The Industrial Real Estate Market in Krakow and Appraising the Market Value of Industrial Properties*

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Key words: industrial properties appraisal, cracowian real estate market, market value, the system of conditional equations.

SUMMARY

The article presents the analysis of industrial properties in Krakow in the range of office, warehouses and production spaces. There are raised basic issues that occur during investment process like small number of land development plans, not suitable developed technical infrastructure, difficulties and time-consuming terms accompanying the investment process. The next part of the article shows the examples of rental rates and offered prices for commercial properties that allow calculating an average capitalization rates for various types of commercial spaces in Krakow. The industrial properties, because of their variety and complexity, are the subject of special algorithms of market value appraisal. Using the system of conditional equations in valuation, there is possible to consider all types and quantity of particular properties components and additionally their attributes values. In a result we receive the most probable market value of valuated industrial property.

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1. INTRODUCTION

Analyzing the industrial real estate market in Krakow, there's noticed high growth of investment interests in this filed of economy. The biggest influence on that situation is attributed to Poland's accession to European Union, which contributed to barter growth, foreign capital inflow and various fields of market development.
Most of studies concern changes taking place in residential market, as well as apartments, built-up areas and un-built areas, what evident from the biggest group of participants on this market. Worth of attention is also commercial and industrial real estate market, because significant changes occurred also on this area in the range of high rate of changes, especially demand for office, warehouse and production space growth.
Krakow, because of his architectonic and antique values, is not on the list of top most industrial cities in Poland. This part of economy developed mainly in postwar time, when the one of the biggest investments in the scale of the country, was called that time Lenin's Steelworks building. Among various branches of industry, that occur on Krakow area and its region, the most popular are: metallurgical, textile industry, food industry, chemical, pharmaceutical, electromechanical and engineering industry.

2. LOCATION ANALYSIS

The industrial buildings location in Krakow is strongly dependent from city's nature, which is first of all the cultural, scientific and sacred centre. All arising investments have to urbanized harmonize with the form of surrounding space. Therefore only few production plants which are situated in the central part of the city. These are objects, which were built dozen years ago, like: Tobacco Plant, currently "Philip Morris", the old historic brewery, situated on more than two hectares land, cracovian power station located on Kazimierz area, currently not exploited in electric energy and heat production
.
One of the Krakow's industrial areas, which gather the most of industrial buildings, is Nowa Huta region, whose advantage is the largest amount of un-built and undeveloped land comparing to other city's districts, deficiency of ecological restrictions and good transport connection with cities from east and north site. Important in this matter is, that these areas, are located in greater distance from the city centre and in unattractive surroundings of steelworks so they are not interesting for residential properties purchasers. The next industrial investment regions are: Rybitwy, Czyzyny, Pychowice, land located along Zakopianska street and Balice.

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In 1997 was formed Cracovian Technology Park, one of six currently running in Poland, whose purpose is creating suitable economic and organizational conditions for local and foreign investors, through tax allowances and access to educated academic staff and qualified experts. The Cracovian Technology Park, being a special economic zone, contains nine sub-zones, whose total area equals about 300 hectares, including around 87 hectares on Krakow territory. The basic purpose is development of advanced technology industry's sector, especially electronics, information technology, telecommunications etc. On the Nowa Huta's region, in Branice sub-zone, there are offered land for industrial plants development. The Czyzyny region, situated in the east part of the city, is reserved for scientific-research and office buildings, considering urban built-up nature.

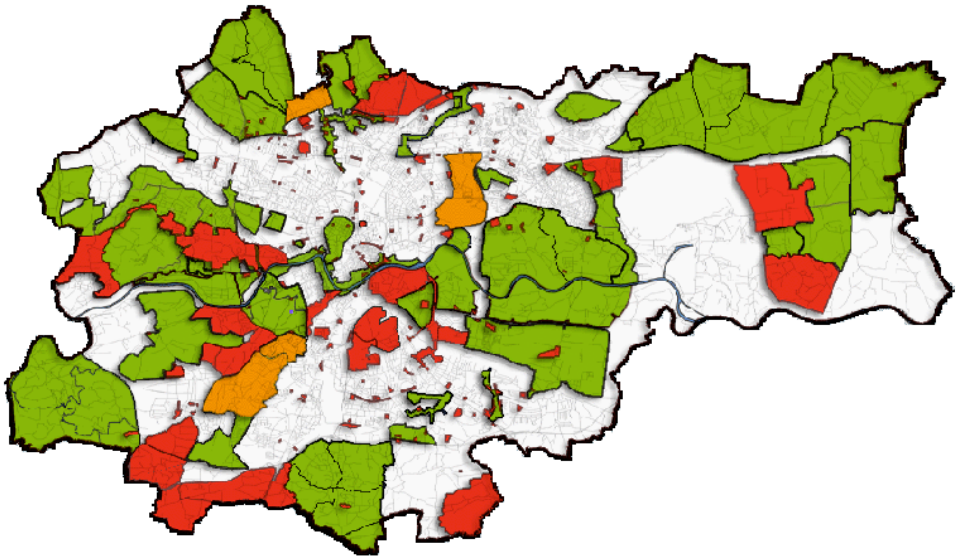
A status of the special economic zone has also the office building in Bronowice, that's why investors can count on public help. The last element, assigned to Cracovian Technology Park, located in the city borders, is Pychowice area, which is situated in south-west part of Krakow. This region is planned for realization of modern industrial production buildings and services.
One point of Krakow strategy concerns necessity the formation of proper condition for investors, which enable investments development and the growth of competitiveness in this range in relation to other polish regions. However there are some problems, which make difficult the realization planned investments. We can classify here most of all the lack of up-to-date site developments plans, insufficient technical infrastructure developed, specific nature of the city, employment costs, public help in relation to tax allowances and simplifications in obtaining permits and necessary documents for beginning investment.

