Results of experiment of correcting for the spherical harmonic coefficients of the EGM2008 based on detailed gravity anomalies data in an area of the North Vietnam

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Keywords: Gravity anomalies, correction of spherical harmonic coefficients, Earth gravitational model, national quasigeoid model

Summary

This paper presents a results of experiment of a correcting for the spherical harmonic coefficients of the EGM2008 based on detailed gravity anomalies data in an area of the North Vietnam with purpose of solving problem "Fill-in" to increase an accuracy of global quasigeoid heights determined from the EGM2008. On base of perfection of Colombo's method with purpose of the correcting for the spherical harmonic coefficients of the EGM2008 on ellipsoid, authors had created software for accomplishing of above mentioned task. The results of experiment carried out with 2006 detailed gravimetric points in an area of the North Vietnam show that a gravity anomalies calculated from the corrected EGM2008 are more near to a ground based gravity anomalies in experimental region. Obtained results allow to orient an usage of the detailed gravity data for construction of highly accurate national quasigeoid model in Vietnam in the future.

INTRODUCTION

For the purpose of high accuracy quasigeoid model building in the territory of Vietnam, the detailed gravimetric collection and measurements are being intensively undertaken. At present on the mainland in the low mountainous and plain areas (average height less than 1500 m) it has been collected and measured additional 65,536 values of detailed gravity accelerations. Detailed measurements of gravity in Vietnam's sea area and mountainous areas with average heights greater than 1500 m are being implemented.

Simultaneously with the detailed gravity measurement, the investigation of methods for calculation of quasigeoid heights is also strongly promoted. According to (Featherstone WE, Kirby JF, 2000), to calculate the highly accurate quasigeoid heights based on gravity anomalies data according to Stokes's integral formula requires a radius of near zone containing the detailed gravity anomalies data must be in the range of $2^0.5$ to $5^0.0$. This is unrealistic for the specific conditions of a country with long and narrow territory like Vietnam in the absence of detailed gravity data in neighboring countries. Meanwhile, for the exploitation of the EGM2008 Earth Gravitational Model to build the national quasigeoid model, Vietnam belongs to the group of "Fill-in" countries, i.e. the detailed gravity data in Vietnamese territory has not been used to calculate the spherical harmonic coefficients of this model (Pavlis Nikolas K, Simon A. Holmes, Steve C. Kenyon, John K. Factor, 2008).

Therefore now, in the process of building the high accuracy national quasigeoid model in Vietnam, the main research direction focuses to solve the "Fill-in" issue on the basis of the correction of the spherical harmonic coefficients of the EGM2008 Earth Gravitational Model from gravity anomalies data collected on Vietnamese territory. This scientific report will present some initial research results in the above research direction.

1. DATA

As presented in (Ha Minh Hoa, 2018), in Vietnam the mountainous areas occupies more than three-fourths of the mainland area of Vietnam, many of which lie on heights of 2 - 3 km. Therefore, in addition to calculating the Faye anomalies, we must also calculate the RTM anomalies.

For the gravity point P with normal height H_P^γ and geodetic latitude B, free air anomaly values are calculated according to the formula:

$$\Delta g_{FA} = g_P - \gamma_N,$$

Where g_P - gravity acceleration is measured on the point P; γ_N - gravity acceleration of the point N corresponds to the point P and lies on the telluroid surface, in addition

$$\gamma_N = \gamma_0 - 0{,}308562.(1+0{,}0007.\cos 2B).H_P^\gamma - 0{,}0723.10^{-6}.(H_P^\gamma)^2 + A_{atm} \quad <mGal>.$$

Atmospheric correction A_{atm} applied to the normal gravity acceleration is calculated by the formula:

$$A_{atm} = 0{,}8658 - 9{,}727.10^{-5}.H_P^\gamma + 3{,}482.10^{-9}.(H_P^\gamma)^2 \quad <mGal>.$$

For WGS84 ellipsoid, the normal gravity acceleration on the ellipsoid surface γ_0 is calculated by the formula:

$$\gamma_0 = 978032{,}5.(1+0{,}0053024.\sin^2 B - 0{,}0000058.\sin^2 2B) \quad <mGal>.$$

