Centre of Space Techniques

Division of Space Geodesy

Application of wavelet analysis to GPS stations coordinate time series

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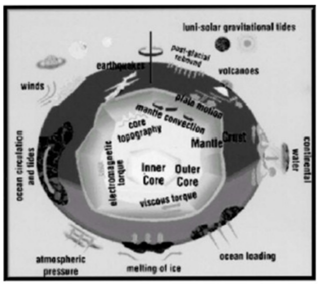
FIG Working Week 2011, Marrakech, Morocco, 18-22 May

Introduction

EARTH : complex system; siege of temporal variations

SPACE GEODESY : science which uses measurements of the artificial satellites which turn around the earth to determine the shape of the earth and its changes in time :

- ➔ Gravity field
- ➔ Geoid (mean sea level)
- ➔ Orientation of the Earth
- ➔ Deformation of the earth's crust
- ➔ …



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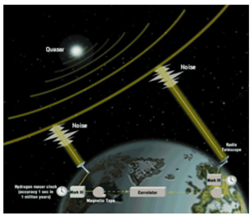
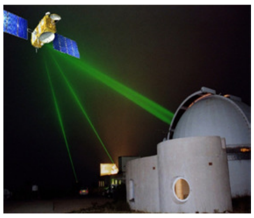
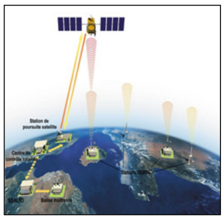
Introduction

Techniques of space geodesy : VLBI, SLR, GPS, DORIS







VLBI

Measures evolution of the phase of the incidental wave (even radiosource) between two radio telescopes.

SLR

Measures the time intervals required for pulses emitted by a Laser transmitter (station) to a satellite.

GPS

Measures satellite- receiver distance deduced from the time between emission and reception.

DORIS

Measures Doppler effect.

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Introduction

The richness and the great quantity of the measurements collected by these systems (VLBI, SLR, GPS and DORIS) Allows today to represent the displacement of the stations in the form of time series of coordinates which require the development of the adequate methods of analysis for a better exploitation.

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Introduction

Statistical methods for the study of the time series in space geodesy

➢ **Signal :**

- **Fourier Spectrum**
- **Wavelets** (Daubechies 1988, Mallat 1989, Johnstone & Silverman 1997)
- **Singular Spectrum Analysis** (Climatic time series, Ghil et al. 2002)

➢ **Noise :**

- **Spectral density and maximum likelihood estimation** (Agnew 1992, Langbein & Johnson 1997, Zhang et al. 1997, Mao et al.1999, Blewitt & Lavallée 2002, Williams 2004)
- **Allan variance** (Time and frequency, Allan 1966)

➢ **Method suggested :**

- **Wavelet transform** ➔ **Multi-resolution analysis**

Objectives

Application of the wavelet transform in the analysis of time series of GPS station coordinates, in order to collect the maximum of exploitable information on the signal (trends and seasonal components) and noise which allows to better apprehend the temporal variability of station movements (stability of stations).

Time series

Time Series :

Sequence of numerical observations X_t measured at successive times **t** (t = 1…, *N*; *N* : length of the series)

$$X_t = m_t + s_t + \varepsilon_t$$

m_t : ***Trend*** *(long-term evolution of the time series)*

s_t : ***Cyclical component*** *(seasonal components)*

ε_t : **Residual component** *(noise affecting the time series)*

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Wavelet Transform

Wavelet proprieties

Function $\Psi(t)$ of $L^2(R)$ (Continuous of real variable and square integrable) :

- ✓ Zero mean : $\int_{-\infty}^{+\infty} \psi(t)dt = 0$
- ✓ Unitary norm: $\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1$
- ✓ Centred around t = 0

Wavelet transform (WT)

$$WT(u,s) = \langle X, \Psi_{u,s} \rangle = \int_{-\infty}^{+\infty} X(t)\,\overline{\psi}_{u,s}(t)\,dt \qquad \psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right), s \in R; s \neq 0$$

u and s : translation and scale factor, and $\overline{\psi}_{u,s}$ the complex conjugate of $\psi_{u,s}$.

Inverse wavelet transform

$$X(t) = \frac{1}{C_\psi} \int_{u \in \Re} \int_{s>0} WT(u,s)\ \psi_{u,s}(t)\,du\,\frac{ds}{s^2} \qquad C_\psi = \int_0^{+\infty} \frac{|FT(\psi(t))|^2}{t} dt < +\infty$$

C_ψ is the standardization coefficient and FT is the Fourier transform.

