Derivation of Seawater Depth from Atmospheric Pressure in the Near-Shore Zone of Barrier-Lagoon Complex, Lagos State, Nigeria

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Keywords: Atmospheric pressure; seawater depth; Pressure-derived depth; Barrier-Lagoon complex; Near-shore zone

SUMMARY

Generally, water depth measurement, otherwise known as bathymetric survey is usually done using the classical bathymetry such as the use of calibrated sounding rod or the use of acoustic sounding systems (echo sounder, side scan sonar, etc.). However, due to inaccessibility of some water body and unsuitability of some very shallow water for sounding boat to sail through, alternative bathymetric methods such as Satellite Derived Bathymetry (SDB) and Pressure Derived Bathymetry (PDB) have been developed. Relying on the fact that pressure is a function of height/depth, several algorithms for deriving depth from pressure have been developed but little efforts have been made towards ascertaining the reliability of depth generated from pressure data. This would have been of immense advantage for regions where cost of bathymetric survey has limited availability and accessibility of depth information on some sections of their waters. This study therefore evaluates the performance of three algorithms for derivation of seawater depth from pressure in decibar. Fifty (50) years (1969 - 2018) of atmospheric pressure data over the barrier-lagoon complex in Lagos State, Nigeria was acquired at 5 years intervals and variation in Latitude between 6^0 10'N, and 6^0 27'N at five minutes (5') intervals. The atmospheric pressure was converted to hydrostatic pressure. Two algorithms including theory of equivalent observed pressure to depth and hydrostatic equation were evaluated for PDB. Depth generated from the UNESCO 1983 formulation was used as a standard for validation of the results. The results showed that the varied Latitude position did not show appreciable difference in the derived depth within the years of study. The derived seawater depth as compared to the standard UNESCO 1983 formulation showed approximate difference of 0.041m with equivalence of observed pressure to depth in decibars and 0.125m with hydrostatic basic equation in decibar. Simplified PDB models in terms of observed pressure and hydrostatic equation were further developed using least square regression for depth estimation. The estimated residuals ranged from 0.127mm to 0.295mm for PDB model using observed pressure and 0.478mm to 1.078mm for PDB model using hydrostatic equation obtained. Thus, the observed pressure based PDB model performs better than hydrostatic equation based PDB model. It is recommended that further validation study should be conducted utilizing depth from in-situ bathymetric survey as standard for assessment.

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1.0 Introduction

Understanding water dynamics and the processes affecting water depth in oceanic environment is fundamental in evaluating the entire hydrodynamic processes in any coastal system. It also helps resolving a number of environmental problems related to flooding, land management (Pérez-Arlucea *et al*., 2011) and sediment distribution. Therefore, for oceanic observations, the depth at which measurements are made needs to be determined (Lie *et al.*, 2015).

According to IHO publication S-44 4th Edition, water depth determination requires specific knowledge of the medium, underwater acoustics, devices plethora available for depth measurement, complementary sensors for attitude and heave measurement and proper procedures to achieve and meet the internationally recommended standards for accuracy and coverage.

Water depth can be measured directly and indirectly. This include the use of sounding rod or leads on graduated lines directly. The indirect methods include the use of acoustic sounding systems such as echo sounder, side scan sonar, seismic profilers and swathe sounding system to measure the water depth. In spite of these methods, challenges to precisely and economically measure water depth have been so difficult that less than 1% of the world's ocean floors, according to Ellsworth (2019) have been properly mapped to a high degree of accuracy. To enhance coverage, most especially in areas not accessible, other methods used recently for water depth measurement include multibeam sonar from ships, optical remote sensing from aircraft and satellite, satellite radar altimetry (Dierssen and Theberge, 2014), Light Detection and Ranging (LIDAR) technology, high-resolution sonar, Airborne Laser Hydrography, satellite-derived near-shore bathymetry (SDB) (Ellsworth, 2019). 99% water depth data has been predicted using satellite imagery data which approximates the shape of the seafloor using gravitational measurement tools with low accuracy. Unfortunately, these methods require a lot of money and time to get a wide and complete coverage (Ellsworth, 2019).

To get an economical way of determining water depth, this study utilizes atmospheric pressure to develop seawater derived water depth algorithm. Pressure, as a function of depth and the rate of change of sound velocity has been found to impact significantly on the sound velocity variation in seawater. The study involved investigating the suitability of atmospheric pressure as an effective tool for updating the water column depth continually through time in coastal environments. For this purpose, the relationship between atmospheric pressure and depth was considered.