For the purpose of eliminating the effects of the Earth's concave, convex topographic surface characterized by short and very short waves of the geoid surface with wavelengths from 100 m to a few kilometers to the free-air anomaly Δg_{FA}, we use the global digital elevation model with high resolution 3” x 3” to calculate the Faye correction. The Faye anomaly is defined by the formula $\Delta g_F = \Delta g_{FA} + \delta g_F$, where Faye correction is calculated by the integral formula (Heiskanen W.A. and Moritz H., 1967; Forsberg R., 1984):

$$\delta g_F = G.\sigma_{cr}.\iint_\omega \int_{H_P^\gamma}^{H^\gamma} \frac{z - H_P^\gamma}{l^3}.dz.d\omega,$$

where ω - integral region of plane coordinates x, y; l - space distance from point P to the points of integral calculation points in the integral region.

Faye correction number is also calculated and checked according to the prism integral method proposed in the literature (Nagy D., 1966) and completed in (Forsberg R., 1984).

For the purpose of eliminating the effect of the residual terrain masses characterized by average waves of geoid surface with wavelengths ranging from 5 to 20 km in high mountainous areas, in (Forsberg R. and C.C. Tsherning, 1981; Forsberg, R., 1984) it has been proposed to use RTM anomalies defined by the formula $\Delta g_{RTM} = \Delta g_F + \delta g_{RTM}$, where δg_{RTM}- RTM correction. The RTM correction is determined based on the global digital elevation model with the high resolution 3”x 3” and the the global digital elevation model with the average resolution 5’x5’. In addition, to replace the smoothed real terrain surface characterized by the global digital elevation model with the high resolution 3”x 3”, we will use the smoothed average terrain surface by the global digital elevation model with the average resolution 5'x5 '. Based on the proposal in (Omang O.C.D., Tsherning C.C., Forsberg R., 2012) we calculate the RTM correction by the formula (Ha Minh Hoa, 2018):

$$\delta g_{RTM} = -0{,}1967.(H_Q^\gamma - H_P^\gamma),$$

where H_Q^γ - the normal height of the gravity point P, but it lies on the smoothed average terrain surface, in addition to the normal height H_Q^γ is determined from the global digital elevation model with the average resolution 5’x5’.

The experiment area covers the entire northern delta region of Vietnam, occupying an area of 16,700 square kilometers containing 2006 detailed gravity points, in which 1961 points for calculating, 45 points for independent checking. During the experiment, SRTM global digital elevation model with the high resolution 3”x3” is the smoothed real terrain surface and DTM2006.0 global digital elevation model with the average resolution 5’x5’ to be the smoothed average terrain surface.

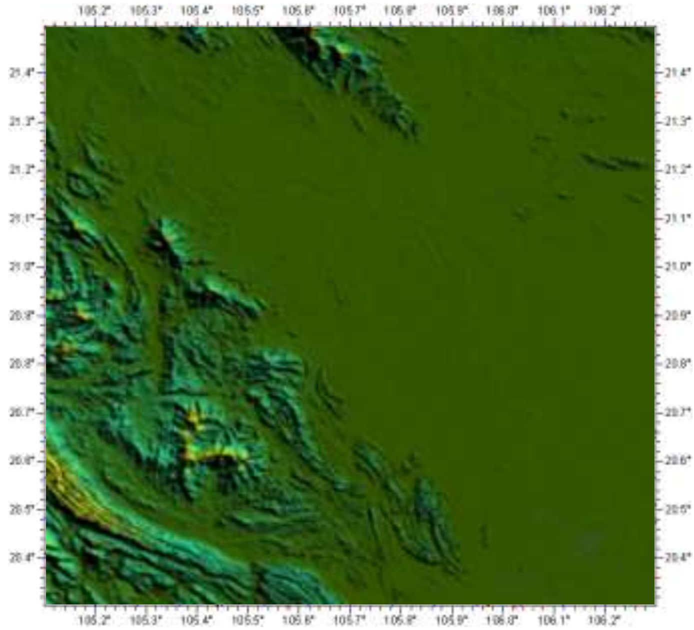

Figure 1. Experiment area

2. APPLIED METHOD

Due to the ellipsoid shape is closer to the shape of the attracting Earth and the ellipsoid harmonic series will converge in space outside the Earth, in the 1990s gravity anomalies have been expanded by spherical functions on the ellipsoidal surface. When correcting the normalized spherical harmonic coefficients $\overline{\overline{C}}_{n,m}, \overline{\overline{S}}_{n,m}$ of the Earth Gravitational Model

