The Meaning of Redundancy
- 3D Topology and Geometric Parameterization

Key words: 3D, data modeling, redundancy, adjustment, BIM

SUMMARY

Database normalization is a standard technique to ensure data integrity. Explicitly specified 3D topology as well ensures geometric consistency. This article explains a stepwise reduction of redundancy in the 3D data model. Whereas the topological normalization is well discussed in literature the geometric normalization is a subject to recent research. This approach of data normalization is motivated by the need for applying geodetic adjustment techniques to measured (observed) three-dimensional building models.

The article examines different usage of the term redundancy and then shows up the normalization of the "polygon soup" data model to an almost redundant free data model. In contrast to existing CAD or GIS data models, the discussed approach does not use any coordinates, instead it utilizes normal vectors to planes. Using this approach, the number of unknowns is significantly reduced, resulting in fewer measurements to be observed. The primary reason for normalization is not to reduce the amount of data being stored but rather to ensure consistency and to allow suitable geometric generalization. In addition efficient adjustment techniques can be applied.

The Meaning of Redundancy - 3D Topology and Geometric Parameterization

1. INTRODUCTION

Gathering three dimensional geometric data of buildings is becoming increasingly important. Virtual 3D City Models and Building Information Models require reliable information about the inside of buildings. Traditional surveying techniques, such as tacheometry, are too time consuming and too expensive. This paper shows how to parameterize the three-dimensional building geometry in a non-standard form that is suitable for fast data acquisition by making use of least squares adjustment. In contrast to existing CAD or GIS data models, the discussed approach does not use any coordinates, instead it utilizes normal vectors to planes. Using this approach, the number of unknowns is significantly reduced, resulting in fewer measurements to be observed.

This article discusses certain aspects of a conceptual three-dimensional data model that was introduced by [Gielsdorf and Gründig 2002]. The methodology of this promising model relies on three main Ideas. Topological and geometrical data are specified separately. Explicit topology is compulsory. Secondly, geometry is parameterized with flat surfaces not coordinates. And thirdly, the observations (measurement, relative geometry) are stored in the project database and are used to determine the parameters of the absolute geometry by applying geodetic adjustment techniques.

This model does not end in itself. The explicit specification of a 3D topology facilitates consistency checks during insert, update or delete operation. A surface based parameterization reduces the number of unknown parameters in the adjustment, by shifting information, like planarity of a face, from the computational geometry domain to the relational domain. [Thompson and Oosterom 2008] emphasis a soaring paradigm: “Note that there is a trend in which the original observations and measurements are more often stored in the (cadastral) database, in addition to the resulting interpretations (parcels)”.

This article discusses the methodology of normalization in the problem domain of 3D geo- and building information. Normalization reduces redundancy. The primary reason for normalization is not to reduce the amount of data being stored but rather to ensure consistency and to allow suitable geometric generalization.

2. REDUNDANCY

If “pieces of information” are exceeding what is necessary or normal, these pieces are called redundant. Redundancy is characterized by similarity or repetition. The meaning of the word ranges from pure duplication of critical components of an engineering system, to ”wasted” space used to transmit certain data, including tautology in spoken language. (Wikipedia)

The GIS/Surveying world uses the term ”redundancy” in context of SDBMS (Spatial Database Management Systems) and in the context of Adjustment Calculation. Database designer try to avoid redundancy. Database Normalization minimizes duplication of information in order to ensure data integrity when data is inserted, updated or deleted. Empirical science like Engineering Surveying, Photogrammetry and Geodesy try to obtain redundant observations in order to increase precision and reliability. The different connotation - negative connotation on the database side and positive connotation on the surveying side - result from different types of information.

A database usually represents *Deterministic Information* (DI). The model of the world does not involve uncertainty and errors. This assumption leads to a contradiction free representation of the real world. Deterministic variables should be stored only once. Here, redundancy means duplication of identical information.

