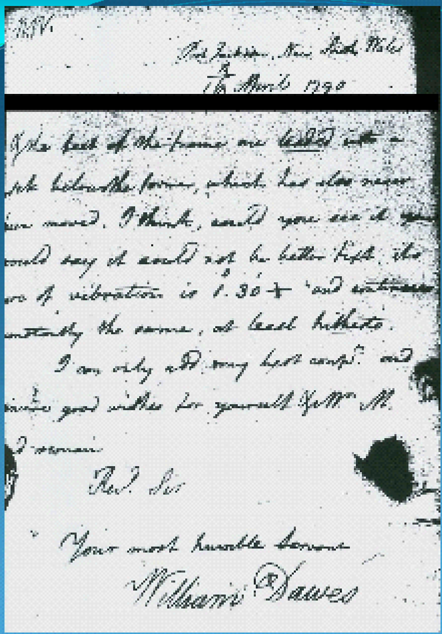
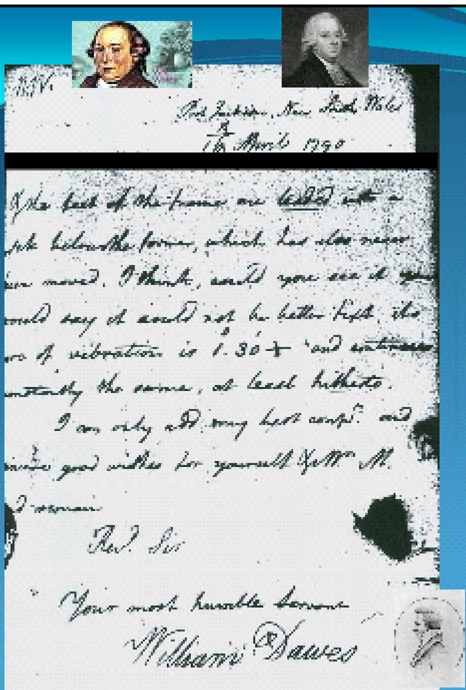


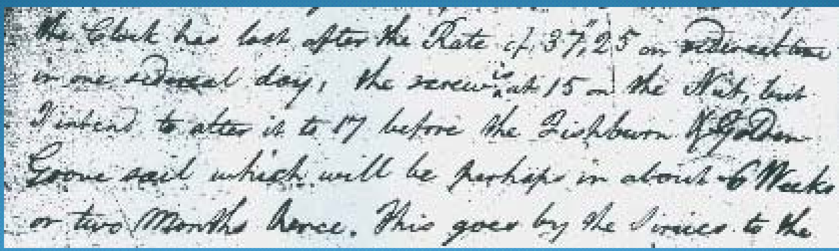
William Dawes' Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER,
Australia


Port Jackson, New South Wales
17th April 1790
Rev. Sir
Your most humble servant
William Dawes

Clock Rate and setting of nut

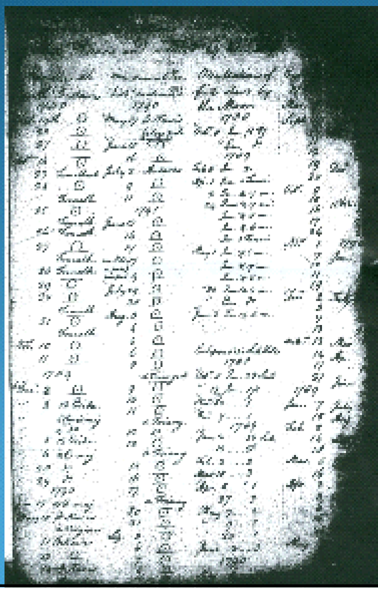
Oct 1, 1788

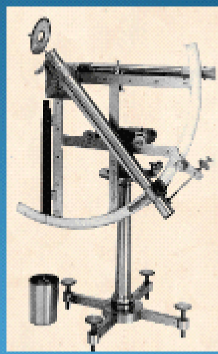

The Clock has lost after the Rate of 37",25 on sidereal time
in one sidereal day, the screw is at 15 on the Nut, but
I intend to alter it to 17 before the Fishburn & Golden
Grove sail which will be perhaps in about 6 Weeks
or two Months hence. This goes by the Sirius to the