2.0 Concept of Depth Estimation from Atmospheric Pressure

The atmospheric pressure is the force exerted by the weight of the Earth's atmosphere, expressed
per unit area in a given horizontal cross-section. Thus, the atmospheric pressure is equal to the weight of a vertical column of air above the Earth's surface, extending to the outer limits of the atmosphere.
Pressure is determined by the weight of the overlying water column per unit area at a particular depth. Although pressure used to be influenced by density which increases with depth, the relationship between pressure and depth has been considered to be effectively linear. Methods used to determine water depths from pressure can be one of the following:

i) Derivation of Depth from Observed Pressure

One convenient and common approximation to derive depth from observed pressure is to use the pressure in decibar (dbar) as equivalent to the depth in meters (Greenawaya *et al*., 2021). This can be expressed as:

$$Z_{Obs} \approx P \tag{1}$$

where

P = pressure in decibars
Z_{Obs} = water depth in meters from observed pressure

ii) Derivation of Depth from Hydrostatic Equation

The principle of hydrostatic equilibrium state that the pressure at any point in a fluid at rest is due to the weight of the overlying fluid. If P is the pressure and Z is the depth (height), the pressure P is proportional to the height,Z, of the column of fluid and as such can be calculated using the basic hydrostatic equation expressed as:

$$P = \rho g Z \tag{2}$$

Where P is pressure, ρ is water density, Z is water depth or height below the free surface of the liquid and g is acceleration due to gravity needed to convert the element of mass (ρdZ) into the force (weight) and it adds to the unit area beneath it.

iii) Conversion of hydrostatic equation and the Knudsen-Ekman Equation of State (EoS) for seawater to water depth

Ocean pressure varies with depth and position in latitude on earth. Pressure can also be converted to water depth using the hydrostatic equation and the Knudsen-Ekman Equation of State (EoS) for seawater. These Equations require gravitational acceleration variation as a function of both latitude and water depth, as well as setting salinity to 35 and the temperature to 0°C (Saunders and Fofonoff, 1976).

The hydrostatic equation and the Knudsen-Ekman Equation of State (EoS80) for seawater to water depth in line with Saunders and Fofonoff (1976) relation, takes complete formulation as:

$$Z = \frac{C_1 P + C_2 P^2 + C_3 P^3 + C_4 P^4}{g(\phi) + 1.092 \times 10^{-6} P} + \frac{AD}{9.8} \tag{3}$$

Where

$$C_1 = +\ 9.72659$$
$$C_2 = \ -2.2512 \times 10^{-5}$$
$$C_3 = \ +2.279 \times 10^{-10}$$
$$C_4 = -\ 1.82\ \times 10^{-15}$$

$g(\emptyset)$ is gravity in ms^{-2} at a function of latitude as given by the international formula for gravity. P is the hydrostatic pressure in decibar. ΔD is geopotential anomaly in J/kg and it accounts for the difference in temperature and salinity structure from the standard ocean depth (Leroy and Parthiot, 1998).

A least squares polynomial of fourth order in pressure was fitted to the Equation to give an expression which eliminate the need for computing logarithms with negligible loss of precision.

3.0. Materials and Methods

The study area is barrier-lagoon complex lying in the south-western part of Nigeria coast. Approximate location lies between latitude 6°10'N – 6°27'N of the Equator and longitudes 2°36'E – 4°20'E of the Greenwich meridian with an area of 5697.006Km^2. The study area is as depicted in Figure 1.

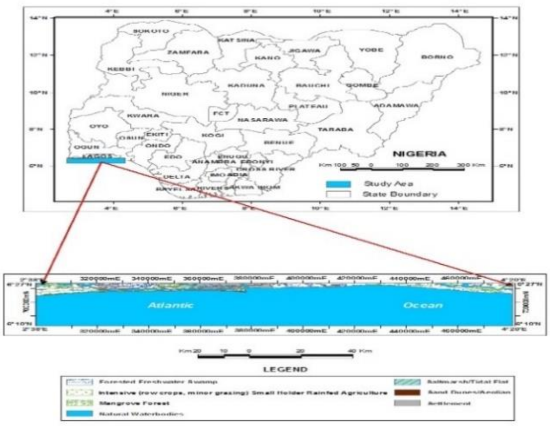

Figure 1: Study Area on Nigeria Map

The barrier-lagoon complex has been wave energy-dominated coasts (Laïbi, *et al.*, 2014), controlled mainly by the combined effect of tides and waves (Dodet *et al.*, 2013). Its semi-diurnal tide ranging is around 1 m generally according to Sexton and Murday (1994). The temperature varies between 26ºC and 32.7ºC (Ezenwa *et al.*, 1990). The prevailing wind has been mainly from south-west throughout the year with an estimated average wind speed of 3.425 m/s recorded in barrier-lagoon complex environment during the fifty year study period 1969 to 2018.

