FIG working week 2012 - Rome

SIMA – Raw Data Simulation Software for the Development and Validation of Algorithms for GNSS and MEMS based Multi-Sensor Navigation Platforms

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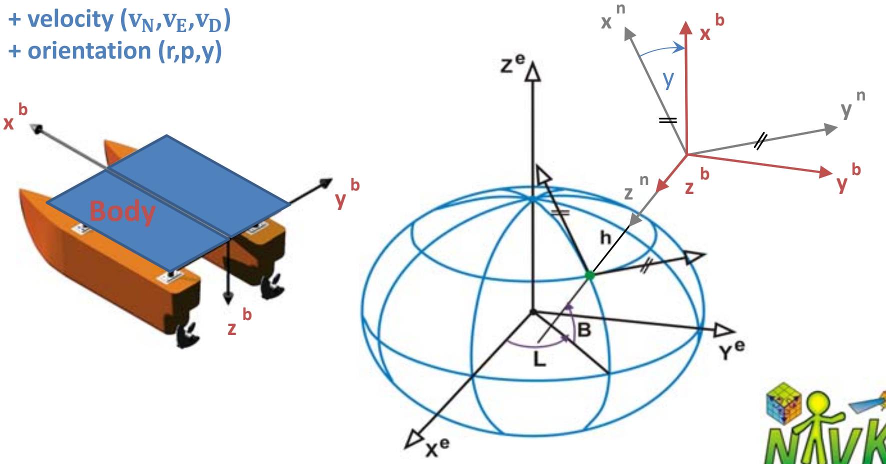
NAVIGATION STATE & FRAMES

Navigation state vector: $\mathbf{y}(t) = [(B, L, h)^e | (v_N, v_E, v_D)^n | (r, p, y)_b^n]^T$

position (B,L,h)
+ velocity (v_N,v_E,v_D)
+ orientation (r,p,y)



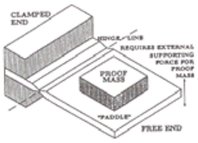
SENSORS FOR ROBUST AND GLOBAL APPLICATIONS

1.) GNSS

References: Inertial Space or e-frame

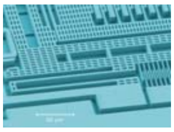
2.) Accelerometers

References: Inertial Space and Gravity Field



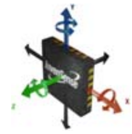
3.) Gyroscopes

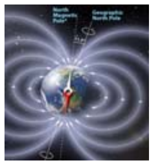
References: Inertial Space



4.) Magnetic field sensors

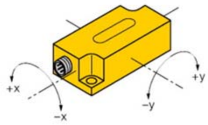
References: Earth Magnetic Field





5.) Inclinometers

References: Gravity Field



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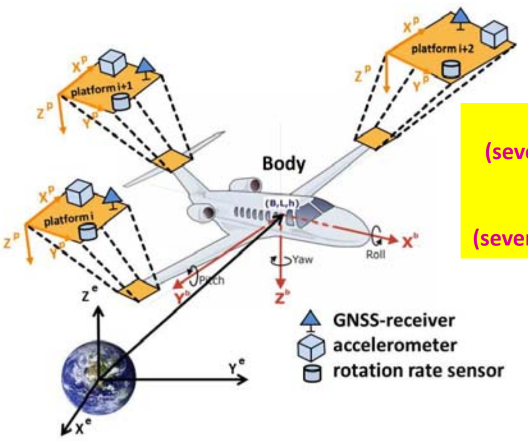
SENSOR CONCEPT FOR STATE ESTIMATION

General concept for robust algorithms & sensor simulation:



„Multiplatform-"
(several platforms (p) navigate one body (b))
and
„Multisensor-Leverarm-" – Concept
(several coordinated sensors on each platform)