Models from Measurements are realized with *Stochastic Information* (SI). Due to the inherent stochastic properties of observations the model involves information about precision and correlation of all parameters. Although diverse observations are used to describe the same quantity in the real world, every observation is stored, since every measurement is a subject to errors or mistakes. Here redundancy can be considered as statistical degree of freedom [Mikhail 1976]. Any surveying engineering application benefits from the usage of redundant information by estimating precision and reliability.

This article describes how the reduction of deterministic redundancy leads to a data model that supports input, update and deletion of three-dimensional topologic-geometric data and supports redundant measurement. Since the amount of unknown variables is lessened this allows for efficient algorithms concerning stochastic information (adjustment calculation).

3. 3D TOPOLOGY IN SPATIAL DATAMODELS

3.1 Topological Models

In spatial information models the terms “geometry” and “topology” are closely linked. From the mathematical point of view the three-dimensional Euclidian space $\mathbb{R}^3$ is a topological space.

In context of solid modeling and GIS, geometric information describes position, orientation and shape of objects whereas topology describes relationships between objects. Topologic properties are invariant under certain geometric transformation.

This article discusses some aspects of explicit topological boundary representation of three-dimensional solids. Out of scope are: Topological predicates from a result of geometric operations and topological networks used for routing.

[Mäntylä 1988] substantiates the necessity of a topological model with the following properties of the Euclidian space $\mathbb{R}^3$: The domain of the propertyset (x,y,z) of the metric space is infinit, uncountable and open. It is imposible to map all points of this space in a unstructured way. The structure of space decomposition (voxel, TEN, B-Rep, Half-Space...) is called topology.

[Mäntylä 1988] distinguishes three separate levels of modeling:

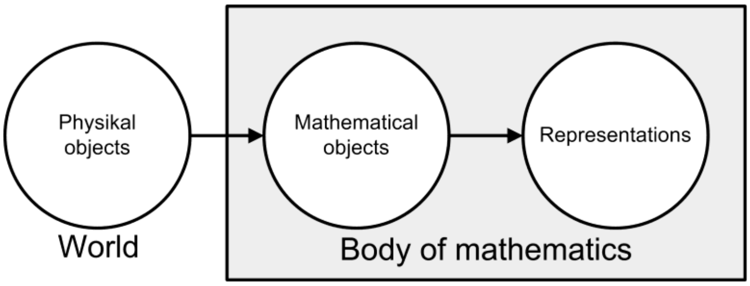


Fig. 1 A three-level view of modeling [Mäntylä 1988]

Physical objects (terrain, buildings, streets, tunnels, parcels) must be identified and the universe of discourse must be restricted, since the real world cannot be mapped in full detail. *Mathematical objects* are a suitable idealizations of the real-world physical objects. Diverse mathematical approaches are discussed in literature (regular cell decomposition, TEN, B-Rep, CSG). [Zlatanova 2004] gives an overview on the history of data models and recent developments. Less effort has been made on the problem of *digital representations*. Only recently [Thompson 2007] addresses the finite precision of spatial data.

Since there is no way to find an explicit representation of shapes in space $\mathbb{R}^3$ an indirect method has to be found. The implicit representation of infinite point-sets can be structured in following classes:

Decomposition models. The point set is represented as a collection of simple objects from a fixed collection of primitive object types with a single (!) implicit or explicit "gluing" operation. Example for decomposition models are: Voxel, Octree, binary space subdivision.

Constructive models. The point set is represented as a combination of primitive point sets together with general construction operations. Example for constructive models are: Boolean set operations on half spaces, constructive solid geometry (CSG).

Boundary models. These vector based models decompose the 3d solid with the relation type "boundary" ∂. Complex n-dimesional Shapes are described indirectly with their $(n-1)$-dimensional boundary. Topology then is the structure that spezifies how objects of lower dimension are combined.