3. THE LOCAL SITE DEVELOPMENT PLANS

The basis of investment planning is the choice of suitable location, which could guarantee satisfying access to technical infrastructure, adequate to projected task needs. However the problem appears already with industrial land searching. Small amount of valid land development plans make significant limitations during acceptable investment areas establishing. The decision of zoning regulations obtainment, in this situation, takes even one year in Krakow, what often discourages potential investors to take a risk connected with waiting time for that decision and additionally the lack of reliability about its settlements.

Possession of valid land development plans in Krakow is the ground for optimal land development, order preservation and cultural assets protection. The objective of plans passing, among other things, is speeding up the processes of activity coordination in spatial development, but in harmony with respect for monuments and environmental protection rules
.
At the end of 2006, valid land development plans were covering the 3 505 hectares, what is 10.7 % of total city's area and 62 plans had the planning procedure started. According to situation in January 2008, plans covered the region of 4 419 hectares, which is about 13% of total city's area, whereas the drafting plans were relating to 13 420 hectares area, which is 38 % city's area. *The map of local land development plans for Krakow* (picture 1) presents the regions of valid or drafting plans. Using green color are marked regions where the plans are

drafting, red color - regions with valid plans and orange color - regions, where the plans are passed but resolutions are not legally valid yet.



Picture 1. The map of land development plans in Krakow

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The source: City Hall of Krakow, Spatial Development Office

According to data collected by City Hall of Krakow, in 2006 there were given 63 planning permissions for industrial buildings, what shows the much bigger interest comparing to last years, when 60 decisions were given in 2005 and 8 decisions in 2004. The amount of given building permissions were 13 in 2006, while in earlier years it was 80 in year 2005 and 6 in year 2004. It shows, that the biggest growth of industrial investments was in 2005, but the interest among real estate market participants, definite by the amount of given planning permissions, was significant in year 2006, what can have a result in great number of realized industrial buildings in 2007.

Most of regions, where industrial investments are already executed or lands located in special economic zone borders have land development plans. The problem related to insufficient number of local plans has influence also for prices of industrial lands. The difficulty is in value settlement when there's no specified land use of sold properties. There's only a possibility of relation to the study on the conditions of the district development, which settlements are binding during local plans preparing, relation to cadastre database, including information about sort of land use of particular plot and the area of property which is the subject of appraisal. It seems logical that a person who's buying a land of great area, over several hectares, in the region where are industrial buildings, in districts which are typically developed in that direction, won't be using it for residential purposes because of environmental limitations or unattractive neighborhood, and much more certain is that this land won't have agricultural use, specially if it's located in the city's borders. We could come to conclusion that in spite of local plan's lack and accepting the situation showed in cadastre, where particular land is an agricultural, it can be used in the future for industrial development.

In Krakow, the biggest un-built industrial properties possible for industrial gaining have around 20-30 hectares of total areas. Even lands, offered by Cracowian Technology Park are from 14 to 31 hectares. That is significant barrier for investors, who have plans of future expansion of their building and activity progress.

4. THE TECHNICAL INFRACTURE AND PUBLIC HELP

Despite the fact, that the city has full technical infrastructural equipment as water pipe and sewerage network, telecom network, electrical power supply, gas network, the regions located on the outskirts of the city, where are large plots, there's not always an access to that kind of technical infrastructure.

The big part plays most of all transport network and its quality. Location of industrial plants along main roads, airports or railways reduces costs of material and products transport and contribute to time saving and are profitable for clients. Therefore, more and more investors move to suburban areas, at a distance to several kilometers from the city, where the better and quicker access, located near highways is.

Because of fast economic development and growing transport needs, all the time there are hold works in different parts of the city relating to new roads, viaducts, overpass or ring ways construction, which would improve traffic in the city. The solution is also ring ways construction, around the city, which make possible transit of bigger vehicles without necessity going through the city centre.

Cities, which are interested in high class investors possessing, preparing the public's help plans, which could be competitive in relation to offers of other cities or regions. The most often founding problem during investments realization is getting proper documentations in administrative offices in suitable period of time. Unfortunately, too complicated rules of proceeding by economic entities service and lengthy in time law procedures has negative influence on new projects development.

The important aspect in the range of public help is tax allowances. Investors, in the regions of Technology Park, obtain tax allowances relating to income tax from physical person or income tax from legal person due to new workplaces and investment's costs

.