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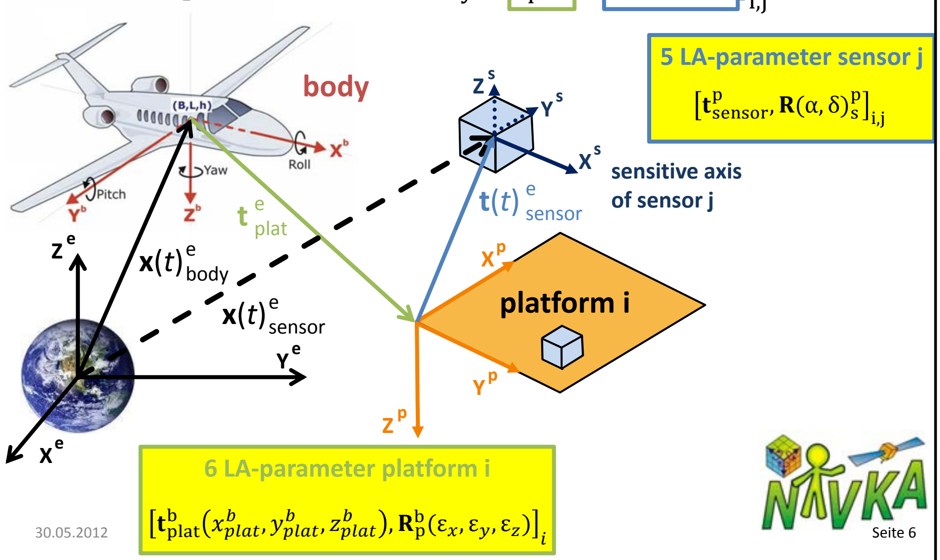
SIMA: SIMULATION OF MULTISENSOR ARRAYS

- Numerical comparison of optimized sensor platforms
- Numerical proof of functionality of new platforms
  - with redundant sensors
  - with sensors in motion
- Further system tests:
  - Can additional parameters be estimated?
  - Filter reaction on gross errors?
  - Filter reactions on different trajectories?

- Simplified implementation because true numerical values are known
- Reference state known from trajectory model

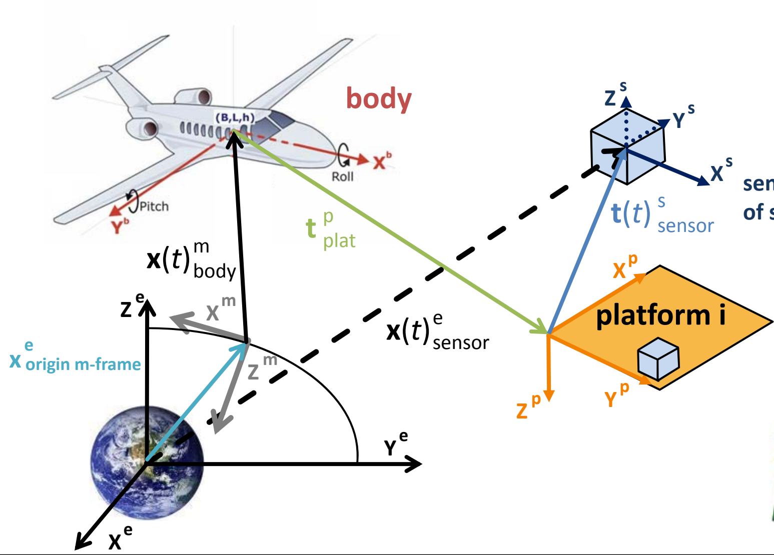
SIMA LEVER ARM CONCEPT & PARAMETRIZATION

$$\left[\mathbf{x}(\mathrm{t})^{\mathrm{e}}_{\mathrm{sensor}} = \mathbf{x}(\mathrm{t})^{\mathrm{e}}_{\mathrm{body}} + \boxed{\mathbf{t}^{\mathrm{e}}_{\mathrm{plat}}} + \boxed{\mathbf{t}(\mathrm{t})^{\mathrm{e}}_{\mathrm{sensor}}}\right]_{\mathrm{i,j}}$$



SIMA LEVER ARM CONCEPT & M-FRAME

$$\left[\mathbf{x}(t)^{e}_{sensor} = \mathbf{x}^{e}_{m\_origin} + \mathbf{R}^{e}_{m}\mathbf{x}^{m}_{body} + \mathbf{R}^{e}_{m}\mathbf{R}(t)^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{plat} + \mathbf{R}^{e}_{m}\mathbf{R}(t)^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}(t)^{p}_{s}\mathbf{t}^{s}_{sensor}\right]_{i,j}$$