To estimate sea water depth, fifty (50) years atmospheric pressure covering the periods between 1969 and 2018 at five years interval over the barrier-lagoon complex in Lagos State, Nigeria was used. Based on variation in Latitude between 6^0 10'N, and 6^0 27'N at five minutes (5') intervals, depth was derived from Equation (1) using atmospheric pressure in decibar to be approximately equivalent to depth in metres. According to Greenawaya *et al*. (2021), one-metre column of seawater produces a pressure of about one decibar (0.1 atmosphere). For one decibar to be equal to a depth of 1 m, g (acceleration due to gravity) was approximated to be equivalent to 9.81 m/s^2 and ρ =1028 kg/m^3.

Depth was also derived from Equation (2) using hydrostatic (gage) pressure in decibar which indicates the difference between the absolute pressure and the local atmospheric pressure as expressed in the following relation:

$$P_{gage} = P_{abs} - P_{atm} \quad (4)$$

Where P_{gage} signifies the hydrostatic (or gage) pressure within a liquid at a given depth, P_{abs}, absolute pressure approximately and P_{atm}, atmospheric pressure.

From hydrostatic pressure P of a water column with known mean density, the depth Z_{Hyd} can be obtained according to the hydrostatic basic equation (Tortell and Awosika, 1996):

$$Z_{Hyd} = \frac{P}{\rho g} \quad (5)$$

where P signifies the hydrostatic pressure, ρ, the density of the liquid (water density for Lagos marine area (1028 kg/m^3), g, the gravitational acceleration (9.81 m/s^2) and Z, the depth (or height) of the liquid (water).

According to Fofonoff and Millard (1983), the depth Z in the ocean can be obtained from pressure and latitude by solving the hydrostatic equation (3) to give:

$$Z_s = \frac{9.72659\,P - 2.2512 \times 10^{-5} P^2 + 2.279 \times 10^{-10} P^3 - 1.82 \times 10^{-15} P^4}{g(\emptyset) + 1.092 \times 10^{-6} P} \quad (6)$$

and

$$g(\emptyset) = 9.780318(1.0 + 5.2788 \times 10^{-3} sin^2\emptyset + 2.36 \times 10^{-5} sin^4\emptyset) \quad (7)$$

Where $g(\emptyset)$ was as expressed by Saunder's and Fofonoff (1976) and Anon (1970) with trigonometric substitutions for $sin(2\emptyset)$.

The complete formulation of Fofonoff and Millard (1983), for $Z_s(P, \emptyset)$ takes the form,

$$Z_s(P, \emptyset) = \frac{9.72659\,P - 2.2512 \times 10^{-5} P^2 + 2.279 \times 10^{-10} P^3 - 1.82 \times 10^{-15} P^4}{9.780318(1.0 + 5.2788 \times 10^{-3} sin^2\emptyset + 2.36 \times 10^{-5} sin^4\phi)} \quad (8)$$

Where $Z_s(P, \emptyset)$ signifies the universal expression of depth referred to as the standard ocean depth at an ideal medium in temperature T = 0 °C and salinity S = 35‰).

Adding Leroy and Parthiot (1998) correction term to Equation (8) results to:

$$Z = Z_s(P, \emptyset) + \frac{\Delta D}{9.8} \quad (9)$$

ΔD, according to Saunders (1981), was a dynamic height correction taking account of the physical conditions in the water column. $\Delta D/9.8$ was Leroy and Parthiot (1998) correction term to an area and can be expressed as:

$$\delta f_i = \Delta D/9.8 \quad (10)$$

Equation (9) becomes:

$$Z = f(P, \emptyset) + \delta f_i(P) \quad (11)$$

and

$$\delta f_i(P) = \delta f_0(P) = P/(P+1) + 5.7 \times 10^{-2} P \quad (12)$$

$f(P, \emptyset) \equiv Z_s(P, \emptyset)$ giving by Equation (8) where $f(P, \emptyset)$ stands for the standard ocean and $\delta f_i(P)$ stands as a simple corrective term applicable to a particular area of the World between 60° N and 40° S. $\delta f_0(P)$, as expressed by the Equation (12) according to Leroy and Parthiot (1998) shows results of the study in the open oceans.

Therefore, Equation (11) was used as a standard to derive depths from atmospheric pressure along the barrier-lagoon complex due to the following reasons:

i. The equation, according to Leroy and Parthiot (1998), was found to represent all open oceans situations within better than ± 0.8 m. with two exceptions: North Eastern Atlantic area between 30° and 35 °N, and Circumpolar waters around the Antarctic.

ii. According to Saunders (1981), the quadratic expression has been a practical and accurate conversion equation.

iii. The formulas were practical and enabled simplified procedure of rapid evaluation of depth (or pressure) in real time by excluding the procedure of integration over a specific volume.

iv. The formulation was simple and gives departures smaller than ± 0.03 m in all situations.