5. THE MARKET RESEARCH IN RELATION TO RENTAL RATES OF OFFICE, WAREHOUSES AND PRODUCTION SPACES

In the industrial properties appraisal very important is the market analysis in reference to unit rental rates due to rent or lease various kinds of surfaces depending from type of valuated properties components, including warehouse, office, productive, social spaces or storage yards. To present mutual relationship between unit rental rates and unit offered prices for particular types of surfaces, there are showed below their averages values, which occur on local real estate market in each of city's districts.

Table no.1 Commercial premises – exemplary of lease offers

no	Location	Property's description	Unit rent [PLN/m^2]
PODGORZE			
1	Al.Pokoju	4 usable rooms, kitchenette, bathroom, usable area of premises: 78m^2.	53
2	Zakopianska	The offer of office space in new building office building of total office area: 1200m^2.	36
3	Plac Bohaterow Getta	The offer of office space in new building office building of total office area: 800m^2.	55
4	Prokocim	Commercial premise situated on ground floor, including 3 rooms, usable area: 98m^2.	31
5	Rybitwy	Office premise on first floor, high standard, usable area: form 50 to 700m^2.	40
6	Borek Fałecki	Office spaces in new building, with high standard, total usable area: 1300 m^2.	45
NOWA HUTA			
1	Centralna	Premise located on second floor, after general overhaul, including 4 rooms, usable area: 70m^2.	29
2	Fatimska	Premise including 4 rooms, usable area: 80m^2.	18
3	Czyzyny	Office premise with high standard, usable area: 70m^2.	22
4	Os.Strusia	Commercial premise, situated on second floor of office block, usable area: 80m^2.	45
KROWODRZA			
1	Wybickiego	Office premise with high standard and total usable area: 750m^2.	55
2	Prądnicka	Office promise situated on loft, including 3 rooms, board-room, kitchenette, 3 bathrooms, mezzanine, with high standard, usable area: 200m^2.	51
3	Wrocławska	Premise including 4 rooms, situated on two levels: ground floor and basement storey, high standard, usable area: 100m^2.	50
4	Konarskiego	Premise situated on ground floor of tenement house, including 4 rooms, with high standard, usable area: 70m^2.	45
5	Kazimierza Wielkiego	Premise situated in new office block, with high standard and usable area: 150m^2.	45
6	Al.Słowackiego	Premise situated on third floor prestigious office block, high standard, usable area: 350m^2.	80
7	Krowodrza	Premise including 7 rooms, usable area: 219m^2.	40

Table no.2 Commercial premises – exemplary offers for sale

no	Location	Property's description	Price [PLN/m^2]
PODGORZE			
1	Ruczaj	Commercial premise situated on ground floor, high standard, usable area: 31m^2.	7 097
2	Podwawelskie	Premise situated in the series of commercial premises, including one room and bathroom, usable area: 27m^2.	6 481
3	Kalwaryjska	Office premise in tenement house, usable area: 120m^2.	6 375
4	Mitery	Office premise, usable area: 45m^2.	6 667
5	Kalwaryjska	Premise situated on first floor – annex, in three-storied tenement house, usable area: 97m^2.	6 216
NOWA HUTA			
1	Bienczyce	Commercial premise, usable area: 52m^2.	4 038
2	Kombatantow	Premise situated in commercial building, on first floor, usable area: 300m^2.	5 833
3	Bienczyce	Premise located in new commercial complex, on first floor, usable area: 80m^2.	3 563
KROWODRZA			
1	Bronowicka	Premise situated on ground floor of residential building from 2007, high standard, usable area: 65m^2.	9 000
2	Balicka	Premise situated on high ground floor, with terrace, after overhaul, usable area: 51m^2.	7 451
3	Rusznikarska	Premise located on ground floor, high standard, usable area: 82m^2.	6 707
4	Rusznikarska	Premise situated on ground floor, usable area: 140m^2.	7 857
5	Krowodrza	Premise situated on ground floor of residential building, including 2 rooms and 3 bathrooms, usable area: 88m^2.	6 500

The Srodmiescie district was omitted in the analysis, because of its specific nature, not related to office spaces being in industrial properties. The largest number of commercial premise's rent offers, from among three researching districts, is registered in Krowodrza. It is region that is very popular among people renting office spaces because of location nearby city centre and many new buildings of commercial function, with high standard and good excess for clients. The average unit rate of rent is 52 PLN/m^2 (with standard deviation 13 PLN/m^2) and average unit offered price is 7 503 PLN/m^2 (with standard deviation 1 001 PLN/m^2). Great interest is focused also on premises in Podgorze district, especially along main roads, surrounded by premises with similar function and easy transport access. The average unit rate of rent is 43 PLN/m^2 (with standard deviation 9 PLN/m^2) and average unit offered price is 6 567 PLN/m^2 (with standard deviation 338 PLN/m^2). The smallest demand for office premises are in Nowa Huta region, where mostly buildings of productive and warehouse function are located. Large distance from city centre and bigger commercial buildings has influence on lower rates of rent, where the average rental rate if 28 PLN/m^2 (with standard deviation 12 PLN/m^2) and average unit offered price for similar commercial premises is 4 478 PLN/m^2 (with standard deviation 1 197 PLN/m^2).

Basing on the results presented below we can calculate, that average return rate for commercial premises, for all analyzed districts, is 0.08.
In further part there was made an analysis of unit rental rates and average unit offered prices for warehouse and productive spaces. Researches are related to Krowodrza, Podgorze and Nowa Huta districts.

