I, **b** Network II, **c** Network III, **d** Network IV

4.2 Observations

If random errors are independent and identically distributed (iid), the Gauss-Markov model is called as homoscedastic in robust statistics (Carroll and Ruppert 1982). However, the random errors' variances in vertical networks are changing depending on distance levelling lines. The model is called as heteroscedastic (Carroll and Ruppert 1982; Hekimoglu and Berber 2001). To investigate the effect of these concepts on the CDA, we produced two types of random errors of observations for the networks as follows.

a) Random errors are heteroscedastic (Type 1)

We produced random error vectors with normal random error generator in MATLAB v.5.1 software for one initial and present epochs as fallows:

$$\mathbf{e}_k = [\, e_{1k} \;\; e_{2k} \ldots e_{nk} \,]^T, \quad k = 1, 2 \;, \tag{31}$$

where e_{jk} $(j = 1, 2, \ldots, n)$ comes from the different normal distributions $N(\mu_{jk} = 0,\; \sigma^2_{jk} = \sigma^2 L_j)$, $\sigma = 1\,mm / \sqrt{1\,km}$ is standart deviation and L_j is the distance of levelling line. Then, the observation vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ are obtained by using (23) and (24). For stochastic model, we selected the distance of levelling lines in the interval of 0.92–3.55 km. Two epoch's observations distances are the same, so the weight matrices of observations are equal to each other as follows:

$$\mathbf{P}_1 = \mathbf{P}_2 = \mathrm{diag}\,(\, L_0 / L_1 \,,\; L_0 / L_2 \,, \ldots ,\, L_0 / L_n \,)\,, \tag{32}$$

where L_0 is the unit distance (1km) of levelling line.

b) Random errors are homoscedastic (Type 2)

In this type, the distances of levelling lines are chosen the same for all the networks, i.e., $L_j = 1$ km. So, random errors of the observations come from the same distribution and are independent, i.e., the model is homoscedastic.

4.3 Analysis

To obtain a mean success rate of the CDA for a given magnitude interval, a deformation kind and n_d number of deformed point in a vertical network, we produced randomly 2500 different working samples by applying mentioned above procedures. Then, the observation vectors of two epochs were free adjusted individually using the same design matrices and the same weight matrices for two epochs in the vertical network. All the estimated variance factors for the working samples are approximately equal to 1 mm as expected, because there are not any ouliers in the observations. The model test which checks the null hypothesis $H_0 : E\{ s_{01}^2 \} = E\{ s_{02}^2 \}$ was applied to the samples to verify whether they were used for the deformation analysis (%95 confidence level is here used for all statistical tests).

We used two localization methods for checking the results obtained from them. Applying both methods, we computed the mean success rates according to (25).

5. RESULTS

The same results were obtained from two localization methods performed to the same working samples. Therefore, the results from one of two methods are presented only as the mean succes rate of the CDA.

First, to obtain whether the CDA produces deformations for the networks, we used 2500 samples which do not have any deformations. As shown in Table 2, the analysis may produce deformations with the rate of %6.3 on the average.

Table 2. The mean succes rates in the case of no added deformations for the vertical networks

Network type	1				2			
Network number	I	II	III	IV	I	II	III	IV
The mean succes rates	6.7	6.0	6.3	6.8	6.3	6.2	5.8	5.9

5.1 The Vertical Networks of the Type 1

The results of the mean success rates for Type 1 are shown in Table 3 and Table 4. The mean success rates under the 51% are shaded in tables to compare them with the others easily.

5.1.1 For random deformations

In the networks I, II, III and IV, the mean success rates are decreased while the number of deformed point n_d is increased for all of the intervals. For the interval of 10σ–50σ, the mean success rates are quite bigger than the ones for the intervals of 3σ–6σ and 3σ–10σ. The networks I and II have the same number of points, but different degrees of freedom. For the intervals of 3σ–6σ and 3σ–10σ, the mean success rates of the network I are bigger than the ones for the network II since the degrees of fredoom are increased.

The networks III and IV have the same number of points, but different degrees of freedom. For the intervals of 3σ–6σ and 3σ–10σ, the mean success rates of the network III are bigger than the ones which are obtained for network IV because of the increasing of the degrees of freedom.

Moreover, the networks II and III have different number of points, but the same degrees of freedom. Although the number of observation of the network III is smaller than the ones of the network II, the mean success rates of the network III are larger than the ones of the network II for the interval of 3σ–6σ and 3σ–10σ for $n_d \leq 5$.

5.1.2 For influential deformations

The mean success rates were obtained for only all plus (uplift) and only all minus (subsidence) deformations. The features of the mean success rates gained for RD are the same for ID. However, the mean success rates are decreased related to the ones that are computed for the RD when $n_d > 2$. The results of ID were not given in Table 3 and 4.

