Theoritical Process on forces extenuating the equilibrium Tide: An Overview

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Key words: Tide Generating Forces; Tidal Potential, Moon-Earth-Sun System

ABSTRACT

The process of oceanic tides has been experimented and considered by humanity as far back as the beginning of development. Perceptibly, early researchers have noticed the linking between high and low water, the position of the moon and the sun. Owing to the consistency of tidal phenomena, it became closely connected with the flow of time as the varying name tides indicate. The reputable equilibrium theory due to Newton clarifies well the forces that generate the tides. This study manages the derivation of the equilibrium Tide which emanates from tide generating forces and extends to tidal potential. The main objective of this study is to look into the mathematical algorithms that give birth to equilibrium tide. The tidal potential is consequential from the gravitational attraction of masses of the moon and the sun. from this potential tide generating forces the expression for the equilibrium Tide is generated.

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1.0 INTRODUCTION

The main goal in sea tidal elements and analysis is to determine a scientific form of the gravitational attraction on the global ocean caused by the presence of the moon and the sun (Haigh et al., 2011). The procedure of maritime tides has been tested and considered by mankind as far back as the start of development. Noticeably, early scientists have seen the linkage between the high and low water, the position of the moon and the sun. Owing to the consistency of tidal phenomena, it became closely connected with the flow of time as the varying name tides indicate. The equilibrium hypothesis established by Newton (Coughenour *et al.*, 2009; Rahman, *et al.*, 2017; Zilrn *et al.*, 2017). Roosbeek, (1996) clarifies well the forces that generate the tides. The observed dominant semi diurnal periodicity of ocean tide was well clarified by Newton's equilibrium theory of tides (Chao and Ray, 1997; Bryden *et al.*, 2007). The history of earth tide generating potential (TGP) can be trace back to Doodson, (1921) when he first achieved a precise representation of the tide generating potential (TGP) by harmonic series (Kudryavtsev, 2004). Consequent developments were done by (Cartwright and Tayler, 1971; Büllesfeld, 1985; Tamura, 1987, 1993; Xi and Hou, 1987), the most recent and up to date most exact consonant improvements of the TGP have been made by (Kudryavtsev, 2004). They introduced some modification in relation to methods used by previous researchers which include the use of an improved spectral analysis technique to obtain expansions of the relevant functions to Poisson series with the terms' amplitudes and arguments being high-degree polynomials of time (as opposed to classical Fourier analysis where the terms' amplitudes are constants and the arguments are linear functions of time). The made used of the most up-to-date planetary and lunar ephemerides of DE/LE-405, -406 (Standish 1998a in spite of the fact that Newton found the true astronomic nature of tide, it was Laplace (Hendershott, 1981) who developed the first principals of a hydrodynamic equation of ocean tides. The tide generating force in terms of Newton's equilibrium tide was contained in Laplace's tidal equation as the forcing function (Hough, 1898; Büllesfeld, 1985). In this study a pleasant derivation of the equilibrium tide is addressed and the derivation of tidal potential and tide generating forces are presented in detail.

2.0 Tidal Generating Force

The geophysical phenomenon produced by the tidal force is known as the tide. The surface of the Earth's sea deviates from a spherical shape, leading to the tides. The tidal force is the impact of gravity on a body, caused by the presence of a secondary body. On Earth, therefore the tidal force is caused by the Moon and the Sun, the tidal force and its potential are produced by the combined effect of the gravity of the earth and the moon (Matsuda *et al.*, 2015),. The numerical description of the tidal force is clear and the discussion will be presented later in the literature

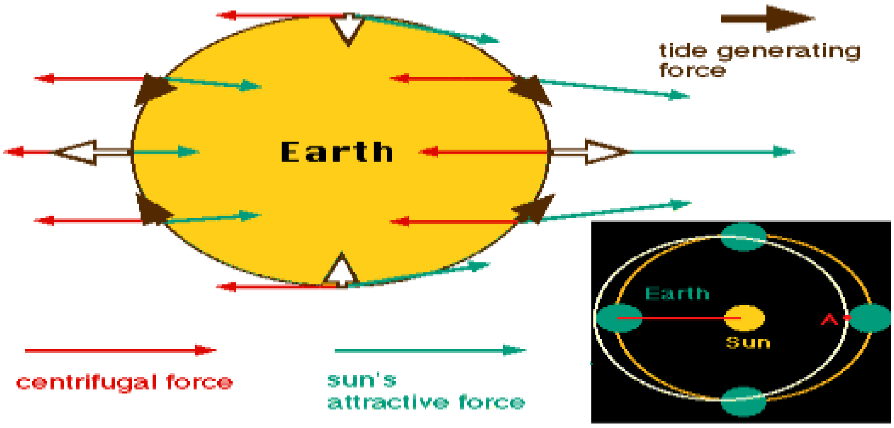

Figure2.1 The tide generating force
Source: - http://oceanography.asu.edu/Oc_Nov14_pos.pdf_(29/10/2018)