Table no.3 Warehouse and productive spaces – exemplary of lease offers.

no	**Location**	**Property's description**	**Unit rent** [PLN/m^2]
PODGORZE			
1	Wielicka	Premise situated on ground floor, in front of the building is a parking, usable area: 250m^2.	28
2	Konopnickiej	Warehouse premise, including one room, kitchenette and bathroom, usable area: 150m^2.	30
3	Przemyslowa	Warehouse premise, with parking for trucks and ramp, usable area: 381m^2.	31
4	Wielicka	Warehouse premise, with parking in front of the building, usable area: 300m^2.	27
NOWA HUTA			
1	Nowa Huta	3 warehouse rooms, usable area: 280m^2.	15
2	Makuszynskiego	Single-storied building of warehouse-productive function, usable area: 1670m^2.	20
3	Makuszynskiego	Warehouse room, usable area: 170m^2.	20
4	Nowa Huta	Warehouse-productive premise, usable area: 200m^2.	18
5	Nowa Huta, centre	Free-standing building with productive function, usable area of 500m^2.	24
6	Centralna	Warehouse premise, usable area: 450m^2.	20
KROWODRZA			
1	Kuznicy Kollatajowskiej	Warehouse premise, usable area: 148m^2.	20
2	Kuznicy Kollatajowskiej	Warehouse including one room, usable area: 210m^2.	25
3	Imbramowska	Warehouse premise, usable area: 530m^2.	30
4	Bronowice	Warehouse premise with its own parking and a uploading platform, usable area: 75m^2.	39
5	Krowodrza	Warehouse premise, usable area: 130m^2.	28

Table no.4 Warehouse and productive spaces – exemplary offers for sale.

no	Location	Property's description	Price [PLN/m^2]
PODGORZE			
1	Rondo Mateczne	Warehouse premise situated in basement storey, usable area: 43m^2.	4 000
2	Rybitwy	Warehouse building, usable area: 810m^2.	3 333
3	Wielicka	Productive premise, usable area: 600m^2.	2 083
4	Teligi	Warehouse building, usable area: 300m^2.	2 767
NOWA HUTA			
1	Mlodosci	Warehouse premise situated in basement storey, usable area: 213m^2.	1 500
2	Piastow	The building of productive function in very good condition, big parking for trucks, usable area: 1750m^2.	3 714
3	Klasztorna	Warehouse premise in basement storey, usable area:130m^2.:	2 500
KROWODRZA			
1	Filtrowa	Warehouse premise, usable area: 200m^2.	3 500
2	Wroclawska	productive premise, usable area: 205m^2.	4 800
3	Bronowice	Warehouse, usable area: 180m^2.	3 900

The largest number of rental offers of warehouse and productive premises comes from Podgorze, where the average unit rental rate is 29 PLN/m^2 (with standard deviation 2 PLN/m^2) and an average unit offered price is 3 045 PLN/m^2 (with standard deviation 816 PLN/m^2). Nowa Huta distinguish itself by the greatest amount of rental offers in comparison to others regions, but significantly lower amount of offered for sell warehouse or productive properties. An average unit rental rate in this district is 20 PLN/m^2 (with standard deviation 3 PLN/m^2) and an average unit offered price is 2 571 PLN/m^2 (with standard deviation 1109 PLN/m^2). The least developed, in the range of industrial properties, is Krowodrza, where there can be found single offers of rental and sales that type of spaces. An average rental rate is 28 PLN/m^2 (with standard deviation 7 PLN/m^2) and an average unit offered price is 4 067 PLN/m^2 (with standard deviation 666 PLN/m^2).
Basing on that data we can conclude that an average return rate in Podgorze region is 0.11, in Nowa Huta: 0.09 and in Krowodrza district is 0.08.

6. APPRAISING MARKET VALUE OF INDUSTRIAL PROPERTIES

6.1.Introduction

Industrial properties are specific type of properties because of great variety of their components like land, buildings with various function of use including: warehouse, office, productive, social, warehouse halls, storage yards, uploading platforms, machines and facilities permanently attached to the ground, sheds, silos, containers, roads, pavements and small architecture buildings. The number of transaction, relating to industrial properties transfer, is small and prices of sales obtaining in this way contain all property's components, that's why their comparison to valuated property is difficult.

In appraising market value of industrial properties there is a necessity of taking into consideration types and the amount of particular components both comparative properties and valuated property.

Within the framework of doctoral thesis there will be elaborated an algorithm which make possible the comparative analysis of particular components unit prices of sold properties and their transaction prices with valuated property's components, based on restrictive statistical models

.

Each of transactions prices of industrial properties will be written as a sum of ratios of components areas and their price indicators and ratios of market attributes values and their importance coefficients. The estimation of the most probable price indicators of property's components and importance coefficients will be realized by the principle of the smallest sum of squares of deviations to approximate values of these parameters.

Price indicators will be estimated on the base of market research in the range of unit rental rates. In the result there will be calculated the most probable price indicators of particular property's components and importance coefficients.