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LEVER ARM CONCEPT – SENSOR VELOCITY & ACCELERATION

$$\left[\frac{\partial}{\partial t}\mathbf{x}(t)^{e}_{sensor} = \dot{\mathbf{x}}(t)^{e}_{sensor}\right.$$

$$= \mathbf{R}^{e}_{m}\dot{\mathbf{x}}(t)^{m}_{body} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{plat} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{t}^{s}_{sensor}$$

$$\left. + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\mathbf{t}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\dot{\mathbf{t}}^{s}_{sensor}\right|_{i,j}$$

$\mathbf{t}^{p}_{plat} = \text{constant}$

$\mathbf{R}^{b}_{p} = \text{constant}$

$$\left[\frac{\partial}{\partial t}\dot{\mathbf{x}}(t)^{e}_{sensor} = \ddot{\mathbf{x}}(t)^{e}_{sensor}\right.$$

$$= \mathbf{R}^{e}_{m}\ddot{\mathbf{x}}^{m}_{body} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{plat} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\dot{\mathbf{\Omega}}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{plat}$$

$$+ \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{t}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\dot{\mathbf{\Omega}}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{t}^{s}_{sensor}$$

$$+ \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\mathbf{t}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\dot{\mathbf{t}}^{s}_{sensor}$$

$$+ \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\mathbf{t}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\mathbf{\Omega}^{s}_{ps}\mathbf{t}^{s}_{sensor}$$

$$+ \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\dot{\mathbf{\Omega}}^{s}_{ps}\mathbf{t}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\dot{\mathbf{t}}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\dot{\mathbf{t}}^{s}_{sensor}$$

$$\left. + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\mathbf{\Omega}^{s}_{ps}\dot{\mathbf{t}}^{s}_{sensor} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{R}^{p}_{s}\ddot{\mathbf{t}}^{s}_{sensor}\right]_{i,j}.$$

Necessary parameters for observation modeling:

$\mathbf{x}^{m}_{body}, \dot{\mathbf{x}}^{m}_{body}, \ddot{\mathbf{x}}^{m}_{body}, \mathbf{R}^{m}_{b}, \mathbf{\Omega}^{b}_{mb}, \dot{\mathbf{\Omega}}^{b}_{mb}, \mathbf{t}^{p}_{plat}, \mathbf{R}^{b}_{p}, \mathbf{t}^{s}_{sensor}, \dot{\mathbf{t}}^{s}_{sensor}, \ddot{\mathbf{t}}^{s}_{sensor}, \mathbf{R}^{p}_{s}, \mathbf{\Omega}^{s}_{ps}, \dot{\mathbf{\Omega}}^{s}_{ps}$

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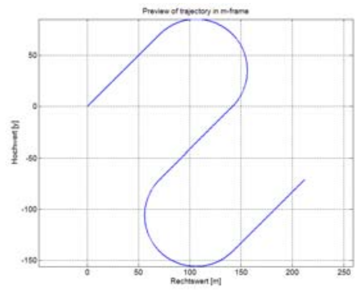
TRAJECTORY GENERATION

- Trajectory parameters:

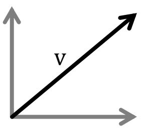
$$\mathbf{x}(t)^{m}_{body} \quad \dot{\mathbf{x}}(t)^{m}_{body} \quad \ddot{\mathbf{x}}(t)^{m}_{body} \quad \mathbf{R}(t)^{m}_{b} \quad \mathbf{\Omega}(t)^{b}_{mb} \quad \dot{\mathbf{\Omega}}(t)^{b}_{mb}$$

- Standard models: straight line, circle, helix, in rest, rotating, 2D-trajectory



- Example: Body orientation in a circle

$v_x^m = -\sin(v\,t/R)$



$v_y^m = \cos(v\,t\,/\,R)$

$$y(t)^m_b = \operatorname{atan}\left(\frac{v_y}{v_x}\right)$$

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GNSS OBSERVATIONS

- GNSS position

$$\left[\mathbf{x}^{e}_{GNSS-pos} = \mathbf{x}(t)^{e}_{body} + \mathbf{LA}(t)^{e}_{pos} = \mathbf{x}(t)^{e}_{body} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{t}^{b}_{plat} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{sensor}\right]_{i,j}$$

$$\left[\mathbf{l}^{e}_{GNSS-pos}\right]_{i,j} = \left[\mathbf{x}^{e}_{GNSS-pos} + \mathbf{n}^{e}_{GNSS-pos}\right]_{i,j}$$