(EGM), it is developed to harmonize at order n and level m thanks to the gravity data, the spherical harmonic coefficients are modified as follows:

$$\tilde{\bar{\bar{C}}}_{n,m} = \bar{\bar{C}}_{n,m} + \delta\bar{C}_{n,m}; \quad \tilde{\bar{\bar{S}}}_{n,m} = \bar{\bar{S}}_{n,m} + \delta\bar{S}_{n,m},$$

where corrections $\delta\bar{C}_{n,m};\delta\bar{S}_{n,m}$ into normalized harmonic coefficients $\bar{\bar{C}}_{n,m},\bar{\bar{S}}_{n,m}$ are alculated according to the following formula (Erol B., 2012):

$$\begin{Bmatrix} \delta\bar{C}_{n,m} \\ \delta\bar{S}_{n,m} \end{Bmatrix} = \frac{a_e^2}{4\pi.GM.(n-1)}.\iint_\sigma \left(\frac{\rho}{a_e}\right)^n .\overline{\delta\Delta g}(\rho,B,L).\begin{Bmatrix} \cos mL \\ \sin mL \end{Bmatrix}.\bar{P}_{n,m}(\sin B).d\sigma, \qquad (1)$$

and

$$\overline{\delta\Delta g}(\rho,B,L) = \overline{\Delta g(\rho,B,L)} - \Delta g_{EGM}(\rho,B,L),$$

where a_e - the semimajor radius of the ellipsoid; ρ - the geocentric radius vector of the integral calculation point; $\overline{\Delta g(\rho,B,L)}$ - the mean of the gravity anomalies at the integral calculation point; $\Delta g_{EGM}(\rho,B,L)$ - global gravity anomaly values is determined from the EGM is being corrected at the integral calculation point; GM - Earth's gravitational constant; B.L – geodetic coordinates of the integral point.

For following usage the EGM with corrected spherical harmonic coefficients $\tilde{\bar{\bar{C}}}_{n,m}$, $\tilde{\bar{\bar{S}}}_{n,m}$ will be called as " the corrected EGM".

The quadratic method proposed by Colombo O.L. in (Colombo, O.L., 1981) has been widely used in the determination of spherical harmonic coefficients according to the gravity data, in addition the spherical harmonic coefficients are defined on the sphere. To use Colombo O.L.'s quadratic method to calculate the corrections $\delta\bar{C}_{n,m};\delta\bar{S}_{n,m}$ according to the formula (1) on the ellipsoid surface, we have modified this formula as follows (Ha Minh Hoa, Nguyen Tuan Anh, 2015):

$$\begin{Bmatrix} \delta\bar{C}_{n,m} \\ \delta\bar{S}_{n,m} \end{Bmatrix} = K_n.\sum_{i=1}^{N} \bar{\chi}_{n,m}^{i}.\sum_{j=1}^{M} \overline{\delta\Delta g}(i,j).\left[\begin{Bmatrix} A(m) \\ -B(m) \end{Bmatrix}.\cos mL_j + \begin{Bmatrix} B(m) \\ A(m) \end{Bmatrix}.\sin mL_j\right], \qquad (2)$$

where

$$K_n = \frac{a_e^2}{4\pi.G.M.(n-1)}$$

$$\bar{\chi}_{n,m}^{i} = \left(\frac{\rho_i}{a_e}\right)^n.\frac{\bar{P}_{n,m}(\sin B_i).\Delta\sigma_{i,j}}{q_n},$$

$$q_n = \begin{cases} \beta_n^2 \quad khi \quad 0 \le n \le n_{\max}/3, \\ \beta_n \quad khi \quad n_{\max}/3 < n \le n_{\max}, \\ 1 \quad khi \quad n_{\max} < n, \end{cases}$$