The geometrical parameters of industrial properties are: the area of land property , usable areas of particular types of buildings, the usable area and cubature of warehouses and productive halls, the usable and cubature of warehouse sheds, usable cubature of silos and containers, lengths and load capacities of gantries, load capacity of unloading platforms, the areas of storage yards, lengths and the amount of ground territorial development, the area of roads and pavements and the range of small architecture. Additionally there should be considered the industrial properties attributes like: location, transport access, the parcels size, buildings technical conditions.

For characteristics of industrial real estate market and their statistical analysis there will be established followed symbols:

- $S_1, S_2, ...S_n$- usable areas showed in [m^2] for particular types of building, storage yards, roads and pavements,
- $V_1, V_2,.....$ - usable cubature showed in [m^3] for particular warehouses and productive halls or containers and silos,
- $L_1, L_2, ...$ - lengths showed in running meters for particular elements of ground territorial development or small architecture,
- $c_{S1}, c_{S2},.....c_{Sn}$- price indicators for particular types of buildings or storage yards, roads and pavements, related to their areas,
- $c_{V1}, c_{V2},.....$- price indicators for particular productive and warehouse halls or containers and silos, related to their cubature,
- $c_{L1}, c_{L2},.....$- price indicators for particular elements of ground territorial development or small architecture, related to their length,
- $a_1, a_2, ...a_j$- values of attributes used in industrial properties appraisal,
- $k_1, k_2, ...k_j$- importance coefficients of attributes used in industrial properties appraisal,
- $C_1, C_2,.....$- total transaction prices for industrial real estates,

6.2.Function conditions for transaction prices.

As it was mentioned above, the number of transaction, related to this type of real estate, is much lower from the number of price indicators and attributes influence on property value, that's why transaction price should comply with some function conditions, which consider geometrical parameters and their market attributes.

To statistical analysis it is assumed, that each transaction price for industrial property should be linear function of ratios of areas, cubature or lengths and price indicators for particular buildings and facilities of the industrial property and ratios of market attributes values and their importance coefficients. Considering all assumed symbols, the function condition for each of transaction price will have a form:

$$S_1 ? c_{S1} + ... + S_i ? c_{Si} + V_1 ? c_{V1} + ... + L_1 ? c_{L1} + ... + a_1 ? k_1 + ... + a_j ? k_j = C_T \qquad (1)$$

If the number of transaction prices is bigger from unity, than there should be considered the system of conditional equations, where the estimated parameters are the price indicators and the importance coefficients.

Approximated values of price indicators for particular components of industrial properties will be estimated on the base of analysis of unit rental rates for particular types of spaces as office, warehouse, productive, social. For properties, where there is no possibility of price indicators estimation, there should be defined the costs of components reconstruction.

Approximated values of price indicators will be calculated using the most probable capitalization rate, defined by formula:

$$\hat{R}_i = \frac{\hat{d}_i}{\hat{c}_i} \qquad (2)$$

where $(\hat{d}_i)$ means average unit rental value, $(\hat{c}_i)$ means average unit offered price value.

Estimated values of price indicators $(\tilde{c}_i)$ for considered industrial properties components can be calculated using their predicted unit rental rents (d_i) and calculated capitalization rate:

$$\tilde{c}_i = \frac{d_i}{\hat{R}_i} \qquad (3)$$

Based on conducted analysis in point 6 of this article, the most probable capitalization rates of the industrial real estate market in Krakow, amount to 0.08 for office and commercial premises in all researched districts in Krakow. For warehouses and productive spaces, the capitalization rates are: 0.08 for Krowodrza region, 0.09 for Nowa Huta and 0.11 for Podgorze region.

With reference to properties components, for which there's no possibility of price indicators establishing, based on rental rates analysis, these indicators will be set using the reconstructions costs. The indicator of reconstruction value (w_{WO}), set on the base of the indicatory technique, will correspond to price indicator particular element, $\tilde{c}_i = w_{iWO}$.

The system of conditional equations can be written in the following form:

$$S_1 ? (\tilde{c}_{S1} + ä_{S1}) + ... + S_i ? (\tilde{c}_{Si} + ä_{Si}) + V_1 ? (\tilde{c}_{V1} + ä_{V1}) + ...$$
$$+L_1 ? (\tilde{c}_{L1} + ä_{L1}) + ... + a_1 ? k_1 + ... + a_j ? k_j = C_T \qquad (4)$$

After made-up transformation we get the following system of equations:

$$S_1 ? \ddot{a}_{S1} + ... + S_i ? \ddot{a}_{Si} + V_1 ? \ddot{a}_{V1} + ... + L_1 ? \ddot{a}_{L1} + ... + a_1 ? k_1 + ... + a_j ? k_j = \\ = C_T - (S_1 ? \tilde{c}_{S1} + ... + S_i ? \tilde{c}_{Si} + V_1 ? \tilde{c}_{V1} + ... + L_1 ? \tilde{c}_{L1}) \quad (5)$$

6.3.Approximation of price indicators and importance coefficients.