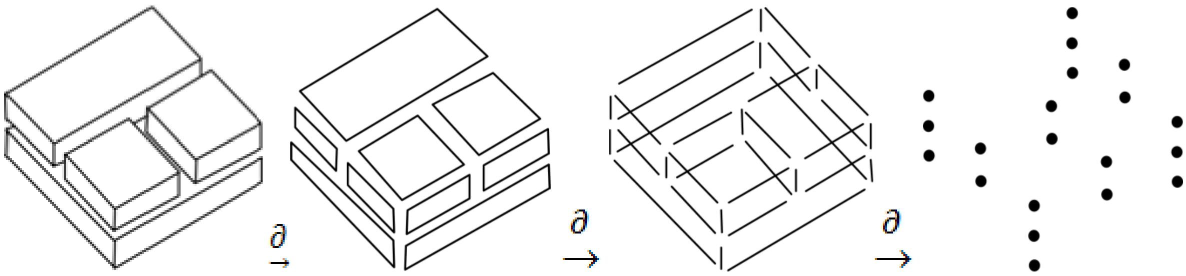


Fig. 2 Boundary Representation [Bradley 2006]

Only the boundary model will be discussed in this article.

3.2 Topological Primitives

Topological primitives are topological objects that describe single, non-decomposed elements of a topological complex. Topological primitives are open and connected point sets.

Node. A node is a 0-dimensional topological object. A node has no boundary.

Edge. An edge is a 1-dimensional topological object. An edge is bounded by two nodes. The interior of an edge is an open and connected point set.

Face. A Face is a 2-dimesional topological object. A Face is bounded by a finite set of edges. The boundary is closed and has no self intersection. The interior of a face is an open and connected point set. Faces are 2-manifolds. If a face is topologically the same like a disk - which means it has no holes – it is called a 2-cell or CW-complex.

Topological Solid. A topological solid is a 3-dimensional topological object. A solid is bounded by faces. The surface (boundary) is closed, orientable, connected and free of intersection. The interior of a topological solid is an open and connected point set.

3.3 Topologic and Geometric Consistency

As mentioned above, the terms “geometry” and “topology” are closely linked. An explicit specified topology helps to ensure geometric consistency. The ISO 19103 says that “the most important productive use of topology is to accelerate computational geometry”. Beyond that, topology allows to ensure consistency during inserting, updating and deleting operations.

A simple example for the usage of topology for consistency checks is the Euler-Poincaré formula. The **number of topologic primitives** in valid solids isinvariant. It is used by the (extended) Euler-Poincaré formula. [Mäntylä 1988]

$$v - e + f = 2(s - h) + r$$

In addition to the check of nodes, edges, faces and solids this simple formula controls the number of rings (hole in a face) and shells.

The **referential integrity of topologic primitives** ensures that the boundary resp. the interior of a topological primitive exit in the model instance. For instance: It can be checked, whether an edge that is supposed to bound a specific face really exists.

Cardinality between topologic primitives provides another validation rule. An edge is bounded by exactly two nodes. A mesh is bounded by three edges or more. In linear models a solid is at least bounded by four faces.

Topologic Integrity of a ring is ensured if the ring is closed (first vertex equals last vertex) and if distinct vertices are used.

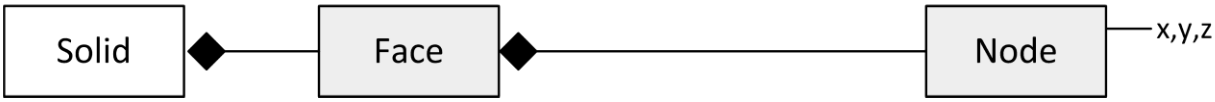
Geometric Integrity validates whether all vertices are in the same the plane and whether edges are non-intersecting. If faces have additional interior inner boundaries (holes) these rings have to lie in the same plane like the exterior ring, and the interior rings are not allowed to intersect and must follow the orientation rules of the data model. Furthermore interior rings must not overlap nor intersect nor touch the exterior ring.

In general one can state, that the algorithms for validating geometric integrity are more expensive than those that are ensured by the data model or the DBMS. Topological encoding transfers certain algorithms from computational geometry to combinatorial algorithms which are well supported by relational data bases.