v. The formula has been used in the deep vehicle localization problem, where the final accuracy desired was only $\pm$ 1 m (Leroy and Parthiot, 1998)

vi. Zaburdaey and Gaisky (2002) applied the Equation of State (EoS) for seawater to the Black Sea to deduce practical formulas for the conversion of pressure into depth and vice versa. The error of these relations for the standard Black Sea (whose salinity is equal to 22.2 at a temperature of +9°C from the surface to the bottom) does not exceed ±0.2m and ±0.2dbar respectively. The difference between the practical and actual depths in winter and summer periods does not exceed ±0.35m for depths varying within range 0–2000m.

vii. Sea-Bird SEASOFT software uses the depth empirical formula in UNESCO (1983) Technical Papers in Marine Science No. 44 for calculating depth from pressure (Sea-Bird Electronics, 2002).

viii. For any particular location, difference between a depth calculated assuming the standard ocean and using a measurement of the actual density of the ocean depend on how different the ocean has been from the standard ocean, which generally, has been found to be small (Greenawaya *et al*., 2021).

Also, the theory of equivalent of observed pressure to depth and hydrostatic equation were evaluated for the Pressure Derived Bathymetry (PDB). Furthermore, relying on the relationship between the depth from observed pressure, the hydrostatic equation and EOS of sea water, a simplified pressure-derived depth model was developed. The generated depths were validated using the UNESCO 1983 formulation as a standard.

4.0. Results and Discussion

4.1. Variation in Latitude on Pressure-Derived Depth.

Depth was derived at Latitude 6^0 10’, 6^0 15’, 6^0 20’, 6^0 25’ and 6^0 27’ at 5’ interval using Saunders and Fofonoff’s (1976) relation. The reason was to see the impact of varied Latitude on derived depth since pressure also varies with latitude; the result of which has been presented in Figure 2.

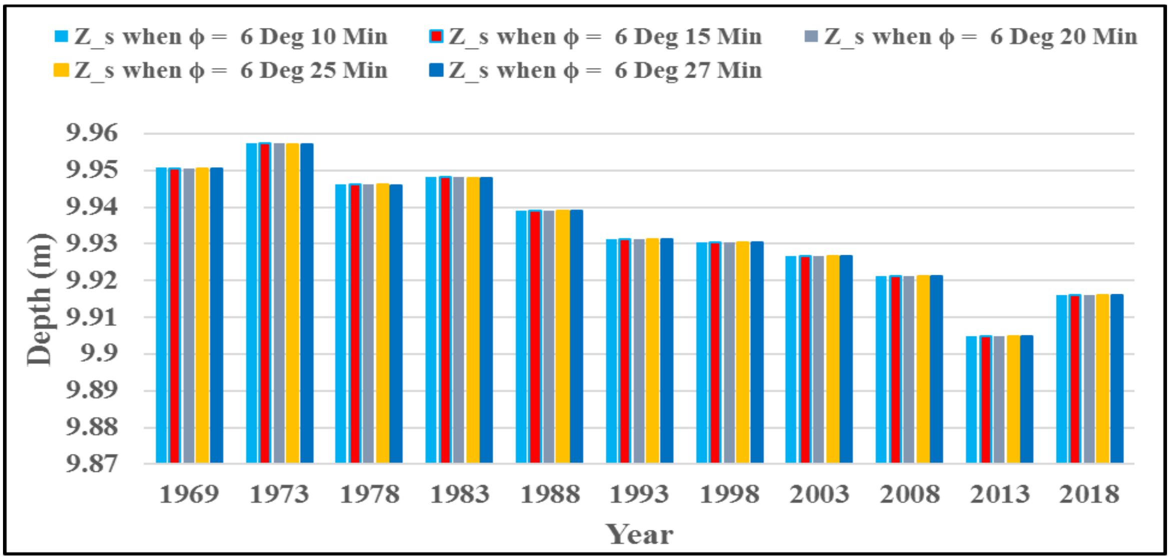

Figure 2: Temporal trend of pressure-derived Depth at Different Latitude from 1969 to 2018.

The derived depth did not show appreciable difference within the year as can be seen in Figure 2 in spite of the variation in latitude used.

Furthermore, the trend of depth derived from hydrostatic equation, Equation of state (EOS) and the theory of equivalence of observed pressure to depth from 1969 to 2018 were presented in Figure 3.