Table 3. The mean success rates for RD in the networks I and II (Type 1)

Magnitude of deformation	Number of Deformed Points														
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
	NETWORK I (n=36, u=16 , f=21)														
3σ-6σ	65.2	46.8	37.8	30.8	24.3	19.4	15.3	11.2	8.0	4.6	2.5	1.4	0.6	0.1	0
3σ-10σ	82.3	68.4	63.0	55.8	51.6	43.6	40.8	31.9	26.3	18.6	12.6	7.7	3.6	0.7	0
10σ-50σ	94.8	93.3	93.7	93.6	93.6	92.8	90.2	86.1	79.0	65.7	52.4	35.8	20.6	7.2	0.3
	NETWORK II (n=27, u=16 , f=12)														
3σ-6σ	47.9	30.3	20.5	16.3	11.8	8.6	6.4	4.1	3.0	1.8	1.0	0.5	0.1	0.1	0
3σ-10σ	71.0	56.3	47.6	37.6	30.0	27.3	23.2	17.9	12.4	9.4	5.8	3.0	1.7	0.4	0
10σ-50σ	94.0	94.1	93.8	93.9	93.2	90.2	87.2	82.0	73.8	60.6	49.2	35.1	21.3	10.0	0.2

Table 4. The mean success rates for RD in the networks III and IV (Type 1)

Magnitude of deformation	Number of Deformed Points														
	1	**2**	**3**	**4**	**5**	**6**	**7**		**1**	**2**	**3**	**4**	**5**	**6**	**7**
	NETWORK III (n=19, u=8 , f=12)								**NETWORK IV** (n=15, u=8 , f=8)						
3σ-6σ	78.2	65.8	56.3	44.2	21.9	3.7	0		66.1	48.8	38.4	26.0	12.9	2.9	0
3σ-10σ	88.0	81.3	75.2	66.4	44.7	13.2	0.1		82.3	69.8	62.8	49.1	31.7	11.4	0.2
10σ-50σ	94.9	93.3	94.9	90.4	75.4	32.6	1.1		93.9	94.8	92.8	85.0	67.4	34.8	1.7

5.2 The Networks of the Type 2

The mean success rates for Type 2 are shown in Table 5 and Table 6. The mean success rates under the 51% are shaded in tables to compare them with the others easily.

5.2.1 For random deformations

The mean success rates for 3σ–6σ and 3σ–10σ are quite bigger than the ones that were obtained from the Type 1. As seen the results from Table 5 and Table 6, the features of the mean success rates obtained for the networks of Type 1 are the same for Type 2.

5.2.2 For influential deformations

The mean success rates for 3σ–6σ and 3σ–10σ are bigger than the ones that were achieved from the Type 1. The results of ID are not presented in Table 5 and 6.

Table 5. The mean success rates for RD in the networks I and II (Type 2)

Magnitude of Deformation	Number of Deformed Points														
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
	NETWORK I (n=36, u=16 , f=21)														
3σ-6σ	84.2	78.1	74.4	69.2	63.0	59.7	53.2	44.8	36.1	21.9	13.2	6.6	2.3	0.6	0
3σ-10σ	90.8	87.6	83.4	82.7	79.3	76.3	71.2	65.8	55.2	41.9	27.6	18.6	8.6	2.6	0
10σ-50σ	93.4	93.7	94.6	93.9	93.1	93.0	91.7	88.2	82.1	70.0	53.7	38.6	20.8	8.0	0.2
	NETWORK II (n=27, u=16 , f=12)														
3σ-6σ	73.7	61.6	52.5	45.3	39.9	35.0	27.4	23.6	16.5	10.4	6.4	3.6	1.3	0.2	0
3σ-10σ	84.2	77.8	72.3	67.3	61.4	55.9	52.9	43.8	37.3	27.8	17.0	10.8	5.9	1.8	0
10σ-50σ	94.4	93.0	94.2	94.6	92.6	90.5	88.7	82.4	75.0	63.7	49.9	34.8	20.2	7.9	0.1

Table 6. The mean success rates for RD in the networks III and IV (Type 2)

Magnitude of Deformation	Number of Deformed Points														
	1	**2**	**3**	**4**	**5**	**6**	**7**		**1**	**2**	**3**	**4**	**5**	**6**	**7**
	NETWORK III (n=19, u=8 , f=12)								**NETWORK IV** (n=15, u=8 , f=8)						
3σ-6σ	91.1	88.1	86.4	77.0	43.0	8.2	0		83.6	76.5	69.9	54.1	30.4	7.9	0.2
3σ-10σ	92.4	90.6	91.3	85.4	62.8	19.3	0.5		90.6	86.6	81.7	71.8	47.8	18.9	1.0
10σ-50σ	94.0	94.4	94.6	93.8	78.8	34.0	1.2		93.6	93.5	94.0	88.4	71.4	35.0	2.4