William Dawes

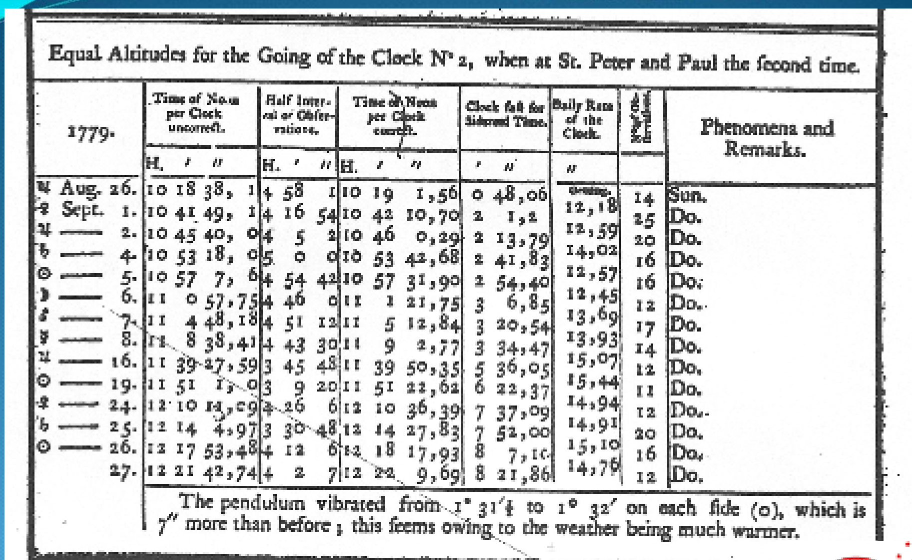
1762 - 1836

Dawes' List of Observations





Gravity Observation by Cook



Equal Altitudes for the Going of the Clock N° 2, when at St. Peter and Paul the fecond time.

1779.	Time of Noon per Clock uncorrect. H. ′ ″	Half Interval of Obfervations. H. ′ ″	Time of Noon per Clock correct. H. ′ ″	Clock flow for Sidereal Time. ′ ″	Daily Rate of the Clock. ″	[illegible]	Phenomena and Remarks.
♃ Aug. 26.	10 18 38, 1	4 58 1	10 19 1,56	0 48,06	gaining.	14	Sun.
♀ Sept. 1.	10 41 49, 1	4 16 54	10 42 10,70	2 1,2	12,18	25	Do.
♃ —— 2.	10 45 40, 0	4 5 2	10 46 0,29	2 13,79	12,59	20	Do.
♄ —— 4.	10 53 18, 0	5 0 0	10 53 42,68	2 41,83	14,02	16	Do.
☉ —— 5.	10 57 7, 6	4 54 42	10 57 31,90	2 54,40	12,57	16	Do.
☽ —— 6.	11 0 57,75	4 46 0	11 1 21,75	3 6,85	12,45	12	Do.
♂ —— 7.	11 4 48,18	4 51 12	11 5 12,84	3 20,54	13,69	17	Do.
☿ —— 8.	11 8 38,41	4 43 30	11 9 2,77	3 34,47	13,93	14	Do.
♃ —— 16.	11 39 27,59	3 45 48	11 39 50,35	5 36,05	15,07	12	Do.
☉ —— 19.	11 51 13, 0	3 9 20	11 51 22,62	6 22,37	15,44	11	Do.
♀ —— 24.	12 10 14,09	4 26 6	12 10 36,39	7 37,09	14,94	12	Do.
♄ —— 25.	12 14 4,97	3 30 48	12 14 27,83	7 52,00	14,91	20	Do.
☉ —— 26.	12 17 53,48	4 12 6	12 18 17,93	8 7,10	15,10	16	Do.
27.	12 21 42,74	4 2 7	12 22 9,69	8 21,86	14,76	12	Do.

The pendulum vibrated from 1° 31′¼ to 1° 32′ on each fide (o), which is 7″ more than before; this feems owing to the weather being much warmer.

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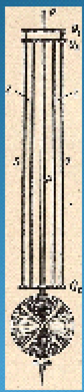
Astronomical Observations [made at ...] for determining the going of the Clock

sent thither by the Royal Society

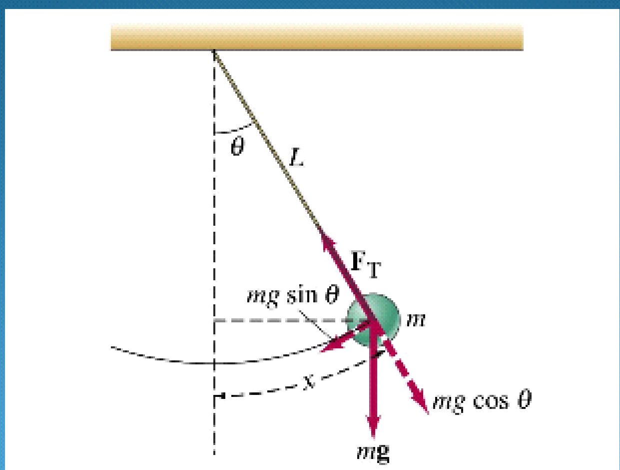
in order to find the Difference of Gravity between the Royal Observatory at Greenwich and the place where the Clock was set up [.....].