- GNSS velocity

$\mathbf{t}^{p}_{sensor} = \text{constant}$

$\mathbf{R}^{p}_{s} = \text{constant} = \mathrm{I}$

$$\left[\dot{\mathbf{x}}^{e}_{GNSS-vel} = \dot{\mathbf{x}}(t)^{e}_{body} + \mathbf{LA}(t)^{e}_{vel} = \dot{\mathbf{x}}(t)^{e}_{body} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{t}^{b}_{plat} + \mathbf{R}^{e}_{m}\mathbf{R}^{m}_{b}\mathbf{\Omega}^{b}_{mb}\mathbf{R}^{b}_{p}\mathbf{t}^{p}_{sensor}\right]_{i,j}$$

$$[\mathbf{l}^{e}_{GNSS-vel}]_{i,j} = [\dot{\mathbf{x}}^{e}_{GNSS-vel} + \mathbf{n}^{e}_{GNSS-vel}]_{i,j}$$

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GNSS OBSERVATIONS

- Raw data observation equations:
  - pseudorange

$$[l_{PR,k}]_{i,j} = \left[\left|\mathbf{x}^{e}_{\mathrm{GNSS-pos,j}} - \mathbf{x}^{e}_{\mathrm{sat,k}}\right| + \mathrm{c}\left(\Delta \mathrm{t}_{\mathrm{GNSS,j}} - \Delta \mathrm{t}_{\mathrm{sat,k}}\right)\right]_{i,j} + \Delta\mathrm{Ion} + \Delta\mathrm{Trop} + n_{PR}$$

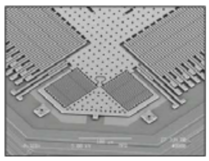
  - phase

$$\left[\mathrm{l}_{\phi,\mathrm{k,Li}}\right]_{\mathrm{i,j}} = \left[\left|\mathbf{x}^{e}_{\mathrm{GNSS-pos,j}} - \mathbf{x}^{e}_{\mathrm{sat,k}}\right| + \mathrm{c}\left(\Delta \mathrm{t}_{\mathrm{GNSS,j}} - \Delta \mathrm{t}_{\mathrm{sat,k}}\right) - \left(\lambda_{\mathrm{Li}} \mathrm{N}^{\mathrm{k}}_{\mathrm{Li}}\right)_{t0} - \left(\lambda_{\mathrm{Li}} \mathrm{D}^{\mathrm{k}}_{\mathrm{Li}}\right)_{ti}\right]_{\mathrm{i,j}}$$
$$-\Delta\mathrm{Ion} + \Delta\mathrm{Trop} + n_{\phi}$$

  - Doppler

$$[\mathrm{l}_{\Delta \mathrm{f}}]_{i,j} = \left[\mathrm{f}_{\mathrm{sat}}\left(1 + \frac{\left(\dot{\mathbf{x}}^{e}_{\mathrm{sat,k}} - \dot{\mathbf{x}}^{e}_{\mathrm{GNSS-vel}}\right)\mathbf{r}^{e}}{\mathrm{c}}\right) - \mathrm{f}_{\mathrm{sat}}\right]_{i,j} + n_{\Delta \mathrm{f}}$$

ACCELEROMETER OBSERVATIONS



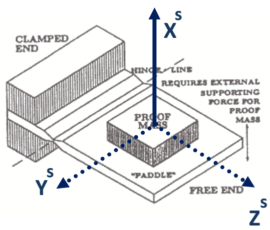
- Navigation equation in the inertial frame:

$$\mathbf{a}^{\mathrm{i}}_{\mathrm{acc}} = \frac{\partial}{\partial \mathrm{t}} \dot{\mathbf{x}}^{\mathrm{i}}_{\mathrm{acc}} - \mathbf{g}^{\mathrm{i}}_{\mathrm{acc}}$$

- Navigation equation in the earth frame:

$$\mathbf{a}^{\mathrm{e}}_{\mathrm{acc}} = \frac{\partial}{\partial \mathrm{t}} \dot{\mathbf{x}}^{\mathrm{e}}_{\mathrm{acc}} - \mathbf{g}(\mathbf{x}^{\mathrm{e}}_{\mathrm{acc}})^{\mathrm{e}}_{\mathrm{acc}} + 2\boldsymbol{\Omega}^{\mathrm{e}}_{\mathrm{ie}} \dot{\mathbf{x}}^{\mathrm{e}}_{\mathrm{acc}} + \boldsymbol{\Omega}^{\mathrm{e}}_{\mathrm{ie}} \boldsymbol{\Omega}^{\mathrm{e}}_{\mathrm{ie}} \mathbf{x}^{\mathrm{e}}_{\mathrm{acc}}.$$