The tide generating force is the summation of the gravitational and centrifugal force. the centrifugal force is the same at each point on the surface of the earth, while the gravitational force differs as shown in figure 1.1. this shows that the tide generating force differs in intensity and direction over the earth's surface. Its vertical component is negligibly small against gravity and its effect on the ocean can be disregarded but Its horizontal component produces the tidal currents which result in sea level variations as shown in Figure 1.2 below.

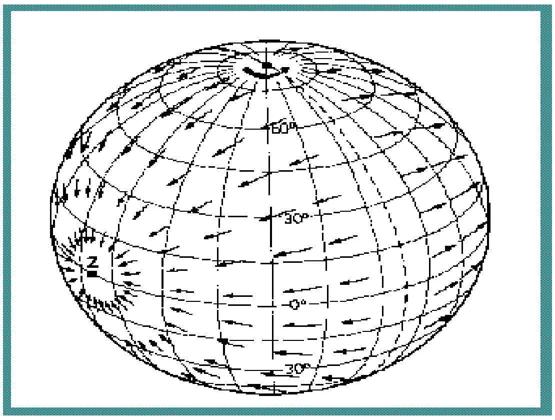

Figure 2.2 A snapshot of the tide generating force when the moon is over the point marked Z (the "zenith")
Source: - http://oceanography.asu.edu/Oc_Nov14_pos.pdf_(29/10/2018)

The gravitational force exerted by a celestial body (moon, sun or star) is directly proportional to its mass but inversely proportional to the square of the distance.

$$F = G\,\frac{m_e m_l}{R_l^2} \qquad (1)$$

The mass of the Sun is equivalent to some 332,000 Kg earth masses, while the moon mass is equivalent to only 1.2 percent of the mass of the Earth. The mean distance between the Sun and the earth is 149.5 million km while the mean distance between the earth and the moon is only 384,000 km. If the gravitational force of the sun and moon are to be compared, it will be discovered that the sun's enormous mass easily makes up for its larger distance to Earth, to the extent that the gravitational force of the sun felt on earth is about 178 times than that of the moon. As a result, the moon's movement around the earth does not seriously distort the earth's orbit around the sun (Pugh, 1996) .Table 2.1 shows the basic astronomical constant of the earth, Sun and Moon Mass.

Table 2.1 Basic Astronomical constants of the Moon, Earth, and Sun

Planet		Value	Unit	Symbol
MOON	mass	7.35×10^{22}	Kg	M_1
	Mean radius	1737.5	Km	
	Mean distance from Earth	384,400 = 60.3	Km Earth radii	$\bar{R}_1$
	Orbital eccentricity	0.0549		e_1
Earth				
	Mass	5.9722×10^{24} = 81.3	Kg lunar masses	M_e
	Equatorial radius	6378.137	Km	
	Mean Distance from the sun	149,600,000 = 23,460	Km earth radii	$\bar{R}_s$
	Mean distance from Earth's centre to Earth -Moon Mass centre.	4671	Km	
	Orbital eccentricity	0.0167		e_e
Sun				
	Mass	1.9884×10^{30} = 332,964	Kg Earth Masses	Ms
	Radius	695,500	Km	

The absolute pull of gravity exerted by the sun and the moon does not actually produce tided, but by the differences in the gravitational fields produced by the two bodies (the sun and earth or the moon and earth) as shown in Figure 2.1, as a result of moon closeness to the earth than the sun, the gravitational force field of the moon varries much more strongly over the surface of the earth than the gravitational force field of the sun. Subsequently, the sun's tide generating force is just around 46% of that from the moon and other celestial bodies don't apply a significant tidal force.