The Shelton Astronomical Regulator Clock

Error Budget



Linearity issue

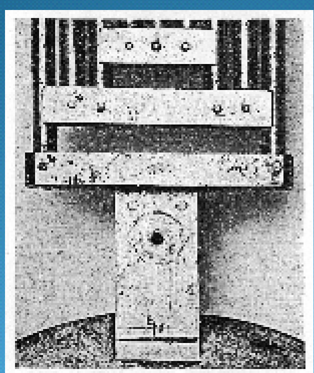


Pendulum equation

$$T = 2\pi \sqrt{\frac{L}{g}} \left(1 + \frac{1}{2^2} \sin^2 \frac{\theta_M}{2} + \frac{1}{2^2} \frac{3^2}{4^2} \sin^4 \frac{\theta_M}{2} + \cdots \right)$$

$$g = (4\pi^2 L/T^2) * (1 + \Delta)^2$$

Setting the Pendulum length



A Regulator Nut

Solve for Pendulum Lengths with EGM 2008

Mean 39.111₂ inches, Std dev of mean 0.002 inch (51 microns)

The Resolution, Captain Cook and William Wales													L Solved
Places	Date	Clock B gains on sidereal time		Latitude		Lat D.dd	Longitude		EGM 2008 normal gravity	Add Gravity Anomaly	clock rate sec	Obsv'd swing arc D.d	with obsvd arc from vert
Greenwich	March 1772	0	5.03	51	28.67N	51.48	0	0	9.81197137	0.00010	5.03	1.88	39.1308
Madeira	July 1772	0	-36.60	32	33.5N	32.56	17	11.25W	9.79525478	0.00015	-36.6	1.67	39.1031
Cape of Good Hope	Nov 1772	-1	15.48	33	55.25S	33.92	18	23.25E	9.79638135	0.00010	-75.48	1.63	39.1428
Dusky Bay	April 1773	0	4.07	45	47.55	45.79	166	18E	9.80586841	0.00000	4.07	1.58	39.1124
Point Venus	August 1773	-1	28.42	17	29.55	17.49	210	25.5E	9.78494586	0.00000	-88.42	1.65	39.1083
Queen Charlotte's Sound	Dec 1773	0	-21.12	41	6S	41.1	174	18.5E	9.80263555	0.00000	-21.12	1.63	39.1189
Point Venus	May 1774	-1	22.64	17	29.55	17.49	210	25.5E	9.78494586	0.00000	-82.64	1.58	39.1034
Queen Charlotte's Sound	Oct 1774	0	-15.58	41	6S	41.1	174	18.5E	9.80263555	0.00020	-15.58	1.63	39.1139
Tierra del Fuego	Dec 1774	0	36.52	55	22S	55.37	70	1.33W	9.81534269	0.00015	36.52	1.63	39.1172
Cape of Good Hope	April 1775	0	-42.21	33	55.25	33.92	18	23.25	9.79638135	0.00010	-42.21	1.65	39.1125

Length change for one rev of nut

From length ratio to length difference:

$$L_0/L_1 = T_0^2 / T_1^2$$

$$L_1 - L_0 = L_1 - (L_1 * T_0^2 / T_1^2)$$

$$= L_1(1 - T_0^2 / T_1^2)$$

Estimate L with g/π^2

Result: One revolution of nut is 0.0258 inch of length change for the pendulum

London pendulum length or not?

From normal gravity ratio to pendulum period ratio:

$$T_L^2 / T_S^2 = g_S / g_L$$

$$T_L = T_S \sqrt{(g_S / g_L)}$$

Conclusion: William Dawes pendulum is too short by one revolution of the regulator nut.