β_n - Pellinen's parameter calculated by the formulas presented in (Pellinen L.P., 1969; Meissl P., 1971; Sjoberg L.E., 1980); n_{max} - the maximum expandsion order of the EGM model, the area of the standard square (i, j) is given by the formula:

$$\Delta\sigma_{ij} = \Delta L.\left[\sin B_i - \sin(B_i + \Delta B)\right],$$

where

$$A(m) = \begin{cases} (\sin m.\Delta L)/m.\Delta L \quad khi \quad m \ne 0, \\ 1 \quad khi \quad m = 0, \end{cases}$$

(3)

$$B(m) = \begin{cases} (\cos m.\Delta L - 1)/m.\Delta L \quad khi \quad m \ne 0, \\ 0 \quad khi \quad m = 0. \end{cases}$$

In the fomulas (2), (3) the cos(mX) và sin(mX) values with any X variable are computed as the following formulas:

$$\cos(mX) = 2.\cos X \cdot \cos[(m-1)\cdot X] - \cos[(m-2)\cdot X],$$
$$\sin(m.X) = 2.\cos X \cdot \sin[(m-1)\cdot X] - \sin[(m-2)\cdot X].$$

In formula (2) the value

$$\overline{\delta\Delta g}_{(i,j)} = \overline{\Delta g}_{(i,j)} - \Delta g_{EGM(i,j)},$$

where $\overline{\Delta g}_{(i,j)}$ is the mean value of the gravity anomalies of the cell (i, j) calculated at the focal point with average latitude $\overline{B}(i,j)$ and the average longitude $\overline{L}(i,j)$ from gravity anomalies at the 4 points of cell according to the bilinear interpolation equation, and $\Delta g_{EGM(i,j)}$ is the gravity anomaly at the focal point is defined from EGM model.

In order to efficiently compute the corrections of the spherical harmonic coefficients of the corrected EGM under formula (2), we represent this formula as:

$$\begin{Bmatrix} \delta\overline{C}_{n,m} \\ \delta\overline{S}_{n,m} \end{Bmatrix} = K_n \cdot \sum_{i=1}^{N} \overline{\chi}^i_{n,m} . H_i(m), \qquad (4)$$

where the 2x1 column vectors for each i row (i = 1,2, ..., N) are defined as the formula:

$$H_{i\,(m)} = \sum_{j=1}^{M} \delta\Delta\bar{g}(i,j).\left[\begin{Bmatrix} A(m) \\ -B(m) \end{Bmatrix}.\cos mL_j + \begin{Bmatrix} B(m) \\ A(m) \end{Bmatrix}.\sin mL_j\right].$$

More complicated problem is to define the normalized associated Legendre function $\bar{P}_{n,m}(\sin B_i)$. In (Ha Minh Hoa, 2014), the normalized associated Legendre function $\bar{P}_{n,m}(\sin B)$ is calculated as follows:

When m = 0, n = 2,...,n_{max}:

$$\bar{P}_{n,0}(\sin B) = \sqrt{2.n+1}.P_n(\sin B),$$

where the Legedre function $P_n(sinB)$ is defined by the retrieved expression:

$$P_n(\sin B) = \frac{2.n-1}{n}.\sin B.P_{n-1}(\sin B) - \frac{n-1}{n}.P_{n-2}(\sin B).$$

The initial values used to compute by the retrieval expression including $P_0(\sin B) = 1,\ P_1(\sin B) = \sin B,\ P_2(\sin B) = \frac{3}{2}.\sin^2 B - \frac{1}{2}....$

When m ≠ 0, n =2,..., n_{max}:

$$\bar{P}_{n,m}(\sin B) = \sqrt{2.(2.n+1).\frac{(n-m)!}{(n+m)!}}.P_{n,m}(\sin B),$$

where the associated Legedre function $P_{n,m}(sinB)$ is defined by the retrieved expression:

$$P_{n,m}(\sin B) = \frac{2.n-1}{n-m}.\sin B.P_{n-1,m}(\sin B) - \frac{n+m-1}{n-m}.P_{n-2,m}(\sin B) \qquad (5)$$

With $m \le n-2$:

The initial values used to compute by the retrieval expression are:

$$P_{1,1}(\sin B) = -\cos B,\ P_{2,1}(\sin B) = 3.\sin B.\cos B,\ P_{2,2}(\sin B) = 3.\cos^2 B,$$

$$P_{3,1}(\sin B) = \frac{3}{2}.\sin^3 B - \frac{3}{2}.\sin B,\ P_{3,2}(\sin B) = 15.\cos^2 B.\sin B,$$

$$P_{3,3}(\sin B) = 15.\cos^3 B.$$

For the associated Legendre function $P_{n,m}(sinB)$, when m > n, $P_{n,m}(sinB)$ = 0. Formula (5) can be used only for the associated Legendre functions at $m \le n-2$. For example, where *n = 181*, we can only compute Legendre functions up to *m = 179*. For the associated Legendre function with *m = n - 1*, we define the function $P_{n,n-1}(sinB)$ as the formula:

$$P_{n,n-1}(\sin B) = (2n-1).\sin(B).P_{n-1,n-1}(\sin B). \qquad (6)$$

For the associated Legendre function with *m = n*, we define $P_{n,n}(\sin B)$ as the following formula:

$$P_{n,n}(\sin B) = -(2n-1).\cos(B).P_{n-1,n-1}(\sin B). \qquad (7)$$

When calculating the correction numbers $\delta\bar{C}_{n,m}$; $\delta\bar{S}_{n,m}$ with defined n and m, we have to define N values $\bar{\chi}^i_{n,m}$ (i = 1,2,...,N), in addition to each index i we have to define the normalized associated Legendre function $\bar{P}_{n,m}(\sin B_i)$ to the base vertex of standard cell with the geodetic latitude B_i. This requires very large computational time when calculating the corrected numbers $\delta\bar{C}_{n,m}$; $\delta\bar{S}_{n,m}$ with the indices n, m determined by formula (4).

To speed up the computation time, with the determined geodetic latitude B, we find that to determine the associated Legendre function $P_{l,m}(\sin B)$, we need to know two quantities: $P_{l-1,m}(\sin B)$ and $P_{l-2,m}(\sin B)$. Supposing that we organize the direct file (the symbolic PL file) with the ordinal number of the record 1 to (n_{max}+1). Therefore the index m corresponds to the record m + 1. In each m + 1 record, we store two values $P_{l-1,m}(\sin B)$ and $P_{l-2,m}(\sin B)$ for the purpose of calculating the associated Legendre function $P_{l,m}(\sin B)$ by the formulas (5), (6), (7). Once we have calculated $P_{l,m}(\sin B)$, we store two values $P_{l,m}(\sin B)$ and $P_{l-1,m}(\sin B)$ in this record and for the subsequent calculation of the function $P_{l+1,m}(\sin B)$ and so on. The above calculation chart is used when we calculate the same number of corrections $\delta\bar{C}_{n,m}$; $\delta\bar{S}_{n,m}$ at the same order n. At the n small order, only a few records (temporarily called empty records) contain two values $P_{l,m}(\sin B)$ and $P_{l-1,m}(\sin B)$, the remaining records are empty. As n increases, the number of non-empty records increases.

With the above calculation diagram, we calculate the corrections of the normalized spherical harmonic coefficients by cumulative sum. Then the formula (4) is expressed in the form as follows:

$$\begin{Bmatrix} \delta\bar{C}_{n,m} \\ \delta\bar{S}_{n,m} \end{Bmatrix} = \begin{Bmatrix} 0.0 \\ 0.0 \end{Bmatrix} + K_n \cdot \bar{\chi}^{1}_{n,m} . H_1(m) + K_n \cdot \bar{\chi}^{2}_{n,m} . H_2(m) + ... + K_n \cdot \bar{\chi}^{i}_{n,m} . H_i(m) + \\ + ... + K_n \cdot \bar{\chi}^{N}_{n,m} . H_N(m). \qquad (8)$$