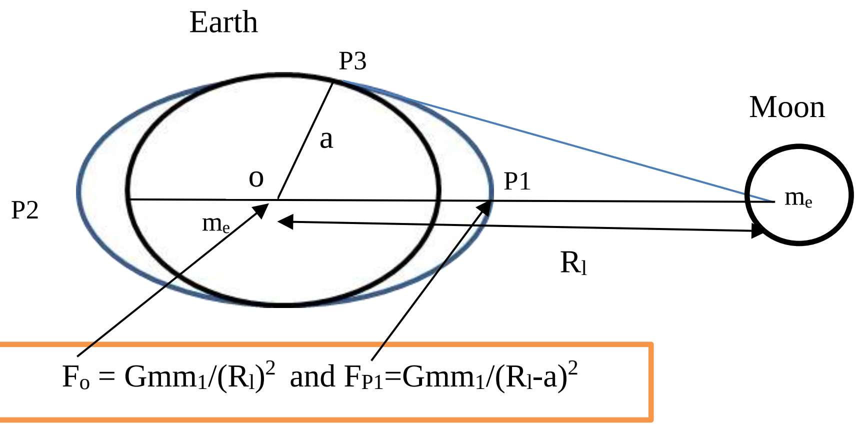

Figure 2.3 Position of the Earth - Moon System
Source http://www.incois.gov.in/documents/ITCOocean/ITC001/ppts/L3-Generation%20of%20Tides.pdf (10/20/2018)

Difference (P1 – O) is the Tide generating Force (TGF) = $2G\dfrac{mm_1a}{(R_l)^3}$

At P2 the force away from the moon is = $2G\dfrac{mm_1a}{(R_l)^3}$

While at P3 the force is directed towards O

Considering the earth – moon system a particle of Mass m at P1 on the surface of the earth as shown in figure 2.3, therefore, force toward the moon is given by

$$F_{P1=}\frac{Gm}{(R-a)^2} \qquad (2)$$

Where G is the gravitational universal constant, ($6.67 \times 10^{-11} Nm^2Kg^{-2}$).

As a result of all point of the earth travel around the center of mass of the earth – moon system in circles, and have the same radius, the distribution of centrifugal force is uniform. Therefore, the centrifugal force which is the force at the center of the earth at point O is given by

$$F_O = \frac{Gm}{R^2} \tag{3}$$

The Difference Between the forces is the tide producing force at point P_1,
Therefore is,

$$TPF = F_{P_1} - F_o$$

$$TPF = \frac{Gm}{(R-a)^2} - \frac{Gm}{R^2}$$

Where TPF is Tide Producing Force, which gives the net force toward the moon.

$$TPF = \left[\frac{1}{(R-a)^2} - \frac{1}{R^2}\right]Gm$$

$$= \left[\frac{1}{\frac{(R-a)^2}{R^2}} - 1\right]\frac{Gm}{R^2} = \frac{Gm}{R^2}\left[\frac{1}{\left(1-\frac{a}{R}\right)^2} - 1\right]$$

The Expression $\left[\frac{1}{\left(1-\frac{a}{R}\right)^2}\right]$ can be expanded using binomial theory, therefore the force at P_1 becomes

$$TPF_{P_1} = \frac{2Gma}{R^3} \tag{4}$$

Comparable thought demonstrates that, for a particle at P_2, the gravitational attraction is excessively week to balance the centrifugal force, which gives rise to a net force away from the moon with the same strength as the force at P_1 and is given by

$$TPF_{P_2} = -\frac{2Gma}{R^3} \tag{5}$$

. It is also easy to prove that the net force at P3 is directed towards the centre of the earth by making use of the approximation sin (∅) = $\frac{a}{R}$, the force at P3 is also given by

$$TPF_{P_3} = \frac{Gma}{R^3} \tag{6}$$

3.0 Tide Generating Potential

Gravitational Potential is the work that must be done comparing the force of attraction to remove a particle of unit mass to an endless distance from the body.