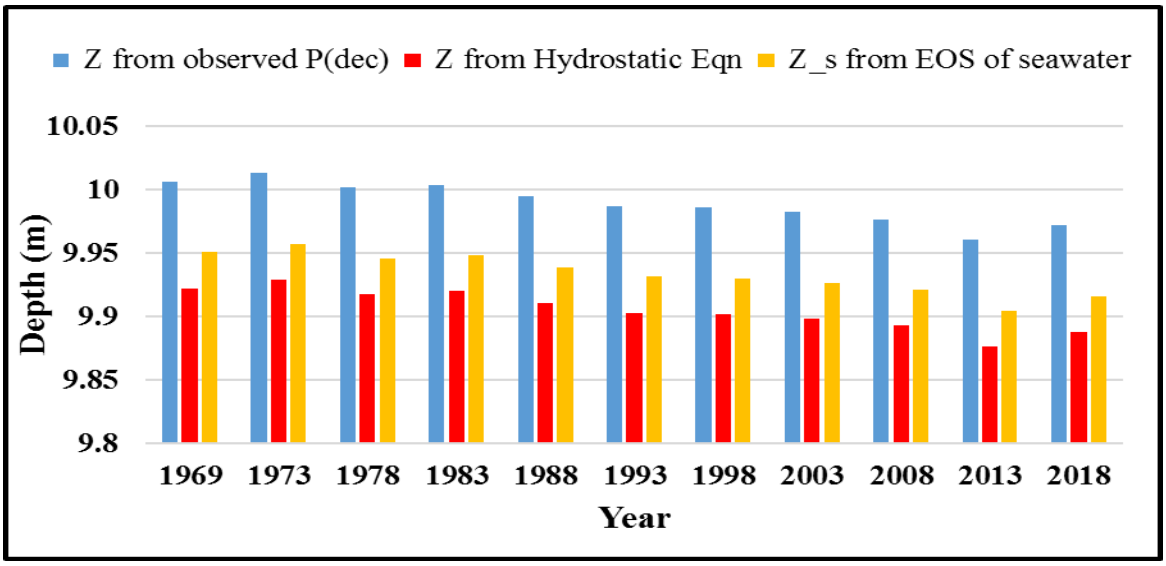

Figure 3: Temporal trend of Derived Depth from hydrostatic equation, Equation of state (EOS) and the theory of equivalence of observed pressure to depth to depth from 1969 to 2018.

Depth estimates from the observed pressure in decibar were higher (with values ranging from about 9.96m to 10.02m) than those obtained from hydrostatic equation whose values ranges from about 9.88m to 9.93m (Figure 3).

4.2 Simplified Pressure Derived Bathymetric (PDB) Model

In order to develop a simplified model for depth estimation from atmospheric pressure, the depth obtained from observed pressure and hydrostatic equation were used. The depths from each of the two approaches were first evaluated using the depth from EOS.

The relationship between the depth derived from observed pressure and equation of state (EOS) for sea water with their relevant corrections were depicted in Table 1.

Table 1: Analysis of depth derived from observed pressure

Year	Z from EOS of Seawater	Z from Observed P	δl_{Obs}	$\delta f_0(P)$	C_{Obs}
1969	10.04729	10.0065	0.04079	0.096666512	0.055876512
1973	10.05398	10.01317	0.04081	0.096725428	0.055915428
1978	10.04269	10.001917	0.040773	0.096626026	0.055853026
1983	10.0447	10.00392	0.04078	0.096643721	0.055863721
1988	10.03559	9.994833	0.040757	0.096563441	0.055806441
1993	10.02781	9.98708	0.04073	0.096494937	0.055764937
1998	10.02689	9.986167	0.040723	0.096486869	0.055763869
2003	10.02321	9.9825	0.040697	0.096454465	0.055757465

2008	10.01753	9.976833	0.04079	0.096404383	0.055614383
2013	10.00122	9.96058	0.04064	0.09626072	0.05562072
2018	10.01243	9.97175	0.04068	0.096359457	0.055679457

Where δl_{Obs} was the correction to be applied to depth from observed pressure in order to convert it to depth from Equation of seawater, while $\delta f_0(P)$ was the correction to the EOS and
$C_{Obs} = \delta f_0(P) - \delta l_{Obs}$

Thus,

$$\delta l_{Obs} = \delta f_0(P) - C_{Obs} = P/(P+1) + 5.7 \times 10^{-2} P - C_{Obs} \quad (13)$$

From the Table 1, we have:

$$\delta l_{Obs} = \text{Z of EOS} - \text{Z of } P_{obs} = \delta f_0(P) - C_{Obs} \quad (14)$$

But according to Leroy and Parthiot (1998):

$$\delta f_0(P) = P/(P+1) + 5.7 \times 10^{-2} P \quad (15)$$

Hence

$$\delta l_{Obs} = P/(P+1) + 5.7 \times 10^{-2} P - C_{Obs} \quad (16)$$

The spread of C_{Obs} from the Table 1 shows more or less a constant variation of 0.00001m. Therefore, C_{Obs} can be approximated to 0.056m.
To simplify seawater depth derivation using observed pressure, this value can be subtracted from Leroy and Parthiot (1998) corrective term and apply to the depth calculated from observed pressure in order to convert it to resultant depth of Equation of state for seawater (EOS).

The relationship between the depth derived from hydrostatic equation and equation of state (EOS) for sea water with their relevant corrections were also shown in Table 2.