The system of conditional equations, showed in formula no. 5, is subjected of the variation analysis, where as a result are obtaining the estimated random deviations (δ_i) for price indicators and importance coefficients of attributes (k_j). Below are presented following matrix symbols:

$$[S] = \begin{bmatrix} S_{11} & ... & S_{i1} & V_{11} & ... & L_{11} & ... \\ S_{12} & ... & S_{i2} & V_{12} & ... & L_{12} & ... \\ ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... \\ S_{1u} & ... & S_{iu} & V_{1u} & ... & L_{1u} & ... \end{bmatrix} \qquad [\ddot{a}] = \begin{bmatrix} \ddot{a}_{S1} \\ ... \\ \ddot{a}_{Si} \\ \ddot{a}_{V1} \\ ... \\ \ddot{a}_{L1} \\ ... \end{bmatrix} \quad (6)$$

$$[a] = \begin{bmatrix} a_{11} & a_{21} & ... & a_{j1} \\ a_{12} & a_{22} & ... & a_{j2} \\ ... & ... & ... & ... \\ a_{1u} & a_{2u} & ... & a_{ju} \end{bmatrix} \qquad [k] = \begin{bmatrix} k_1 \\ k_2 \\ ... \\ k_j \end{bmatrix} \quad (7)$$

$$[\ddot{A}C] = \begin{bmatrix} C_1 - (S_{11}\tilde{c}_{S1} + ... + S_{i1}\tilde{c}_{Si} + V_{11}\tilde{c}_{V1} + ... + L_{11}\tilde{c}_{L1} + ...) \\ C_2 - (S_{12}\tilde{c}_{S1} + ... + S_{i2}\tilde{c}_{Si} + V_{12}\tilde{c}_{V1} + ... + L_{12}\tilde{c}_{L1} + ...) \\ ..... \\ C_u - (S_{1u}\tilde{c}_{S1} + ... + S_{iu}\tilde{c}_{Si} + V_{1u}\tilde{c}_{V1} + ... + L_{1u}\tilde{c}_{L1} + ...) \end{bmatrix} \quad (8)$$

Considering above formulated matrixes, the system of conditional equations in form (5) can be written in the following matrix's form:

$$[S][\ddot{a}] + [a][k] = [\ddot{A}C] \quad (9)$$

For analysis for random deviations variation and importance coefficients should be formulated the Lagrange's function in the following form:

$$\emptyset = [\ddot{a}]^T [\ddot{a}] + [k]^T [k] + \ddot{e} ? ([S][\ddot{a}] + [a][k] - [\ddot{A}C]) \quad (10)$$

The minimum of that function is the minimum of variation of random deviations and importance coefficients, what is the necessary conditional to solve the system of equations (9). After calculation for function (10) the matrix of molecular derivatives and importance coefficients and compared to zero matrixes we get the formulas for random deviations estimators and importance coefficients.

The approximated random deviations values (δ_i) and importance coefficients (k_j) can be calculated using the formula:

$$\begin{Bmatrix} [ä] \\ [k] \end{Bmatrix} = \left\{ [S]^T [a]^T \right\} \left\{ [S][S]^T + [a][a]^T \right\}^{-1} [ÄC] \qquad (11)$$

The estimated price indicators values equal the sum of their approximated values and calculated random deviations:

$$c_i = \tilde{c}_i + ä_i \qquad (12)$$

The variations from random deviations and importance coefficients is established by the dependence

$$ó_0^2 = \frac{[ÄC]^T \left\{ [S][S]^T + [a][a]^T \right\}^{-1} [ÄC]}{u} \qquad (13)$$

In relation to variation analysis for estimated price indicators and importance coefficients there can be established variation matrix for them, which is presented by formula:

$$\mathrm{Cov} \begin{Bmatrix} [c] \\ [k] \end{Bmatrix} = ó_0^2 \left([I] - \left\{ [S]^T [a]^T \right\} \left\{ [S][S]^T + [a][a]^T \right\}^{-1} \begin{Bmatrix} [S] \\ [a] \end{Bmatrix} \right) \qquad (14)$$

6.4.Appraising market value of industrial properties

If for appraising industrial property there are set geometrical parameters related to usable area, cubature or length of its components, then matrix of parameters can be written in the following form:

$$[\bar{S}] = \left[\bar{S}_1 \; \bar{S}_2 \; ... \bar{S}_i \; \bar{V}_1 ... \bar{L}_1 ... \right] \qquad (15)$$

Values of appraising property can be written in the following matrix of attributes:

$$[\bar{a}] = \left[\bar{a}_1 \; \bar{a}_2 \; ... \; \bar{a}_j \right] \qquad (16)$$

Considering values of approximated price indicators (c_i) and importance coefficients (k_j), the market value of appraising property will be estimated using the formula:

$$WR = \left\{ [\bar{S}][\bar{a}] \right\} ? \begin{Bmatrix} [c] \\ [k] \end{Bmatrix} \qquad (17)$$

Variation of appraised market value for industrial property is calculated with relation to covariance's matrix (14), with following formula:

$$ó^2 (WR) = \left\{ [\bar{S}][\bar{a}] \right\} ? \; \mathrm{Cov} \begin{Bmatrix} [c] \\ [k] \end{Bmatrix} ? \begin{Bmatrix} [\bar{S}]^T \\ [\bar{a}]^T \end{Bmatrix} \qquad (18)$$

6.5.The example of industrial property valuation.