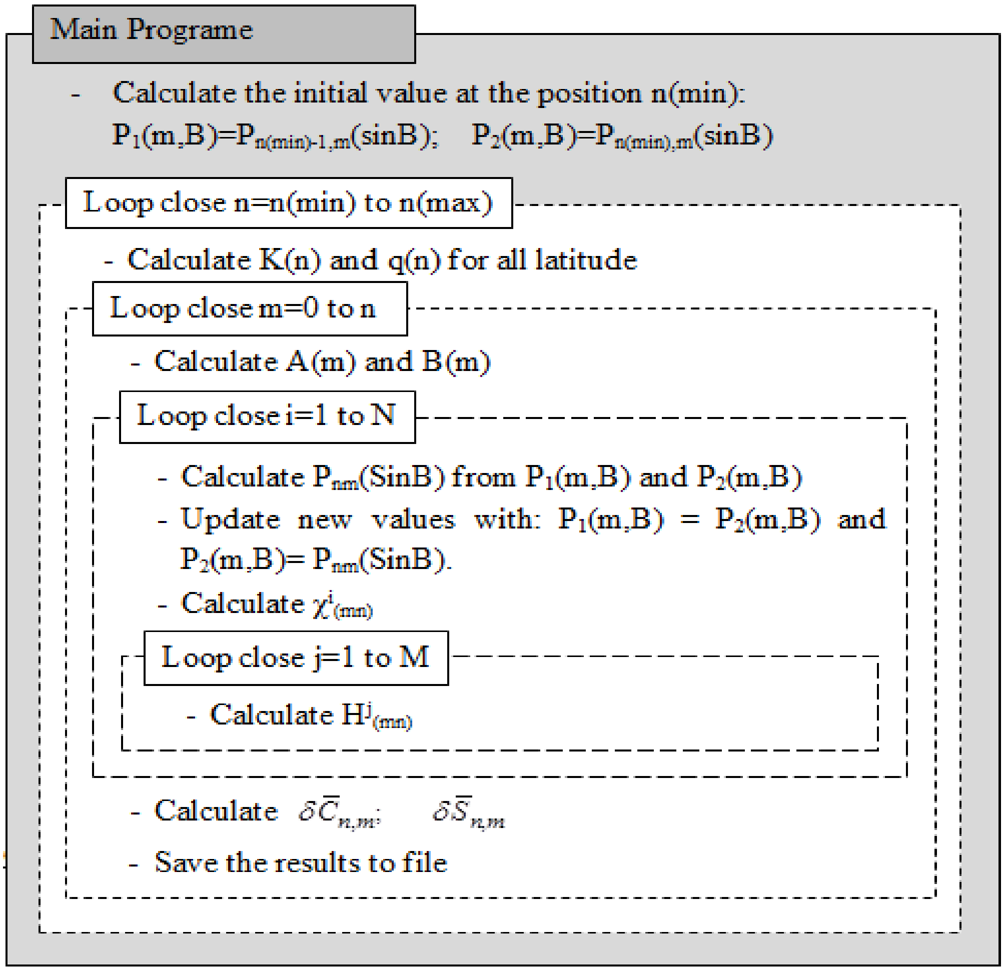

Figure 2. Correction calculation procedure of the spherical harmonic coefficients of EGM model. Based on the gravity anomalies

For each geodetic latitude B_i of the base vertex (i = 1,2, ..., N), we compute the corresponding $K_n \cdot \bar{\chi}^i_{n,m} \cdot H_{i(m)}$ component by formula (8). After computation of all coefficients for normalized spherical harmonic coefficients with n varies from n_{min} to n_{max} and m varies from 0 to n for the each base vertex with geodetic latitude B_i, we re-organize the PL file for geodetic latitude B_{i+1} of the next base vertex. Similarly, after computation of all normalized spherical harmonic coefficients with the order of n varies from n_{min} to n_{max} and the level m varies from *0* to n for each geodetic latitude B_{i+1} we reorganize the file PL for the geodetic latitude B_{i+2} of the next base vertex, and so on until the geodetic latitude B_N.

The method presented above has been applied to correct the EGM2008 model with order $n_{\max} = 2159$ based on RTM anomalies in the experiment area. The software module that implements the calculations process is shown in Figure 1 above.

3. RESULTS

Based on 1961 gravity points, we conduct a test of correction calculation of more than 2 million of the normalized spherical harmonic coefficients of EGM2018 model**.** Since the corrected EGM2008 model, it has been calculated 2006 gravity anomalies $\overline{\overline{\Delta g}}$ on the 2006 detailed gravity points in the experiment area. The test results are shown in Table 1, whereby $\delta\Delta g$ is the difference of ground-based gravity anomalies with the gravity anomalies determined from the uncorrected EGM2008 model, $\overline{\overline{\delta\Delta g}}$ is the difference of ground-based gravity anomalies with the gravity anomalies determined from the corrected EGM2008 model. The chart compares the differences between $\delta\Delta g$ và $\overline{\overline{\delta\Delta g}}$ is shown in Figure 3.