Table 2: Analysis of Depth Derived from Hydrostatic Pressure Equation

Year	Z from EOS Seawater	Z from Hydrostatic Eqn	δl_{Hyd}	$\delta f_0(P)$	C_{Hyd}
1969	10.04729	9.922476469	0.124813531	0.096666512	0.028147019
1973	10.05398	9.929090462	0.124889538	0.096725428	0.028164113
1978	10.04269	9.917931952	0.124758048	0.096626026	0.028132022
1983	10.0447	9.919918133	0.124781867	0.096643721	0.028138146
1988	10.03559	9.910907436	0.124682564	0.096563441	0.028119123
1993	10.02781	9.903219537	0.124590463	0.096494937	0.028095526
1998	10.02689	9.902314203	0.124575797	0.096486869	0.028088928
2003	10.02321	9.898677995	0.124532005	0.096454465	0.02807754
2008	10.01753	9.89305858	0.12447142	0.096404383	0.028067037
2013	10.00122	9.876942055	0.124277945	0.09626072	0.028017225
2018	10.01243	9.888018261	0.124411739	0.096359457	0.028052282

Where δl_{Hyd} was the correction to depth from hydrostatic equation in order to convert it to depth of Equation of Seawater (EOS) and $C_{Hyd} = \delta f_0(P) - \delta l_{Hyd}$

Thus,

$$\delta l_{Hyd} = \delta f_0(P) + C_{Hyd} = P/(P+1) + 5.7 \times 10^{-2}P + C_{Hyd} \tag{17}$$

From Table 2, we have:

$$\delta l_{Hyd} = \text{Z of EOS} - \text{Z of } P_{Hyd} \tag{18}$$

And also

$$\delta l_{Hyd} = \delta f_0(P) - C_{Hyd}$$

But according to Leroy and Parthiot (1998):

$$\delta f_0(P) = P/(P+1) + 5.7 \times 10^{-2}P \tag{19}$$

Hence

$$\delta l_{Hyd} = P/(P+1) + 5.7 \times 10^{-2}P - C_{Hyd} \tag{20}$$

Also, the spread of C_{Hyd} from the Table 2 shows more or less a constant variation of 0.0001m. Therefore, C_{Hyd} can be approximated to 0.0281m.

To simplify seawater depth derivation using hydrostatic equation, this value can be subtracted from Leroy and Parthiot (1998) corrective term (Eqn. 31) and apply to the depth calculated from observed pressure in order to convert it to resultant depth of Equation of state for seawater (EOS).

The relationship between the estimated depth from observed pressure and depth from Equation of state of seawater (EOS) on one hand and between depth from hydrostatic equation and depth from Equation of state of seawater (EOS) on the other hand were examined. The scatter plots were depicted in Figures 4 and 5.