The subject of the appraisal is the developed property – the plot number 577/8 with area of 0,1100 hectares, situated in Wola Filipowska, near Krakow. The appraisal's purpose is the estimation of the market value. There were assigned the following attributes to the objective property:

- location of the property : *good (100)*
- technical condition of buildings: *good (100)*
- the shape of plot: *very good (200)*
- the area of plot – *11 acres*

The components of the property are:

Administrative building

It's the stone not cellared building, with 3 office rooms, toilet and the hall. The total usage area of the building is 66,07 m^2. The technical standard is good.

Productive hall:

The stone construction building. Usage area: 163,64 m^2. The technical standard is good.

Productive hall:

The stone construction building. Usage area: 176,50 m^2. The technical standard is good.

Social - warehouse building:

It's the stone building with good technical standard. There are following rooms in the building:

- office room with usage area of 93,45 m^2
- warehouse with usage area of 51,69 m^2
- the hall with usage area of 120,10 m^2
- warehouse with usage area of 30,76 m^2
- social room with usage area of 32,49 m^2
- boiler room with usage area of 23,96 m^2
- attic storey with area of 202,30 m^2

In the appraisal there were used 4 industrial properties, which are the most similar to valuated property. The information about these properties is showed in the table no. 5.

Table no.5 The base of properties which are used in comparative approach.

lp.	Location of property	The area of the plot [acres]	Location	Technical condition of buildings	The shape of plot	Transaction price [PLN]
1	Mogilany, plot no 326/10, 326/11	14,00	very good (200)	very good (200)	very good (200)	2 677 412
2	Krzeszowice, plot no 1699/10	21,43	good (100)	good (100)	good (100)	1 155 636
3	Kraków, plot no 268/12, 268/19	25,43	very good (200)	good (100)	good (100)	1 905 613
4	Kraków, plot no 181/2, 182/4	113,58	very good (200)	very good (200)	very good (200)	4 390 274
Val. prop.	Wola Filipowska, plot no 577/8	11,00	good (100)	good (100)	very good (200)	

The list of components of the comparable properties.

Transaction number 1 Mogilany, plots no 326/10 and 326/11.

- Transaction price 2 677 412 PLN
- The area of the plot 1 400 m^2
- The administrative building with usage area of 341 m^2 and social – administrative building with usage area of 374 m^2 (total usage area is 715 m^2)

Transaction number 2 Krzeszowice, plot nr 1699/10.

- Transaction price 1 155 636 PLN
- The area of the plot 2 143 m^2
- The service building with usage area of 295 m^2

Transaction number 3 Kraków, plots nr 268/12 and 268/19.

- Transaction price 1 905 613 PLN
- The area of the plot 2 543 m^2
- The warehouse with usage area of 420m^2
- The service building with usage area of 300m^2

Transaction number 4 Kraków, Podgórze, plots nr 181/2 and 182/4

- Transaction price 4 390 274 PLN
- The area of the plot 11 358 m^2
- Warehouse with cubature of 5 872 m^3
- The office part with usage area of 320,94m^2

On the base of presented data related to comparable properties, there can be written the following function conditions:

$$0 \cdot c_{S-M} + 715 \cdot c_{S-B} + 0 \cdot c_{V-P} + 1400 \cdot k_D + 200 \cdot k_P + 200 \cdot k_S + 200 \cdot k_K = 2\,677\,412$$

$$0 \cdot c_{S-M} + 295 \cdot c_{S-B} + 0 \cdot c_{V-P} + 2143 \cdot k_D + 100 \cdot k_P + 100 \cdot k_S + 100 \cdot k_K = 1155\,636$$

$$420 \cdot c_{S-M} + 300 \cdot c_{S-B} + 0 \cdot c_{V-P} + 2543 \cdot k_D + 200 \cdot k_P + 100 \cdot k_S + 100 \cdot k_K = 1\,905\,613$$

$$0 \cdot c_{S-M} + 320{,}94 \cdot c_{S-B} + 5872 \cdot c_{V-P} + 11358 \cdot k_D + 200 \cdot k_P + 200 \cdot k_S + 200 \cdot k_K = 4\,390\,274$$

where:

c_{S-M} – price indicator for warehouses
c_{S-B} – price indicator for office buildings
c_{V-P} – price indicator for productive buildings in relation to cubature
k_D – importance coefficients for *the area of the plot* attribute
k_P – importance coefficients for *location* attribute
k_S – importance coefficients for *the technical condition of buildings* attribute
k_K – importance coefficients for *the shape of plot* attribute
The analysis of rental market and offered prices of similar properties enable determination of the capitalization rate which equals R=0.09 for warehouse and productive areas and R=0.08 for office areas.
Based on the market information about rental rates for commercial premises, there were predicted unit rents for sold industrial properties.