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Multi-resolution analysis

Represent the signal on a limited number of wavelets by varying a scaling factor in a dyadic way $s=2^{-j}, j \in z$ and $u=k.2^{j}, k \in z$

↓

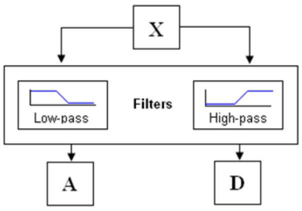
Multi-resolution analysis
(wavelet decomposition)

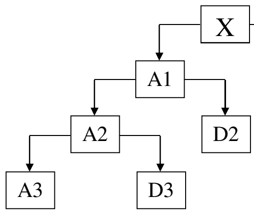
↓

At each level of resolution j we decompose the signal

↓ Information of approximation A

↓ Information of detail D





X=A1+D1
= A2+D2+D1
=A3+D3+D2+D1

Noise signal separation (De-Noising)

De-Noising steps

1. Choose a wavelet and compute the wavelet coefficients WT at level j of the signal

2. Determination of the threshold λ (**VisuShrink** [Donoho and Johnstone, 1994])

$$\lambda = \hat{\sigma}\sqrt{2\log(N)} \qquad \hat{\sigma} = \frac{\text{Med}}{0.6745}$$

σ^2 is the noise variance, N is the length of the series and Med is the median of the absolute values of the first level of decomposition.

3. Thresholding of the wavelet coefficients

Hard thresholding: $T_\lambda^{Hard}(WT) = \begin{cases} W \text{ if } |WT| > \lambda \\ 0 \text{ if } |WT| \le \lambda \end{cases}$

Soft thresholding: $T_\lambda^{Soft}(WT) = \begin{cases} WT - \lambda \text{ if } WT \ge \lambda \\ WT + \lambda \text{ if } WT \le -\lambda \\ 0 \text{ if } |WT| \le \lambda \end{cases}$

4. Reconstruct the signal from the threshold wavelet coefficients

Data Used

- Time series of weekly coordinate residuals of (16) GPS stations provided by AIUB Analysis centre "CODE" of the IGS using BERNESE Software.
- Referred to ITRF2000 and expressed in the local geodetic reference frame (dN : North, dE : East and dH : Vertical)

Acronym	Site	Country	Lat, deg	Long, deg	Data span
EISL	Easter Island	Chile	-27.02	-109.38	1996.1-2003.8
FAIR	Fairbanks	United-states	64.97	-147.52	1996.1-2006
GODE	Greenblet	United-states	38.9	-76.8	1996.1-2006
HRAO	Hartebeesthoek	South Africa	-25.88	27.70	1996.8-2006
METS	Metsahovi	Finland	60.2	24.7	1996.1-2006
NKLG	Libreville	Gabon	0.3	9.7	2000.3-2006
NOUM	Noumea	France	-22.3	166.4	1998.2-2006
NYA1	Ny-Alesund	Norway	78.9	11.9	1998.3-2006
REYK	Reykjavik	Island	64.2	-22.0	1996.1-2006
RIOG	Rio Grande	Argentina	-53.8	-67.8	1997.7-2006
SANT	Santiago	Chile	-33.2	-70.7	1996.1-2006
STJO	St John's	Canada	47.6	-52.7	1996.1-2006
SYOG	Syowa	Antarctica	-69.00	39.58	1999.4-2006
THTI	Papeete	France	-17.58	-149.62	1998.5-2006
TLSE	Toulouse	France	43.6	1.0	2001.2-2006
USNO	Goldstone	United-states	35.3	-116.8	1997.4-2006

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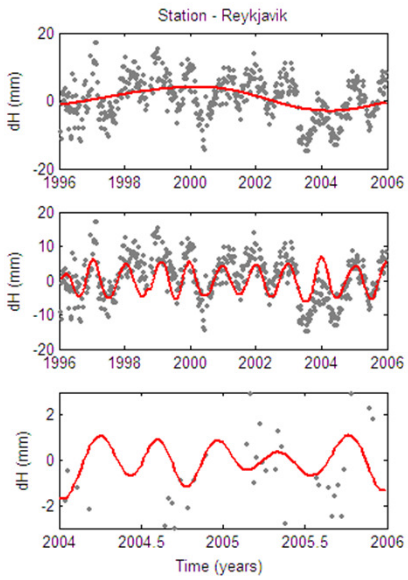
Results and analysis

✓ Using discrete Meyer wavelet :

The approximation signal at level 7
↓
Nonlinear trend

The detail signal at level 5
↓
Annual signal

The detail signal at level 4
↓
Semi-annual signal



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Results and analysis

Amplitudes of annual and semi-annual signals in North, East and Vertical components of studied position stations.