Result:

Sydney Cove gravity value in 1788

979.705 gal

The Appropriate Pendulum Lengths

The Resolution, Cook and Wales			
Places	Date	Adopted pendulum length	1775 Gravity value
Greenwich	March 1772	39.137	9.81364
Madeira	July 1772	39.111	9.79739
Cape of Good Hope	Nov 1772	39.137	9.79504
Dusky Bay	April 1773	39.111	9.80651
Point Venus	August 1773	39.111	9.78561
Queen Charlotte's Sound	Dec 1773	39.111	9.80086
Point Venus	May 1774	39.111	9.78684
Queen Charlotte's Sound	Oct 1774	39.111	9.80211
Tierra del Fuego	Dec 1774	39.111	9.81394
Cape of Good Hope	April 1775	39.111	9.79609

William Dawes' Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER, Australia

THE END

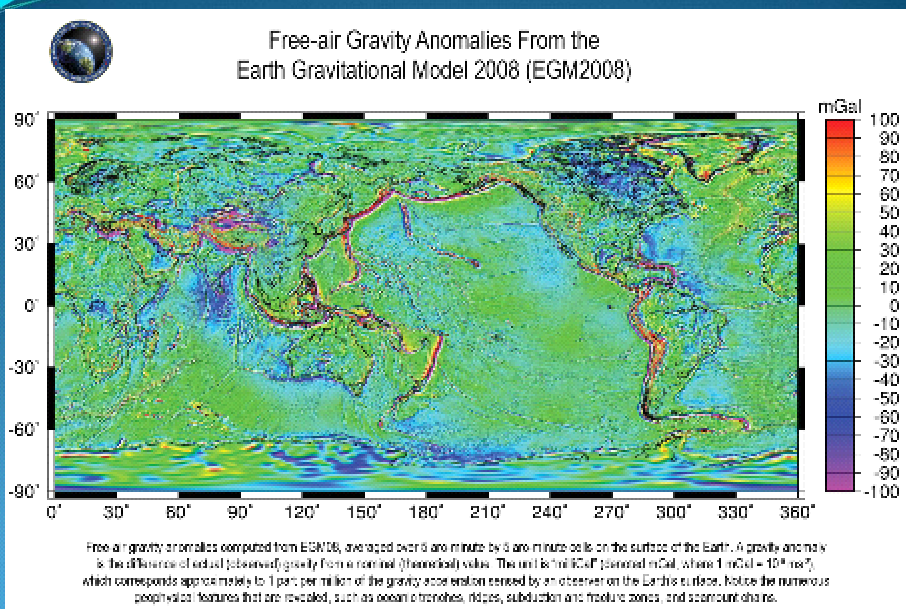
- Supporting background information

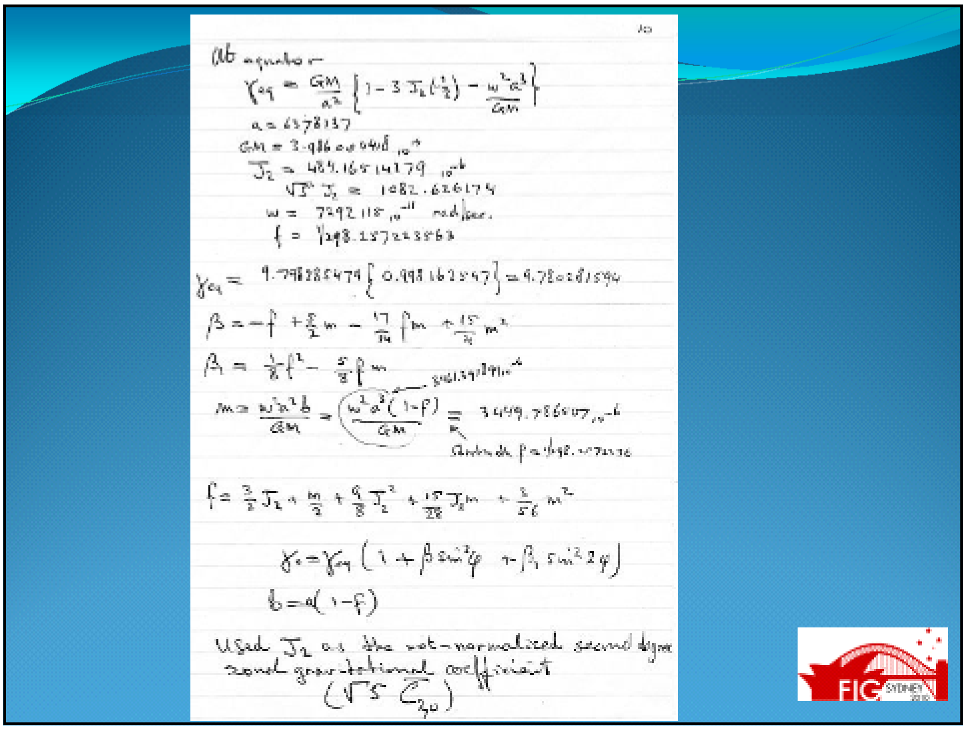
A Six-figure relative gravity value in 1768

Page 405, Philosophical Transactions of the Royal Society, 1771

July 18, 1768. Therefore the force of gravity at Greenwich is to that at King George's Iſland, as 1000000 to 997075. N. M.