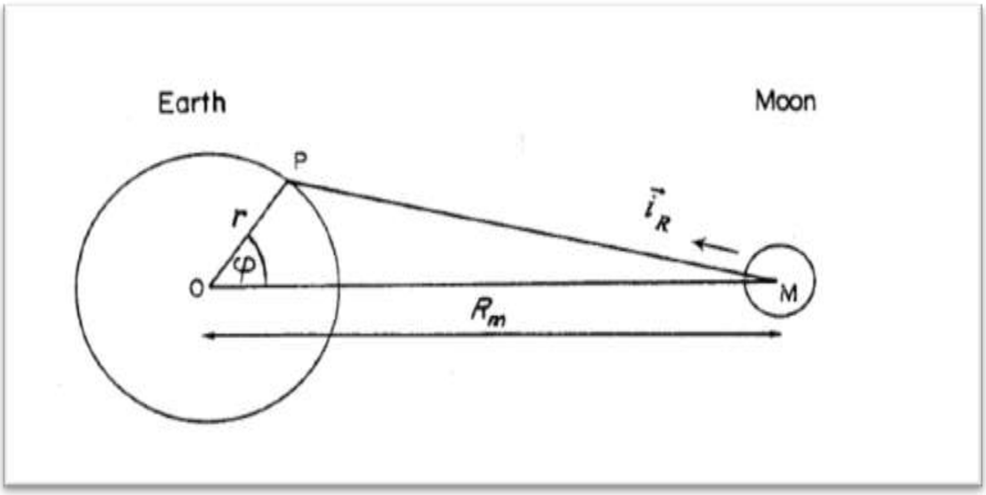

Figure 2.4 Geometry of calculation of the tidal potential.

Since the location of point P on the surface of the earth in Figure 2.4 the gravitational potential can be inscribed in terms of the lunar angle θ, the radius of the earth r and the distance Rm between the earth and the moon, using cosine rule we hae the

following equation

$$PM^2 = r^2 + R^2 - 2rR\cos\theta \tag{7}$$

$$PM = \frac{R}{R}\left(R^2 + r^2 - 2Rr\,\text{Cos}\,\theta\right)^{\frac{1}{2}}$$

$$= R\left(\frac{R^2 + r^2 - 2Rr\,\mathrm{Cos}\,\theta}{R^2}\right)^{\frac{1}{2}}$$

$$= R\left(1 - \frac{2r\,\mathrm{Cos}\,\theta}{R} + \left(\frac{r}{R}\right)^2\right)^{\frac{1}{2}}$$

Taken inverse of PM in order to calculate the tidal potential

$$V = -\frac{Gm}{R}\left(1 - 2\frac{r}{R}\cos\theta + \left(\frac{r}{R}\right)^2\right)^{-1/2} \qquad (8)$$

But here $\left(1 - 2\frac{r}{R}\cos\theta + \left(\frac{r}{R}\right)^2\right)^{-1/2}$ can be expanded using Legendre Polynomial taken

$\frac{r}{R}$ as X

we have $\frac{1}{\left(1 - 2x\,\mathrm{Cos}\,\theta + x^2\right)^{\frac{1}{2}}}$

Therefore $\left(1 - x\left(2\,\mathrm{Cos}\,\theta - x\right)\right)^{-\frac{1}{2}}$

$$= 1 + \frac{1}{2}x\left(2\,\mathrm{Cos}\,\theta - x\right) + \frac{3}{8}x^2\left(2\,\mathrm{Cos}\,\theta - x\right)^2 + \frac{5}{16}x^3\left(2\,\mathrm{Cos}\,\theta - x\right)^3 + \frac{35}{128}x^4\left(2\,\mathrm{Cos}\,\theta - x\right)^4$$

$$\left(1 - 2\frac{r}{R}\cos\theta + \left(\frac{r}{R}\right)^2\right)^{-1/2} = 1 + \frac{r}{R}\cos\theta - \frac{r^2}{2R^2} + \frac{3}{8}\left(\frac{4r^2}{R^2}\cos^2\theta - 4\frac{r^2}{R^3}\cos\theta + \ldots\right)\frac{5}{16}\left[\left(2\frac{r}{R}\cos\theta\right)^3 - \ldots\right]$$

Or,

$$\left(1 - 2\frac{r}{R}\cos\theta + \left(\frac{r}{R}\right)^2\right)^{-1/2} = 1 + \frac{r}{R}\cos\theta + \left(\frac{r}{R}\right)^2\left\{\frac{1}{2}\left(3\cos^2\theta - 1\right)\right\} + \left(\frac{r}{R}\right)^3\left\{\frac{1}{2}\left(5\cos^3\theta - 3\cos\theta\right) + \ldots\right\}$$