4. TOPOLOGICAL NORMALIZATION

In terms of relational databases "normalization" means the reduction of multiple stored pieces of information. This chapter does not provide a new 3D topological data model. The aim is to describe the stepwise reduction of redundancy. One technique though is to replace *composite* relationship (black rhomb) with a shared *aggregate* (white rhomb).

In so-called "topology-free", "polygon soup" or "spaghetti" data models faces are represented as a collection of coordinate triples (x,y,z). The coordinate values are stored in every collection as a *point composite*.



This simple structure is suitable to visualization purposes, since there is no need for searching during rendering. It is obvious, that the storage of the point coordinates is redundant. However, more levels of redundancy can be specified [Fig 3]:

- Node-redundancy (multiple storage of point geometry)
- Edge-redundancy (multiple storage of node-connectivity (edge))
- Face-redundancy (multiple storage of edge-connectivity (face))

– Solid-redundancy (multiple storage of faces)

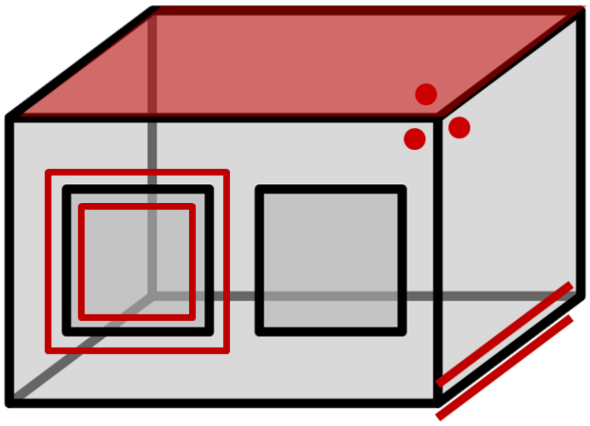


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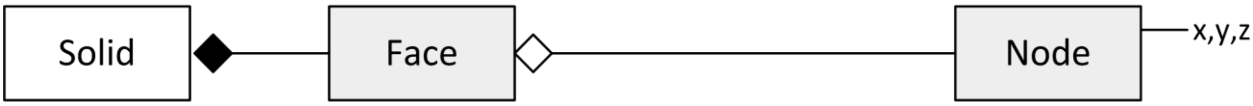
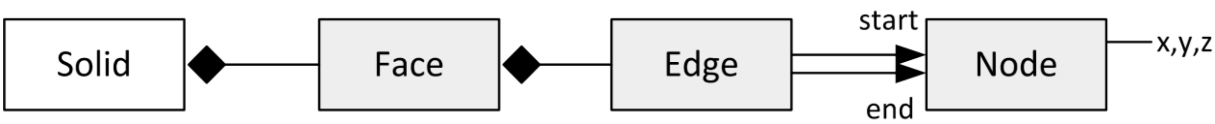
Fig. 3 Levels of redundancy

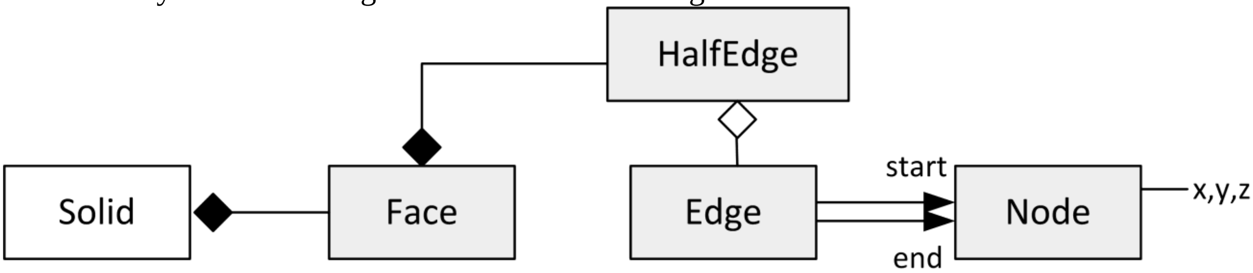
In order to avoid node-redundancy points are uniquely stored in an independent list and are referred by faces. (*shared point aggregation*).