Table 1

Difference	± (0 - 5) mGal	± (5 - 10) mGal	± (10 - 20) mGal	± (20 - 30) mGal	± (30 - 40) mGal
$\delta\Delta g$ (Points)	181	176	677	916	11
$\overline{\overline{\delta\Delta g}}$ (Points)	863	506	427	139	26

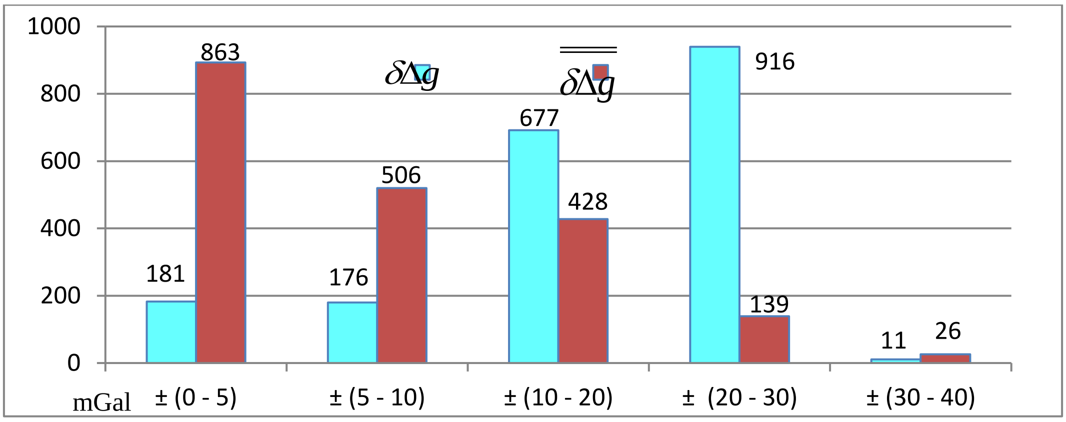

Figure 3. Comparative chart $\delta\Delta g$ and $\overline{\overline{\delta\Delta g}}$ (mGal)

From Table 1 and Figure 3, we show that in total of 1961 differences of $\delta\Delta g$ và $\overline{\overline{\delta\Delta g}}$ which are compared, for the uncorrected EGM2008 model only 357 differences $\delta\Delta g$ (accounting for 18.20%) are in intervals of ± (0 - 5) mGal and ± (5 - 10) mGal, meanwhile the corrected EGM2008 model has 1369 differences $\overline{\overline{\delta\Delta g}}$ (accounting for 69.81%) in the above intervals.

Thererefore, the correction of the normalized spherical harmonic coefficients of the EGM2008 model based on the ground-based gravity anomalies shall make the gravity anomalies $\Delta g_{EGM}(\rho, B, L)$ closer to the ground-based gravity anomalies $\Delta g(\rho, B, L)$.

Root mean square error on 45 independent checking points spread evenly within the experiment area compared to the corrected $\overline{EGM2008}$model of 4,779mGal, compared to the uncorrected EGM2008 model of 15,038mGal. This confirms that gravity anomalies $\overline{\Delta g_{EGM}}$ are determined from the corrected $\overline{EGM2008}$model that is most closely correlated with the ground-based gravity anomalies Δg_{RTM}, i.e., the corrected $\overline{EGM2008}$model is more suitable for gravitational field in the territory of Vietnam.

4. CONCLUSION

For a long and narrow country such as Vietnam, in the context of not having gravity data in neighboring countries, the correction of the normalized spherical harmonic coefficients of the EGM2018 based on ground-based gravity anomalies data on the national territory not only allows to solve the "Fill-in" problem, but also facilitates to improve the accuracy of the global quasigeoid heights obtained from the corrected EGM2008 for national highly accurate quasigeoid model building. Although the experiment area is small, the test results show that the corrected EGM2008 tends to be closer to the gravitational field in the territory of Vietnam. This opens the way for feasibility study to exploit the ground-based gravity anomaly data collected in the territory of Vietnam to improve the accuracy of the EGM2008 model in accordance with the Earth's gravitational field in the national territory and serve the task of national highly accuracte quasigeoid model building in the future./.

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