Thus, the tidal due to the gravitational pull of the moon becomes

$$V = \frac{Gm}{R}\left[1 + \frac{r}{R}\cos\theta + \frac{1}{2}\left(\frac{r}{R}\right)^2\left(3\cos^2\theta - 1\right) + \frac{1}{2}\left(\frac{r}{R}\right)^3\left(5\cos^3\theta - 3\cos\theta\right)\right]$$

$$\therefore V = \frac{Gm}{R}\left(1 + \frac{r}{R}L_1 + \left(\frac{r}{R}\right)^2 L_2 + \left(\frac{r}{R}\right)^3 L_3 ....\right) \qquad (9)$$

Where Ln are Legendre polynomials given by

$$\begin{aligned} L_0 &= 1 \\ L_1 &= \mathrm{Cos}\,\theta \\ L_2 &= \frac{1}{2}\left(3\mathrm{Cos}^2\theta - 1\right) \\ L_3 &= \frac{1}{2}\left(5\mathrm{Cos}^3\theta - 3\mathrm{Cos}\right) \\ &: \\ &: \end{aligned} \qquad 10)$$

The first term in the square bracket in (8) is constant and has no related force, it does not contribute to the tidal force and we may ignore it. The second term produces a uniform force parallel to OM because differentiating with respect to *(r* cos θ) yields a gradient of potential:

$$-\frac{\delta V}{\delta\left(r\cos\theta\right)} = G\frac{m}{R^2} \qquad (11)$$

This is constant and corresponds to the gravity due to the moon at C and is balanced by the centrifugal force.

Lastly, the major tidal potential Ωp is the third term in the square bracket in equation (10) which is given as

$$\Omega_p = -\frac{Gm}{2R^3}r^2\left(3\mathrm{Cos}^2\theta - 1\right) \qquad (12)$$

4.0 The Equilibrium Tide

The Equilibrium Tide is defined as the elevation of the sea surface that would be in equilibrium with the tidal forces if the Earth were covered with water to such a depth that the response of the tidal generating forces would be instantaneous.“

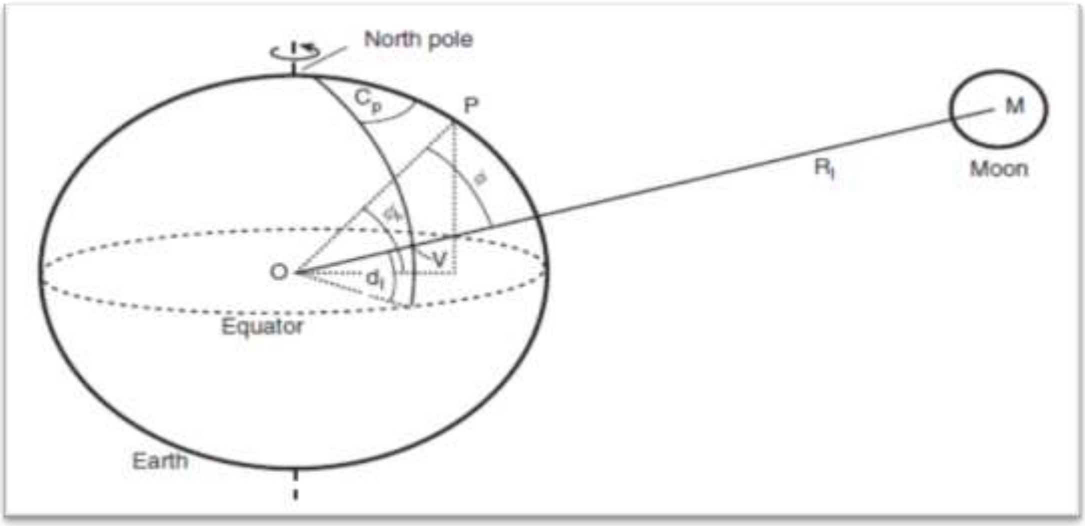

Figure4.1 The three-dimensional location of a point P on the Earth's surface relative to the sub-lunar position. Source

The lunar angle ϕ must be expressed in an appropriate astronomical variable. These are select to be the north latitude of P, ϕ_p, the declination of the Moon north of the equator dl, and the hour angle of the Moon, which is the difference in longitude between the meridian of P and the meridian of the sub-lunar point V, as shown in figure 2.3, by a standard formula in spherical trigonometry the angle ϕ is related to the other angles which are given by

$$\mathrm{Cos}\,\phi = \mathrm{Sin}\,\phi_p \bullet \mathrm{Sin}\,d_1 + \mathrm{Cos}\,\phi_p\,\mathrm{Cos}\,d_1\,\mathrm{Cos}\,C_p \qquad (13)$$