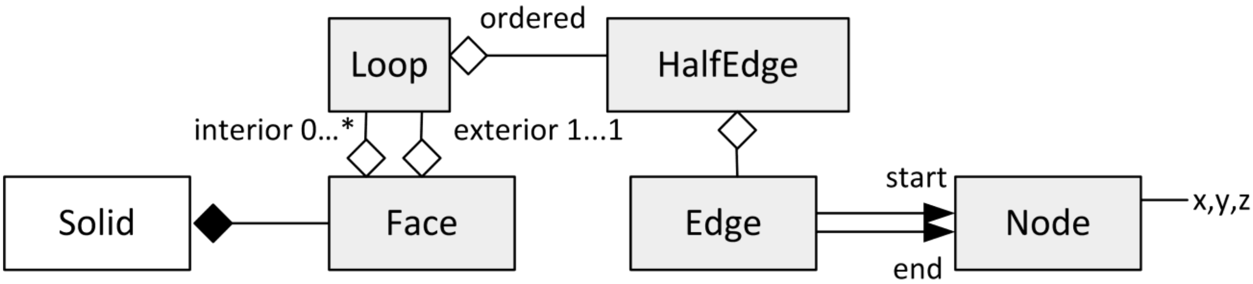
Here every vertex-coordinate is only stored once. As a next step of normalization the entity type ”edge” is introduced, in order to avoid edge-redundancy. Since the ”normalization” of higher-dimensions redundancies must take into account the *orientation* of the topological primitives, the 2:N relationship is specified more detailed with the attributes ”start” and ”end”.



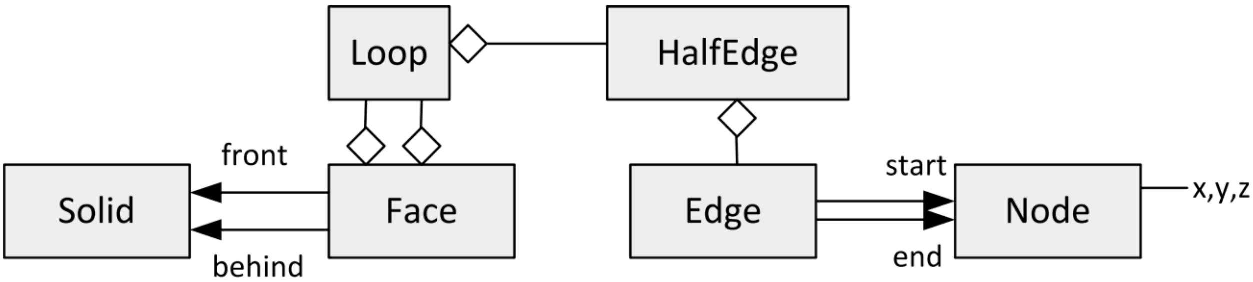
Now a face consists of a list of oriented edges and is stored twice. By switching from edge-*composition* to edge-*aggregation* a ”real-world” edge is only stored once, but is refered by two faces (if the model is a 2-manyfold). Since these two faces refer the same edge, but might be of different orientation, the edge is decomposed in two half edges. A half-edge is entity with a binary orientation flag and a reference to an edge.



A face comprises half edges. If a face has no holes, one (1!) well-ordered set of half-edges (polygon, closed ring, loop) bounds a face. If a face has holes it is bound by one exterior and one or many interior rings. Again, by switching from *composition* to *aggregation* a loop is only stored once.



In this type of data model each solid stores its own faces. As each face separates two solids the reference direction could be changed. Now, the solids are oriented by the face attributes ”behind” and ”front”. Orientation of solids allows numeric distinguishing between the inside and outside.