- For office building $d_B = 24\,PLN/m^2\ per\ month$
- For productive hall $d_P = 3\,PLN/m^3\ per\ month$
- For warehouse building $d_M = 11\,PLN/m^2\ per\ month$

Using the capitalization rate R=0.09 for warehouse and productive building and R=0.08 for office buildings, there were calculated

- For office building $\tilde{c}_{S-B} = 3600\,PLN/m^2$
- For productive hall $\tilde{c}_{K-P} = 400\,PLN/m^3$
- For warehouse building $\tilde{c}_{S-M} = 1400\,PLN/m^2$

The functions conditions are in the form:

$$0\cdot\delta_{S-M} + 715\cdot\delta_{S-B} + 0\cdot\delta_{V-P} + 1400\cdot k_D + 200\cdot k_P + 200\cdot k_S + 200\cdot k_K = 2\,677\,412 - 2\,574\,000$$
$$0\cdot\delta_{S-M} + 295\cdot\delta_{S-B} + 0\cdot\delta_{V-P} + 2143\cdot k_D + 100\cdot k_P + 100\cdot k_S + 100\cdot k_K = 1\,155\,636 - 1\,062\,000$$
$$420\cdot\delta_{S-M} + 300\cdot\delta_{S-B} + 0\cdot\delta_{V-P} + 2543\cdot k_D + 200\cdot k_P + 100\cdot k_S + 100\cdot k_K = 1\,905\,613 - 1\,668\,000$$
$$0\cdot\delta_{S-M} + 320{,}94\cdot\delta_{S-B} + 5872\cdot\delta_{V-P} + 11358\cdot k_D + 200\cdot k_P + 200\cdot k_S + 200\cdot k_K = 4\,390\,274 - 3\,504\,184$$

The matrixes can be written:

$$[S] = \begin{bmatrix} 0 & 715 & 0 \\ 0 & 295 & 0 \\ 420 & 300 & 0 \\ 0 & 320{,}94 & 5872 \end{bmatrix} \qquad [a] = \begin{bmatrix} 1400 & 200 & 200 & 200 \\ 2143 & 100 & 100 & 100 \\ 2543 & 200 & 100 & 100 \\ 11358 & 200 & 200 & 200 \end{bmatrix}$$

$$[\delta] = \begin{bmatrix} \delta_{S-M} \\ \delta_{S-B} \\ \delta_{V-P} \end{bmatrix} \qquad [k] = \begin{bmatrix} k_D \\ k_P \\ k_S \\ k_K \end{bmatrix} \qquad [\Delta C] = \begin{bmatrix} 103\,412 \\ 93\,636 \\ 237\,613 \\ 886\,090 \end{bmatrix}$$

In the result, there were estimated the following values of random deviations and importance coefficients of attributes:

$\delta_{S-M} = 293;$ $\quad c_{S-M} = 1693\ PLN/m^2$

$\delta_{S-B} = 55;$ $\quad c_{S-B} = 3655\ PLN/m^2$

$\delta_{V-P} = 84;$ $\quad c_{V-P} = 484\ PLN/m^3$

$k_D = 31$

$k_P = 80$

$k_S = 10$

$k_K = 10$

The variations from random deviations and importance coefficients is equal:

$$\sigma_0^2 = \frac{103562}{4} = 25890$$

The standard deviation is

$$\sigma_0 = \sqrt{25890} = 161$$

The variation matrix of price indicators and importance coefficients has the following values:

$$\operatorname{cov}\begin{Bmatrix}[c]\\[k]\end{Bmatrix} = 25890 \times \begin{bmatrix} 0{,}0463 & 0{,}0404 & -0{,}0003 & 0{,}0017 & -0{,}2036 & 0{,}0235 & 0{,}0235 \\ / & 0{,}1645 & -0{,}0010 & 0{,}0071 & -0{,}2063 & -0{,}2159 & -0{,}2159 \\ / & / & 0{,}0000 & 0{,}0000 & 0{,}0013 & 0{,}0014 & 0{,}0014 \\ / & / & / & 0{,}0003 & -0{,}0089 & -0{,}0094 & -0{,}0094 \\ / & / & / & / & 0{,}9010 & -0{,}0505 & -0{,}0505 \\ / & / & / & / & / & 0{,}9439 & -0{,}0561 \\ / & / & / & / & / & / & 0{,}9439 \end{bmatrix}$$

Valuated market value of industrial property is estimated using the following matrix:

$$[\bar{S}] = \begin{bmatrix} 192 & 769 & 0 \end{bmatrix} \quad \text{i} \quad [\bar{a}] = \begin{bmatrix} 1100 & 100 & 100 & 200 \end{bmatrix}$$

The market value of industrial property is:

WR = 3 181 378 PLN

The standard deviation of appraised market value is:

$\sigma(WR) = 28\ 071\ PLN$

7. CONCLUSION

In the face of globalization and fast economy growth of malopolska region, very important seems to be using the potential that has a Krakow city in the range of various fields of economy. The chance for constant development is gaining new investors, who perceive more often Krakow as an ideal place for their investment plans realization. Significant, in this matter, is creation suitable conditions enable them to start investment projects. In the aspect of industrial properties, the main issues, which make some limitations in this range and should be resolved, are: the lack of local land development plans, not suitable developed technical infrastructure, and long periods of time needed to get all necessary documentations to start the investment.

On the appraisal surface, industrial properties are very differential and specific group of real estate. In the valuation process appears the problem of choosing the proper comparable unit, which allows to compare transaction prices of sold industrial properties and the amount of their component elements. This kind of properties are rarely observed on sale market, and transaction price include all components of property. The market value of appraised industrial properties should be estimated in relation to special algorithms, which allow considering types and number of particular components of properties and their price indicators. It allows determining the most probable market value of appraised property, with the range to its components and attributes values.

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