6. CONCLUSION

In this paper, we pointed out that the reliability of the conventional deformation analysis for the vertical networks can be measured by using the mean success rates. The reliability of the CDA changes depending on the many factors, such as, the number of unknowns, the degrees of freedom, the number of deformed points, the magnitudes of deformation, the deformation kinds (RD and ID), especially types (heteroscedasticity or homoscedasticity) of the random errors. According to the results of the numerical simulations, if the degrees of freedom and the magnitudes of the deformations increase, the reliability of the CDA increases. On the contrary, when the number of unknowns, number of deformed points increase, the reliability of the CDA decreases. Furthermore, if the random errors come from the same normal distribution, i.e., the model is homoscedastic, much more reliable results are obtained from CDA.

When the points have vertical deformations whose magnitudes are bigger than approximately 10σ, CDA's reliability is high in all vertical networks used in this paper. If the magnitudes are smaller than 10σ, the reliability of the CDA's starts decreasing rapidly. However, this decreasing rate in the mean success rates of Type 1 is bigger than the ones that in Type 2.

Although vertical network II and network III have the same degrees of freedom, the number of unknowns of network III are smaller than network II. When the mean success rates of these networks are compared with each other, the ones of network III are bigger than the ones for network II. It can be interpreted that the number of points must be small to get more reliable results.

Consequently, the reliability of the CDA may increase very much if the points of a vertical network are established, so that the distances of levelling lines are approximately equal to each other. Moreover, in this network, when the assumed number of deformed points and especially the number of points are taking small, the reliability of the CDA increases. So, even if the deformation magnitutes of the points are smaller than approximately 10σ, more reliable results may be obtained since the Gauss-Markov model is homoscedastic.

REFERENCES

Betti B., Biagi, L., Crespi, M., Riguzzi, F., 1999, GPS sensitivity analysis applied to non-permanent deformation control networks, Journal of Geodesy, vol. 73, pp.158-167.

Carroll, R. J. and Ruppert, D.,1982, Robust estimation in heteroscedastic linear models, The Annals of Statistics, vol. 10, no. 2, pp. 429-441.

Chen, Y., Q., 1983, Analysis of deformation surveys-a generalized method, Department of Surveying Engineering, University of New Brunswick, Technical Report No. 94.

Chrzanowski, A, Chen, Y. Q., Secord J., M, 1982, On the analysis of deformation surveys, 4th Canadian Symposium on Mining Surveying and Deformation Measurements, The Canadian Institute of Surveying, Banff, 07-09. 06. 1982, Proceedings.

Hekimoglu, S., and Koch, K-R., 1999, How can reliability of robust methods be measured?, Third Turkish-German Joint Geodetic Days, Istanbul, vol.1, pp.179-196.

Hekimoglu, S., and Koch, K-R., 2000, How can reliability of the test for outliers be measured?, AVN 107, pp. 247-252.

Hekimoglu, S., and Berber, M., Reliability of robust methods in heteroscedastic linear models, IAG 2001 Scientific Assembly, September 1-7 2001, Budapest, Hungary.

Koch, K., R., 1999, Parameter estimation and hypothesis testing in linear models, Springer-Verlag, New York, 2nd Edition.

Liu, Q., W. and Chen Y., Q., 1998, Combining the geodetic models of vertical crustal deformation, Journal of Geodesy, vol. 72, pp.673-683.

Niemeier, W., 1985, Deformationanalyse, Geodaetische Netze in Landes-und Ingeniuervermessung II, Herausg. von H. Pelzer, Konrad Wittwer, Stuttgart, pp. 559-623.

Pelzer, H., 1971, Zur Analyse Geodaetischer Deformationmessungen, Deutsche Geodaetiche

Kommission, Reihe C, No. 164, München.
Teunissen, P. J., 1985, Zero order design: Generalized inverses, adjustments, the datum problem and S-transformations. In: Grafarend E. W., Sanso F (eds), Optimization and design of geodetic networks. Springer, Berlin Heidelberg New York, pp. 11-55.
Welsch, W., Heunecke, O., Kuhlmann, H., 2000, Auswertung Geodaetischer Überwachungsmessungen, Herbert Wichman Verlag, Heidelberg.
Welsch, W., M. and Heunecke, O., (2001), Models and terminology for the analysis of geodetic monitoring observations, The 10th International Symposium on Deformation Measurements, Orange, California, USA, pp. 390-412.