Late: - $\phi_p = \theta'; d_1 = \delta; C_p = \Upsilon$

Substituting the value of Cos ϕ in Equation 11, we have

$$\Omega_p = -\frac{Gm}{2R^3} r^2 \left(3\mathrm{Cos}^2\theta - 1\right)$$

$$\Omega_p = -\frac{3Gmr^2}{2R^3}\left(\mathrm{Cos}^2\theta - \frac{1}{3}\right)$$

$$= -\frac{3Gmr^2}{2R^3}\left(\left(\sin\theta \bullet \sin\delta + \cos\theta\cos\delta\cos\Upsilon\right)^2 - \frac{1}{3}\right)$$

$$= -\frac{3Gmr^2}{2R^3}\left(\left(\sin^2\theta \bullet \sin^2\delta + \frac{1}{2}\cos^2\theta + \cos^2\delta + \frac{1}{2}\cos^2\theta + \cos^2\delta\cos 2\Upsilon + \frac{1}{2}\sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right) - \frac{1}{3}\right)$$

$$= -\frac{3Gmr^2}{4R^3}\left(\left(2\sin^2\theta \bullet \sin^2\delta + \cos^2\theta + \cos^2\delta + \cos^2\theta + \cos^2\delta\cos 2\Upsilon + \sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right) - \frac{2}{3}\right)$$

$$= -\frac{3Gmr^2}{4R^3}\left(\left(2\sin^2\theta \bullet \sin^2\delta + \left(1-\sin^2\theta\right)\left(1-\sin^2\delta\right) + \cos^2\theta + \cos^2\delta\cos 2\Upsilon + \sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right) - \frac{2}{3}\right)$$

$$= -\frac{3Gmr^2}{4R^3}\left(\left(3\sin^2\theta \bullet \sin^2\delta + 1 - \sin^2\theta - \sin^2 + \cos^2\theta + \cos^2\delta\cos 2\Upsilon + \sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right) - \frac{2}{3}\right)$$

$$= -\frac{3Gmr^2}{4R^3}\left[\frac{9\sin^2\theta \bullet \sin^2\delta - 3\sin^2\theta - 3\sin^2\delta + 1}{3} + \cos^2\theta + \cos^2\delta\cos 2\Upsilon + \sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right]$$

$$= -\frac{3Gmr^2}{4R^3}\left[\frac{\left(3\sin^2\theta - 1\right)\left(3\sin^2\delta - 1\right)}{3} + \cos^2\theta\cos^2\delta\cos 2\Upsilon + \sin 2\theta \bullet \sin 2\delta \bullet \cos\Upsilon\right]$$

$$-\Omega_p = \frac{3}{2} rGm\left(\frac{r}{R}\right)^3\left[\frac{3}{2}\left(\sin^2 d_1 - \frac{1}{3}\right)\left(\sin^2\phi_p - \frac{1}{3}\right) + \frac{1}{2}\sin 2d_1 \sin 2\phi_p \cos\Upsilon + \frac{1}{2}\cos^2 d_1 \cos^2\phi_p \cos 2\Upsilon\right]$$

Where d_1 is the lunar declination, which is corresponding to geocentric latitude, ϕ_p is the geodetic latitude of P, and C_p is the angle between the meridian plane and the plane that containing the Earth's axis and P is the hour angle of P. The above equation is used to compute equilibrium tide.