Until now all topological dimensions have been checked for redundant information. The topological data structure has been normalized. However there are diverse ways to structure topological primitives [Ellul 2005]. It is the objective of this article to demonstrate general methods to reduce redundancy in a rigors way.

5. GEOMETRIC NORMALIZATION

Until now geometric representation has not been covered. The prevalent method for specifying location is making use of point representation: Every node is attached with three coordinate values (x,y,z) in order to specify position. Additionally implicit (in the mathematic model) or explicit (stored) assumptions are made on planarity of faces, parallelism of faces and perpendicularity of faces.

The main idea of the discussed approach is to reduce geometric redundancy by replacing point representation with *surface representation*.[Fig. 4]

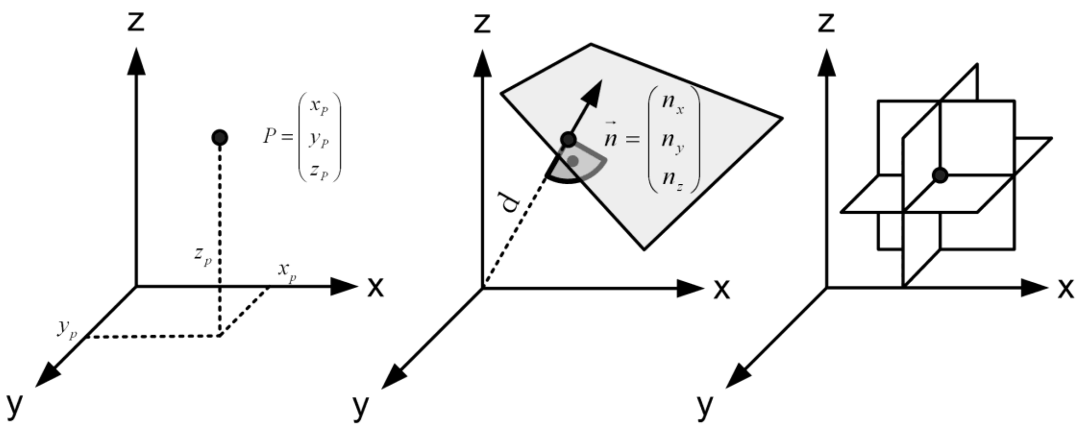


Fig. 4 a) point representation b) surface representation c) plane intersection

A flat surface is represented by its normal vector $\vec{n} = (n_x, n_y, n_z)^T$ and the orthogonal (shortest) distance d to the origin. Since topology is integral part of the data model the (relational) database is able to determine which planes intersect in each node. The point coordinates can be calculated on demand, using Cramer's Rule.

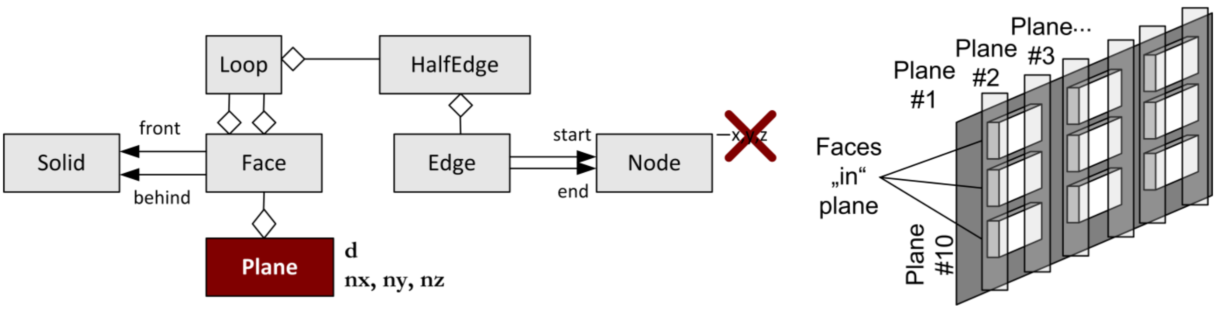
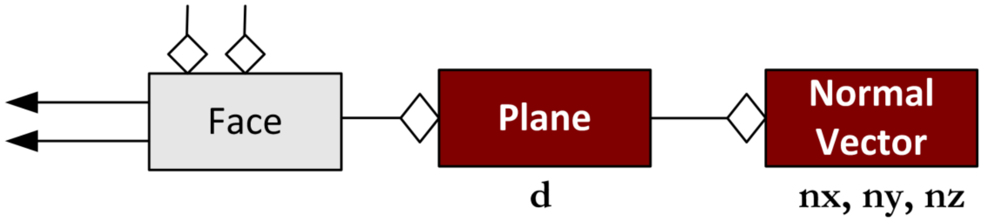