EGM 2008 Anomalies





At equator

$$\gamma_{eq} = \frac{GM}{a^2}\left\{1 - 3J_2\left(\tfrac{1}{2}\right) - \frac{\omega^2 a^3}{GM}\right\}$$

$a = 6378137$

$GM = 3.986004418 \times 10^{14}$

$J_2 = 484.16514179 \times 10^{-6}$

$\sqrt{5}\, J_2 = 1082.626174$

$\omega = 7292115 \times 10^{-11}$ rad/sec.

$f = 1/298.257223563$

$$\gamma_{eq} = 9.798285479\,\{0.998162547\} = 9.780281594$$

$$\beta = -f + \frac{5}{2}m - \frac{17}{14}fm + \frac{15}{4}m^2$$

$$\beta_1 = \frac{1}{8}f^2 - \frac{5}{8}fm$$

$$m = \frac{\omega^2 a^2 b}{GM} = \frac{\omega^2 a^3(1-f)}{GM} = 3449.786507 \times 10^{-6}$$

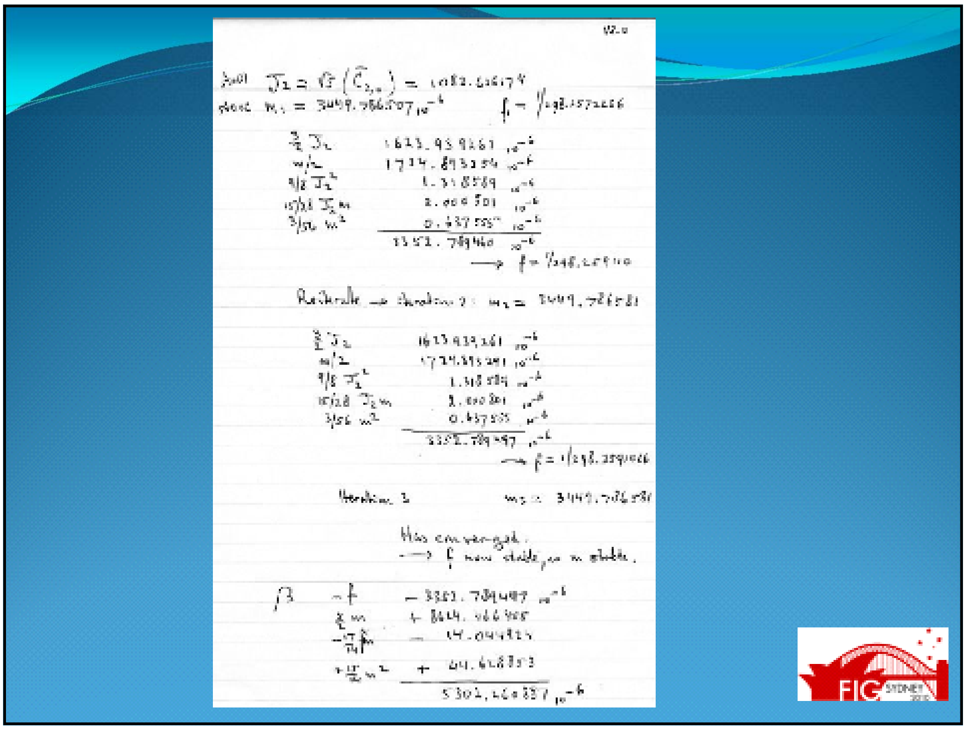
Started with $f = 1/298.257223$

$$f = \frac{3}{2}J_2 + \frac{m}{2} + \frac{9}{8}J_2^2 + \frac{15}{28}J_2 m + \frac{3}{56}m^2$$

$$\gamma_0 = \gamma_{eq}\left(1 + \beta \sin^2\varphi + \beta_1 \sin^2 2\varphi\right)$$

$$b = a(1-f)$$

Used J_2 as the not-normalized second degree zonal gravitational coefficient ($\sqrt{5}\,\bar{C}_{2,0}$)