- Rotation to the s-frame:



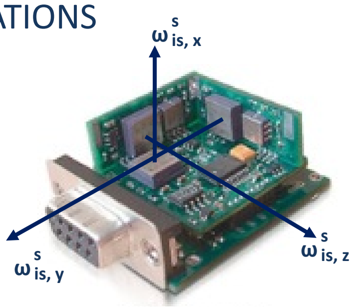
$$\mathbf{a}^{\mathrm{s}}_{\mathrm{acc}} = \mathbf{R}^{\mathrm{s}}_{\mathrm{p}} \mathbf{R}^{\mathrm{p}}_{\mathrm{b}} \mathbf{R}^{\mathrm{b}}_{\mathrm{m}} \mathbf{R}^{\mathrm{m}}_{\mathrm{e}} \mathbf{a}^{\mathrm{e}}_{\mathrm{acc}}$$

$$[\mathrm{a}^{\mathrm{s}}_{\mathrm{acc}}]_{\mathrm{i,j}} = (1 \quad 0 \quad 0) \cdot \mathbf{a}^{\mathrm{s}}_{\mathrm{acc}}$$

- Adding sensor errors:

$$[\mathrm{l}^{\mathrm{s}}_{\mathrm{acc}}]_{\mathrm{i,j}} = [\mathrm{a}^{\mathrm{s}}_{\mathrm{acc}}]_{\mathrm{i,j}} \cdot \kappa^{\mathrm{s}}_{\mathrm{acc}} + \mathrm{b}^{\mathrm{s}}_{\mathrm{acc}} + \mathrm{n}^{\mathrm{s}}_{\mathrm{acc}}.$$

GYROSCOPE OBSERVATIONS



- Gyro observation model:

$$\boldsymbol{\omega}_{is}^{s} = \boldsymbol{\omega}_{ie}^{s} + \boldsymbol{\omega}_{em}^{s} + \boldsymbol{\omega}_{mb}^{s} + \boldsymbol{\omega}_{bp}^{s} + \boldsymbol{\omega}_{ps}^{s}.$$

$$\boldsymbol{\omega}_{is}^{s} = \mathbf{R}_{p}^{s}\mathbf{R}_{b}^{p}\mathbf{R}_{m}^{b}\mathbf{R}_{e}^{m}\boldsymbol{\omega}_{ie}^{e} + \mathbf{R}_{p}^{s}\mathbf{R}_{b}^{p}\boldsymbol{\omega}_{mb}^{b} + \boldsymbol{\omega}_{ps}^{s}.$$

- for one sensor j on platform i:

$$[\omega_{is}^{s}]_{i,j} = (1 \quad 0 \quad 0) \cdot \boldsymbol{\omega}_{is}^{s}.$$

- Adding sensor errors:

$$\left[l_{gyro}^{s}\right]_{i,j} = [\omega_{is}^{s}]_{i,j} \cdot \kappa_{gyro}^{s} + b_{gyro}^{s} + n_{gyro}^{s}.$$

MEMS
3D-Gyroskop
22 x 22 mm

MAGNETIC FIELD OBSERVATIONS

- Magnetic field observation: $\mathbf{m}_{mag}^{s} = \mathbf{R}_{p}^{s}\mathbf{R}_{b}^{p}\mathbf{R}_{m}^{b}\mathbf{R}_{e}^{m}\mathbf{m}_{mag}^{e}(\mathbf{x}_{mag}^{e}, t)$

- World Magnetic Model 2010 from NOAA (National Oceanic and Atmospheric Administration) & BGS (British Geological Survey):

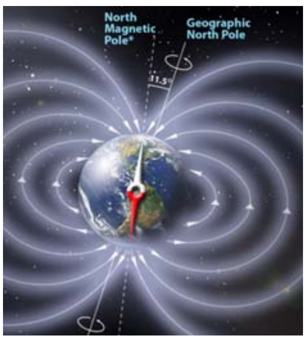
$$X'(\lambda,\varphi',r) = -\frac{1}{r}\frac{\partial V}{\partial \varphi'} = -\sum_{n=1}^{12}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left(g_n^m(t)\cos m\lambda + h_n^m(t)\sin m\lambda\right)\frac{d\,\breve{P}_n^m(\sin\varphi')}{d\varphi'}$$