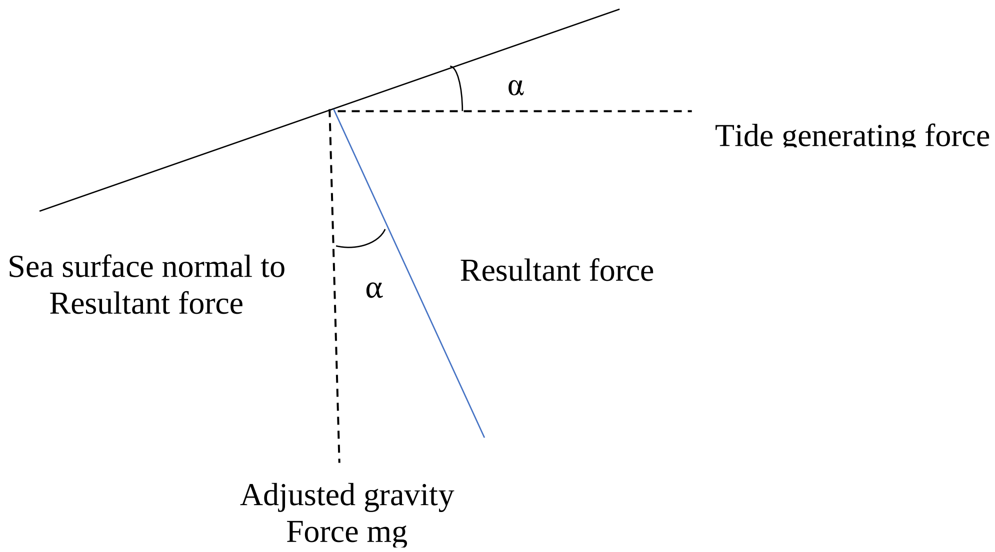

Figure 4.1 The relationship between the equilibrium water surface, the tide generating force, and the normal earth gravity force.

If the tidal generating force, the gravity of P and the equilibrium surface is as shown in figure 2, then

$$\tan\alpha = -\frac{\left(\frac{\Delta\Omega_p}{\Delta x}\right)}{g} = \left(\frac{\zeta\Delta}{\Delta x}\right)$$

So that

$$\frac{\Delta}{\Delta x}\left(g\zeta + \Omega_p\right) = 0$$

and similarly, on the y direction

$$\frac{\Delta}{\Delta x}\left(g\zeta + \Omega_p\right) = 0$$

Integrate over a finite area, if the total volume of water is conserved, then

$$g\zeta + \Omega_p = 0$$

and substitute the tidal potential back finally, the Equilibrium Tide becomes:

$$\zeta = a\frac{m_1}{m_e}\left[C_0(t)\left(\frac{3}{2}\sin^2\phi_p - \frac{1}{2}\right) + C_1(t)\sin 2\phi_p + C_2(t)\cos^2\phi_p\right] \text{----------} \ (13)$$

Where $\zeta =$ the elevation of the free surface and the time dependent coefficient are given by

$$C_0(t) = \left(\frac{a}{R}\right)^3\left(\frac{3}{2}\sin^2 d_m - \frac{1}{2}\right)$$

$$C_1(t) = \left(\frac{a}{R}\right)^3\left(\frac{3}{4}\sin^2 2d_m \cos C_m\right)$$

$$C_2(t) = \left(\frac{a}{R}\right)^3\left(\frac{3}{4}\cos^2 2d_m \cos 2C_m\right)$$

The three coefficients depict the three main types of tidal constituent.

(i) the constituents which are generated as a result of monthly variations in lunar declination d_1 produces the long-period of tidal species. It has a maximum amplitude at the poles and zero amplitude at latitudes 35°- 16’, north and south of the equator.

(ii) The constituent that is modulated at twice the frequency of the lunar declination, and with a frequency close to one cycle per day is the diurnal species when the declination is at maximum it has also a maximum amplitude. Spatially it has a maximum amplitude at 45° latitude and zero amplitude at the equator and the poles. The discrepancies north and south of the equator are in opposite phase.

The semidiurnal species, comprise the constituent with a frequency closed to two circles per day, and is modulated at twice the frequency of the lunar declination, but is has a maximum amplitude at the equator when the declination is zero and zero amplitude at the poles.

5.0 Conclusion

The equilibrium Tide due to the sun can be denoted in an analogous form to Equations (13), with m_m,R_m, and d_m swapped by m_s, R_s, and d_s. The resulting amplitudes are lesser than those of the lunar tides by a factor of 0.46, but the vital explanations are the same.

In the above equation, the period associated to $\cos^2\theta$ is termed as semi – diurnal tides, Semidiurnal tides are those with a period of about half a day, while the term consists of $\sin^2 \theta$ is called the diurnal tides, diurnal tides are those with a period of about a day. and the term connected to $3\sin^2 \theta -1$ are long period tides. For the lunar tidal potential, those periods are nearly 12hours, 24 hours and 14 days respectively. For the solar tidal potential, they are close to 12 hours,24 hours, and 180 days respectively.

The equilibrium Tidal theory is a source for the definition of the harmonic frequencies by which the energy of the observed tides is distributed, and is also imperative as a reference for the observed phases variables. and amplitudes of the tidal constituents. Its practical usage is based on Doodson's development and on the astronomical variables.

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BIOGRAPHICAL NOTES

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