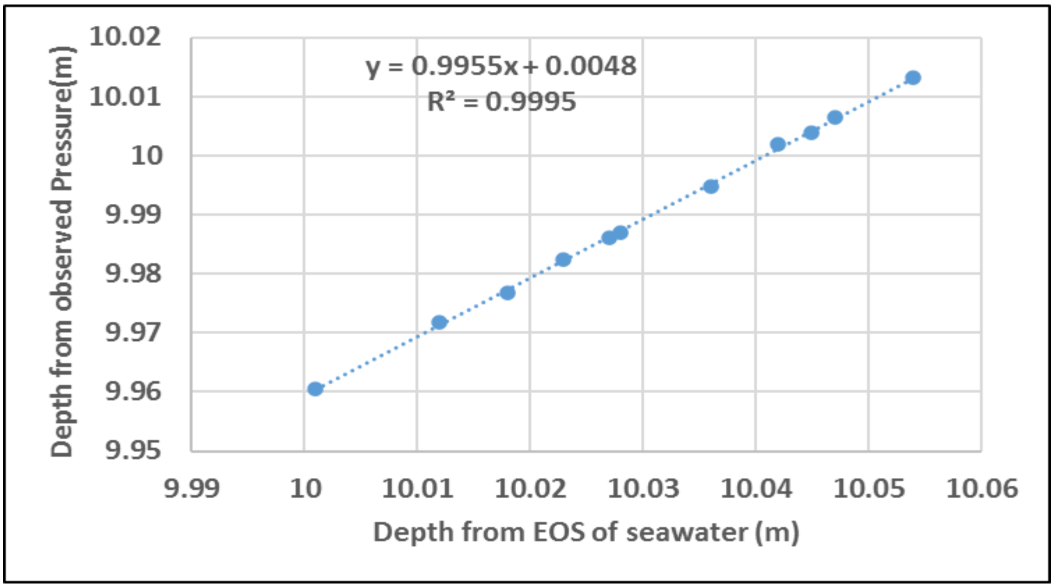

Figure 4: Linear trend between the depth from observed pressure and EOS

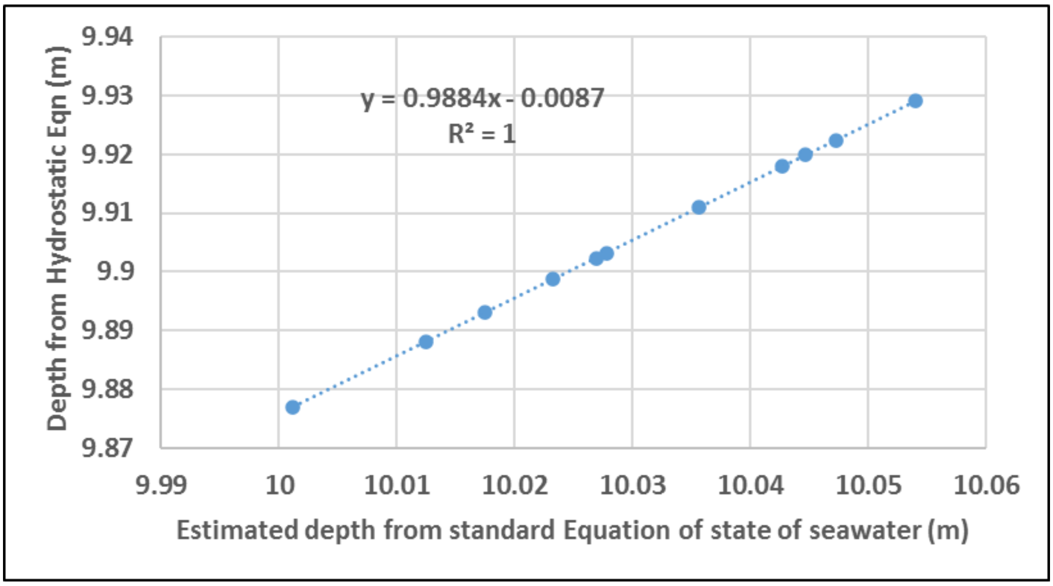

Figure 4: Linear trend between the depth from hydrostatic equation and EOS

With correlation coefficient (R^2) of about 1 in both cases (Figures 4 and 5), it suggests that the depth obtained from the two concepts were highly correlated with the standard equation of state of sea water depth.

Since the approximated value of the correction to the depth from observed pressure (C_{Obs}) which was derived to be 0.041m has been constant throughout the study period, the expression for the corrected depth in terms of observed atmospheric pressure can be given as:

$$Z \approx \quad Z_{Obs} + 0.041 \tag{21}$$

Similarly, the approximated value of the correction to the depth from hydrostatic equation (C_{Hyd}) was derived to be 0.125m and the value has also been constant. Hence, the expression for the corrected depth from hydrostatic equation can be given as:

$$Z \approx \quad Z_{Hyd} + 0.125 \tag{22}$$

Therefore, the depth computed from Equations (21) and (22) were compared with the standard Equation of state of seawater depth.

Estimated corrected depth derived from observed pressure as compared to the standard Equation of state of seawater was as shown in Table 3.

Table 3: Analysis of corrected depth derived from observed pressure

Year	Depth from EOS of Seawater (m)	Corrected depth from observed Pressure (m)	Estimation error (m)
1969	10.04729	10.0475	0.00021
1973	10.05398	10.05417	0.00019
1978	10.04269	10.04292	0.000227
1983	10.0447	10.04492	0.00022
1988	10.03559	10.03583	0.000243
1993	10.02781	10.02808	0.00027
1998	10.02689	10.02717	0.000277
2003	10.02321	10.0235	0.00029
2008	10.01753	10.01783	0.000303
2013	10.00122	10.00158	0.00036
2018	10.01243	10.01275	0.00032

While the corrected depth derived from hydrostatic pressure basic equation as compared to the standard Equation of state of seawater was as shown in Table 4.

Table 4: Analysis of corrected depth derived from hydrostatic pressure basic equation

Year	Depth from EOS of Seawater (m)	Corrected depth from hydrostatic pressure basic equation (m)	Estimation error (m)
1969	10.04729	10.04748	0.00019
1973	10.05398	10.05409	0.00011
1978	10.04269	10.04293	0.00024
1983	10.0447	10.04492	0.00022
1988	10.03559	10.03591	0.00032
1993	10.02781	10.02822	0.00041
1998	10.02689	10.02731	0.00042
2003	10.02321	10.02368	0.00047
2008	10.01753	10.01806	0.00053
2013	10.00122	10.00194	0.00072
2018	10.01243	10.01302	0.00059

As seen in Tables 3 and 4, the error for the corrected depth derived from the observed pressure and the hydrostatic equation ranged from 1.9mm to 3.6mm and 1.1mm and 7.2mm respectively. This indicates a great improvement over the raw estimates from the two approaches.