Fig. 5 Planar faces and paralell planes

How does this shift from point representation to surface representation affect the objective to reduce geometric redundancy?

Planarity of a single face. Planarity of polygons is traditionally seen as a geometric integrity constrained and is checked with run time algorithms up to a certain tolerance. Planarity must be ensured for computational reasons (computational geometry) or visualization purposes. Especially manmade objects are designed with planar surfaces. Using *surface representation* (here: plane based) the discussed model ensures the planarity of a single polygon by means of referential integrity.

Planarity of faces. The *Face-Plane* relationship is of cardinality 1:N. Each face associates exactly one plane. But many faces can associate the same plane. Faces representing the floor, the ceiling or the façade could link the same geometrical entity "plane". Planarity of different faces is ensured by referential integrity.



Parallelism of planes. Fig. 5 illustrates a modern office building. Often planes, which are “carring” diverse faces, are parallel to each other. Plane #1, #2 and #3 are parallel. What does this mean in terms of redundancy? Planes are parallel if they have equal rotation; therefore they have the same normal vector. The idea of the discussed data model is to normalize this redundancy by introducing an entity type *Normal Vector*. Normal vectors are referred by diverse planes (1:N). The translational component *d* remains with the entity type *Plane*. Planarity of different faces is ensured by referential integrity.

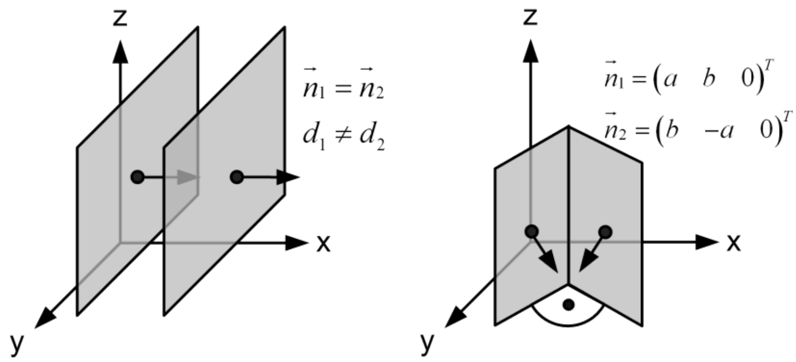
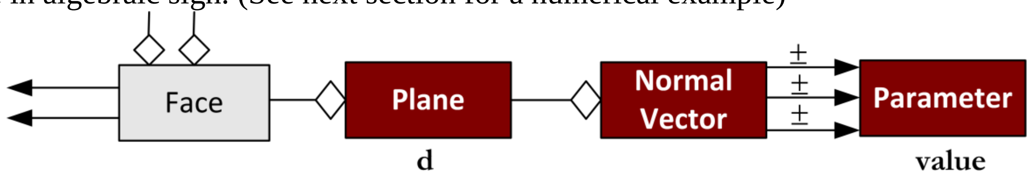


Fig. 6 Parallelism and perpendicularity

Perpendicularity of normal vectors. If, for example, two vertical walls are perpendicular to each other, the normal vectors components are switched in the non-zero elements and the sign of the n_y component changes. Therefore the data model introduces the entity type *Parameter*. The Parameter entity is referred by the normal vector. Since the sign might change, the *Normal Vector – Parameter* association is qualified by a Boolean variable that indicates a change in algebraic sign. (See next section for a numerical example)



This model is called plane-based and geometrically normalized parameterization.