$J_2 = \sqrt{5}\left(\bar{C}_{2,0}\right) = 1082.626174$

$m_1 = 3449.786507 \times 10^{-6}$ $\quad f_1 = 1/298.2572236$

$\frac{3}{2}J_2$	$1623.939261 \times 10^{-6}$
$m/2$	$1724.893254 \times 10^{-6}$
$9/8\, J_2^2$	1.318589×10^{-6}
$15/28\, J_2 m$	2.000501×10^{-6}
$3/56\, m^2$	0.637555×10^{-6}
	$3352.789460 \times 10^{-6}$

$\longrightarrow f = 1/298.259110$

Recalculate → iteration 2: $m_2 = 3449.786581$

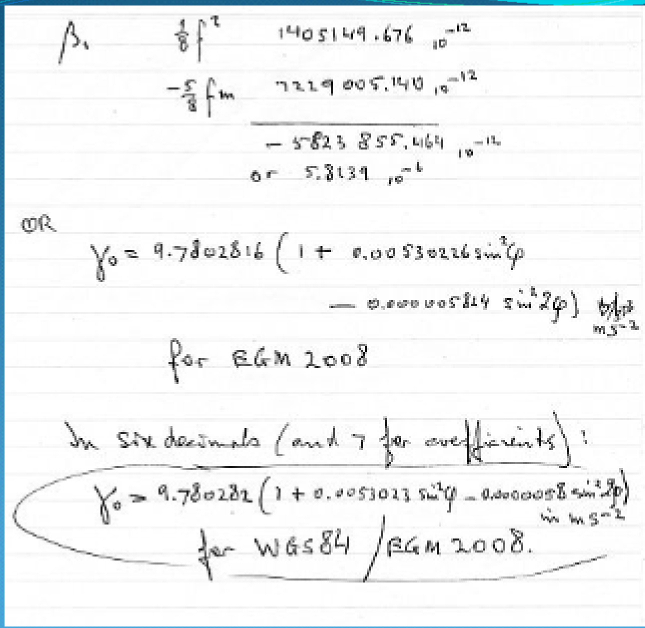
$\frac{3}{2}J_2$	$1623.939261 \times 10^{-6}$
$m/2$	$1724.893291 \times 10^{-6}$
$9/8\, J_2^2$	1.318589×10^{-6}
$15/28\, J_2 m$	2.000501×10^{-6}
$3/56\, m^2$	0.637555×10^{-6}
	$3352.789497 \times 10^{-6}$

$\longrightarrow f = 1/298.2591066$

Iteration 3 $\quad m_3 = 3449.786581$

Has converged.
→ f now stable, so m stable.

β	$-f$	$-3352.789497 \times 10^{-6}$
	$\frac{5}{2}m$	$+8624.466455$
	$-\frac{17}{14}fm$	-14.044924
	$+\frac{15}{4}m^2$	$+44.628353$
		$5302.260387 \times 10^{-6}$



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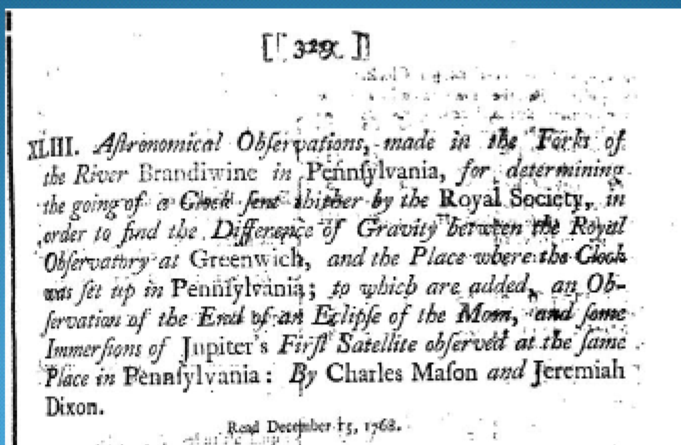

[329.]

XLIII. *Astronomical Observations, made in the Forks of the River* Brandiwine *in* Pennsylvania, *for determining the going of a Clock sent thither by the* Royal Society, *in order to find the Difference of Gravity between the Royal Observatory at* Greenwich, *and the Place where the Clock was set up in* Pennsylvania; *to which are added, an Observation of the End of an Eclipse of the Moon, and some Immersions of* Jupiter's *First Satellite observed at the same Place in* Pennsylvania: *By* Charles Mason *and* Jeremiah Dixon.

Read December 15, 1768.