It could be seen from Figures 4 and 5 that a linear positive relationship exists between the depth from the two methods and EOS, therefore, as the depth from EOS of seawater increases, so does the depth derived from the methods. Hence, a simple linear regression model of the form given by Equation (23) can be used to describe the relationship between the corrected depths to EOS standard as:

$$y = a + bx \tag{23}$$

where y signifies the dependent variable (desired depth from either observed pressure or hydrostatic equation), x, the independent variable or the estimator (corrected estimated depth from either of the methods) while a and b, the model parameter to be estimated.

In order to solve for a and b in Equation (23), a least square regression approach was adopted and the solution yielded the values of a and b to be 0.0323 and 0.9968 respectively for depth estimation in terms of observed pressure, while a and b values were 0.1161 and 0.9885 respectively for depth estimation in terms of hydrostatic equation.

Therefore, the simplified pressure derived bathymetric model for depth estimation in terms of observed pressure was derived from the parameters as:

$$y = 0.9968x + 0.0323 \tag{24}$$

and the simplified pressure derived bathymetric model for depth estimation in terms of hydrostatic equation was derived from the parameters as:

$$y -= 0.9885x + 0.1161 \tag{25}$$

To perform internal validation, Equations (24) and (25) were used to estimate the corrected depth and the estimated errors generated as shown in Tables 5 and 6 respectively for depth estimated from observed pressure and hydrostatic equation.

Table 5: Analysis of depth derived from simplified PDB model in terms of observed pressure

Year	Corrected depth from observed Pressure (m)	Depth from PDB model in terms of observed pressure (m)	Estimation residuals (m)	Error from EOS depth (m)
1969	10.0475	10.04765	0.000148	0.000358
1973	10.05417	10.0543	0.000127	0.000317
1978	10.04292	10.04308	0.000163	0.00039
1983	10.04492	10.04508	0.000156	0.000376
1988	10.03583	10.03602	0.000185	0.000428
1993	10.02808	10.02829	0.00021	0.00048
1998	10.02717	10.02738	0.000213	0.00049
2003	10.0235	10.02372	0.000225	0.000515
2008	10.01783	10.01808	0.000243	0.000546

2013	10.00158	10.00187	0.000295	0.000655
2018	10.01275	10.01301	0.000259	0.000579

Table 6: Analysis of depth derived from simplified PDB model in terms of hydrostatic pressure basic equation

Year (m)	Corrected depth from Hydrostatic Pressure basic equation (m)	Depth from PDB model in terms of hydrostatic pressure basic equation (m)	Estimation residuals (m)	Error from EOS depth
1969	10.04748	10.04803	0.000554	0.000744
1973	10.05409	10.05457	0.000478	0.000588
1978	10.04293	10.04354	0.000606	0.000846
1983	10.04492	10.0455	0.000583	0.000803
1988	10.03591	10.0366	0.000687	0.001007
1993	10.02822	10.029	0.000775	0.001185
1998	10.02731	10.0281	0.000786	0.001206
2003	10.02368	10.02451	0.000828	0.001298
2008	10.01806	10.01895	0.000892	0.001422
2013	10.00194	10.00302	0.001078	0.001798
2018	10.01302	10.01397	0.00095	0.00154

As indicated in Table 5, the estimation residuals ranging from 0.127mm to 0.295mm was obtained for PDB model using observed pressure data while the estimation residuals for PDB model using hydrostatic equation ranges from 0.478mm to 1.078mm as indicated in Table 6. Similarly, when compared to the EOS of sea water depth, errors ranging from 0.317mm to 0.655mm was obtained for observed pressure based PDB model while the error for hydrostatic equation based PDB model ranges from 0.588mm to 1.798mm. This implies that the observed pressure based PDB model performs better than hydrostatic equation based PDB model. However, further validation study utilizing depth from in-situ bathymetric survey as standard for assessment is essential.

5.0 Conclusion

Hydrostatic equation and the Knudsen-Ekman formulation for specific anomaly were used to derive new depths using UNESCO 1983 formulation as a standard for validation of the results. Algorithms were developed to obtain depth from pressure in relation to standard Ocean formula for estimating near-shore seawater depth. This method involved enablement to be able to calculate depth within the derived seawater depth as compared to the standard UNESCO 1983 formulation. The variation in latitude used to derive depth did not show appreciable difference within the years of study. Results showed expected difference of approximately 0.041m with observed pressure in decibars and 0.125m with hydrostatic basic equation. Correlation and regression analysis were used to investigate the strength and direction of the relationship between the variables. From the correlation and regression analysis, it can be concluded that there were significant linear correlation between the derived pressure depth from both theory of equivalence of observed pressure to depth and hydrostatic equation as compared to the standard UNESCO 1983 formulation. This study recommended the need for further validation of these new PDB models using observed bathymetric data. The new models have the ability for fast data gathering over a wide area, thus, adoption of the new PDB model would be highly remarkable for region with limited bathymetric information due to high cost of classical hydrographic survey.

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