- for one sensor j on platform i:

$$\left[m_{mag}^{s}\right]_{i,j} = (1 \quad 0 \quad 0) \cdot \mathbf{m}_{mag}^{s}$$

- Error model:

$$\left[l_{mag}^{s}\right]_{i,j} = \left[m_{mag}^{s}\right]_{i,j} + n_{mag}^{s}.$$



INCLINOMETER OBSERVATIONS

- Observation equation inclinometer:

$$[\theta]_{i,j} = \cos^{-1}\left(\frac{\mathbf{e}_z^{LAV}\mathbf{s}_{inc}^{LAV}}{|\mathbf{e}_z^{LAV}||\mathbf{s}_{inc}^{LAV}|}\right)$$

direction of gravity: $\mathbf{e}_z^{LAV} = (0 \quad 0 \quad 1)^T$

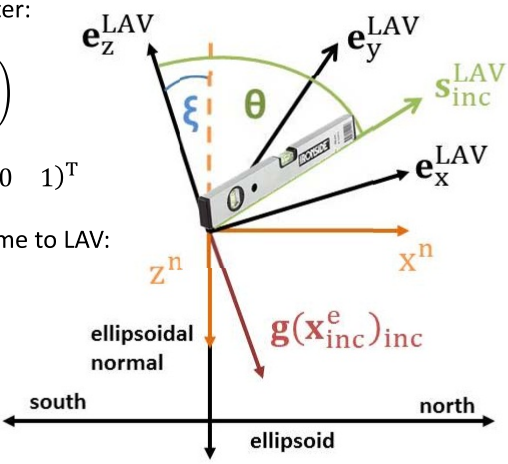
- Rotation of sensitive axis in s-frame to LAV:

$$\mathbf{s}_{inc}^{s} = (1 \quad 0 \quad 0)^T$$

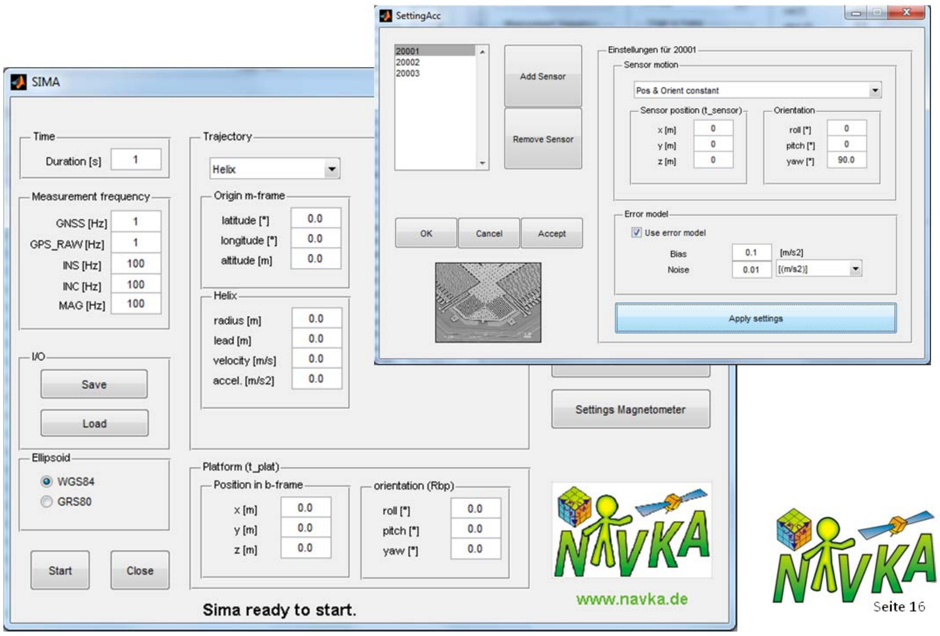
$$\mathbf{s}_{inc}^{LAV} = [\mathbf{R}_n^{LAV}]_{i,j}\mathbf{R}_b^n\mathbf{R}_p^b\mathbf{R}_s^p\mathbf{s}_{inc}^s$$