Station	North (mm)		East (mm)		Vertical (mm)	
	Annual signal	Semi annual signal	Annual signal	Semi annual signal	Annual signal	Semi annual signal
EISL	4.2	4.5	1.5	2.7	8.9	7.1
FAIR	1.8	2.3	3.2	2.4	5.6	4.9
GODE	1.9	1.7	2.4	1.1	4.6	4.9
HRAO	3.1	2.2	1.4	2.1	5.6	4.6
METS	0.8	1.3	1.4	1.5	4.0	4.8
NKLG	1.2	1.9	1.3	1.8	4.7	4.1
NOUM	3.3	3.7	0.8	2.1	8.5	5.3
NYA1	1.3	1.0	1.7	1.0	3.9	5.2
REYK	3.2	1.5	1.3	1.2	6.9	4.5
RIOG	2.1	2.4	2.3	2.1	4.5	5.9
SANT	3.3	2.7	2.6	3.1	6.4	6.7
STJO	1.4	1.7	1.6	2.0	3.8	3.8
SYOG	0.9	1.4	0.9	1.3	2.3	3.9
THTI	3.5	2.5	1.6	1.4	3.8	4.6
TLSE	0.6	0.9	1.9	1.1	2.4	3.1
USNO	2.1	1.2	1.7	1.8	3.3	4.5

⇨The annual and semi annual signals have the largest amplitude in the Vertical component which is in the range of 3-7 mm compared to 1-3 mm in the horizontal components.

Results and analysis

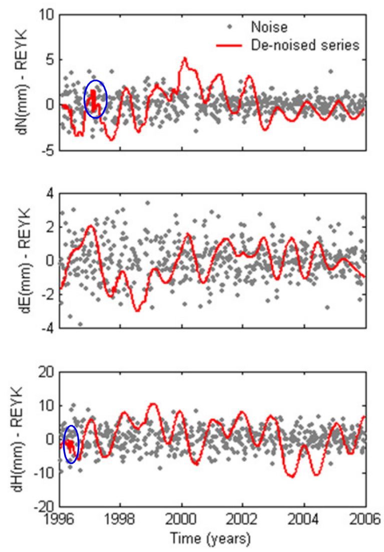
✓ Discrete Meyer wavelet.

✓ Decomposition of the signal at level 4.

✓ VisuShrink method with soft thresholding.

➢ Well filtered and smoothed the signal (De-noised time series) from the noise.

➢ Well identified the fluctuations contained in the studied time series.



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Results and analysis

Station	North [mm]			East [mm]			Vertical [mm]		
	MIN	MAX	STD	MIN	MAX	STD	MIN	MAX	STD
EISL	-9.2	10.0	2.9	-5.1	5.5	1.8	-17.7	17.2	6.1
FAIR	-4.1	3.7	1.3	-3.9	4.2	1.3	-15.2	10.9	3.8
GODE	-3.2	4.2	1.1	-3.9	3.9	1.1	-9.0	12.3	3.2
HRAO	-5.2	6.0	1.7	-4.1	4.7	1.3	-14.6	10.5	3.5
METS	-2.7	3.5	1.0	-4.0	4.3	1.1	-9.0	11.4	3.4
NKLG	-4.3	3.3	1.3	-2.8	2.1	0.9	-16.8	8.2	3.3
NOUM	-6.7	8.0	2.1	-5.0	5.3	1.4	-12.1	12.4	4.1
NYA1	-2.9	2.2	0.8	-3.2	3.5	0.9	-12.3	10.9	3.8
REYK	-3.6	3.7	1.1	-3.8	3.4	1.1	-9.8	9.8	3.2
RIOG	-4.4	4.9	1.6	-5.4	6.8	1.7	-13.7	14.2	4.0
SANT	-5.7	5.4	1.8	-5.2	4.5	1.5	-14.0	10.6	3.5
STJO	-4.1	4.5	1.3	-4.1	4.4	1.1	-12.7	8.1	2.9
THTI	-7.7	8.9	2.2	-4.5	4.1	1.2	-11.7	17.9	4.3
USNO	-3.2	3.1	1.0	-3.2	2.3	1.0	-7.7	10.8	3.1
TLSE	-1.9	1.9	0.6	-1.8	2.0	0.6	-4.8	5.6	2.1
SYOG	-4.0	3.3	1.1	-3.6	3.3	1.2	-11.3	10.8	3.5

⇨The standard deviation (STD) of the noise in the horizontal (North and East) and the Vertical components ranges between 1-2 mm and 2-5 mm, respectively.

⇨The noise level in the Vertical direction is more important compared to the horizontal one.

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Conclusion

The main purpose of this work is to apply the wavelet transform into the analysis of GPS coordinate time series, in order to assess the "noise" which allows to investigate the stability of the stations and on those the "signal" which allows to determine the systematic signals such as trends and seasonal components.

The application of the wavelet transform on weekly solutions of coordinate residuals of 16 well distributed stations, permits to better assess their nonlinear trend and their annual and semi annual signals, and behaves well in the simultaneous identification of the fluctuations contained in the studied time series which are related to parasitic phenomena of observational, instrumental or geophysical origin.

Using the VisuShrink thresholding, based on the mean square error minimization, the obtained results show that the noise level is higher in the Vertical component; it is in the range of 2-5 mm compared to the horizontal components (North and East) which is in the range of 1-2 mm.

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Thank you