- Adding sensor errors:

$$[l_\theta = \theta + c_{inc} + n_{inc}]_{i,j}.$$



SIMA – GUI



EXAMPLE – ATTITUDE HEADING REFERENCE SYSTEM

- Navigation state vector

$$\mathbf{y}(t) = \left[\mathbf{q}_{\boldsymbol{b}}^{\boldsymbol{n}}, \mathbf{b}_{\text{gyro}}^{s}, \mathbf{b}_{\text{acc}}^{s}\right]^{\mathrm{T}}$$

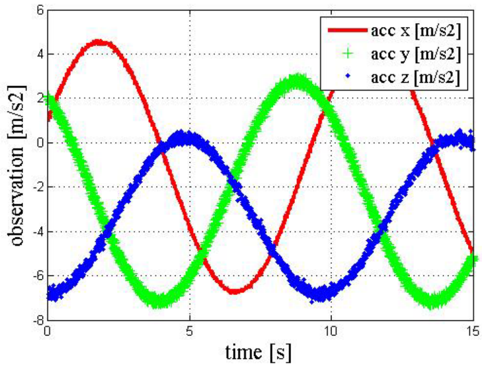
- Trajectory: Body rotates in rest

$$\boldsymbol{\omega}_{mb}^{b} = (10.0 \quad 20.0 \quad 30.0)$$

$$\boldsymbol{R}_{m}^{e}(0{,}0)$$

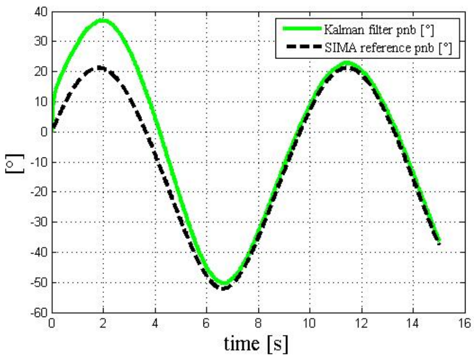
- Accelerometer biases:

$$\boldsymbol{b}_{acc}^{s} = (1.0 \quad 2.0 \quad 3.0)$$

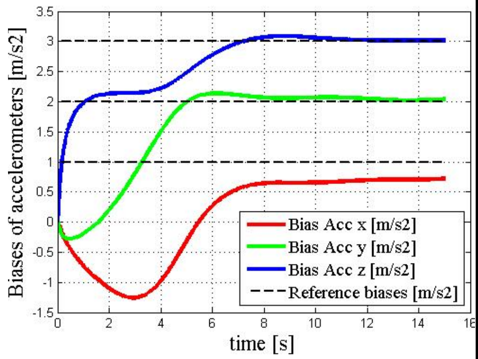


EXAMPLE – ATTITUDE HEADING REFERENCE SYSTEM

Kalmal filtered pitch angle:



Kalmal filtered accelerometer biases



CONCLUSION

- SIMAs features:
  - arbitrary number of different types of sensors
  - freely open platform design
  - consideration of the lever-arm effects
  - modeling of sensorerrors
  - different trajectories
  - known reference data for filter validation
- Perspective:
  - enhanced error modeling
  - adding additional trajectories
  - …

Thank you for your attention!

SIMA available at: www.navka.de

ENHANCED NAVIGATION-ALGORITHMS

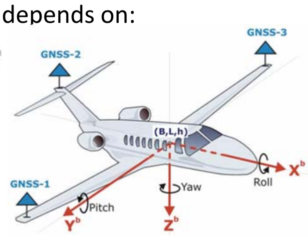
- **Platform optimization** in a similar manner as in the conventional classification in the optimization of geodetical nets:
  - design of 0th order: choise of the appropriate sensor type
  - design of 1th order: choice of optimal sensor position and orientation on platform at given variance for the observations and system state
  - design of 2nd order: choice of optimal observation accuracy at given platform design and variance of the system state
  - design of 3rd order: choice of additional sensors to optimize given platform design

➡ **Sensor raw data simulation tool required**

MULTI-SENSOR-ALGORITHMS DEVELOPMENT

- Sensor design differs in
  - sensor type
  - sensor quantity
  - sensor quality
  - location

- Different sensor designs for different applications depends on:
  - navigation parameters
  - body tractory
  - required accuracy



➡ **Sensor raw data simulation tool required**