6. NUMERICAL EXAMPELS

In this chapter the degree of redundancy is calculated. Here, the model instance is seen as a set of atomic piece of “information”, where each piece has the size 1. One piece of information is either positive or negative.

In terms of *deterministic information* a positive piece needs to be stored whereas a negative piece needs to be checked. The total sum of information equals zero when all data are checked. In terms of *stochastic information* (i.e. Least-Squares-Adjustment) a positive piece is one unknown whereas a negative piece is one condition. The total sum of information equals zero when the so called minimal configuration is present.

6.1 Cube

Point aggregate. A cube needs 9 pieces of information (3 Translation, 3 Rotation, 3 Dimensions). With point representation 8 points have to be stored. Since each point has 3 components this equals 24 pieces of information. Thus the overhead of information (redundancy) can be calculated with 24-9=15. This means, that 15 consistency checks have to be made. These are:

- Planarity of faces (-6)
- Parallelism of opposite faces. Normal vector dot product equals -1 (-3)
- Perpendicularity of pairs of parallel faces (-3)
- Orientation of faces (inside or outside) (-3)

Normalized to Parallelism of Planes. 3 normal vectors need to be stored if the cardinality plane-normal vector is 1:N. 6 translational components *d* must be attached to each plane. So the number of stored values is 3*3 + 6 = 15. There still is a redundancy of 6. So 6 negative pieces of information need to be specified:

- Absolute value of the 3 normal vectors equals 1 (-3)
- Perpendicularity of all three normal vectors (-3)

Normalized to Perpendicularity of normal vectors. In manmade objects like buildings most walls are vertical whereas floor and ceiling are horizontal. With the cube example this means that 2 rotational components are fixed. Under the assumption of being parallel to the z-axis a cube needs 7 pieces of information (3 Translation, 1 Rotation, 3 Dimensions) . Instead of storing three normal vectors 6 values are stored $\left(\alpha_1 = 0; \alpha_2 = 0; \alpha_3 = 1; \alpha_4 = a; \alpha_5 = b; \alpha_6 = 0\right)$ whereas the normal vectors refer to theses values

$$\overrightarrow{n_1} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \overrightarrow{n_2} = \begin{pmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \quad \overrightarrow{n_3} = \begin{pmatrix} \alpha_5 \\ -\alpha_4 \\ \alpha_6 \end{pmatrix} = \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}$$

$$\overrightarrow{n_4} = \begin{pmatrix} -\alpha_1 \\ -\alpha_2 \\ -\alpha_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \overrightarrow{n_5} = \begin{pmatrix} -\alpha_4 \\ -\alpha_5 \\ -\alpha_6 \end{pmatrix} = \begin{pmatrix} -a \\ -b \\ 0 \end{pmatrix} \quad \overrightarrow{n_6} = \begin{pmatrix} -\alpha_5 \\ +\alpha_4 \\ -\alpha_6 \end{pmatrix} = \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix}$$

6 translational components *d* must be attached to each plane. So the number of stored values is 6 + 6 = 12. There still is a redundancy of 5. So 5 negative pieces of information need to be specified:

– $\alpha_1, \alpha_2, \alpha_3, \alpha_6$ are fixed (-4)
– a,b are rotation parameters so $a^2+b^2 = 1$ (-1)

Even with this very simple example of a cube one can find many ways of parameterization. The decision, which type of parameterization is applied, depends on the application.

6.2 Building

Fig.7 illustrates the application of a plane-based parameterization to a building. The model consists of 412 nodes. With a point aggregate model 1236 coordinate values are stored. In the discussed plane based parameterization 104 translational components *d* and 14 α are stored. Under the assumption of coplanarity, paralellism and perpendicularity of vertical walls, the amount of stored geometric information is reduced about 91%.

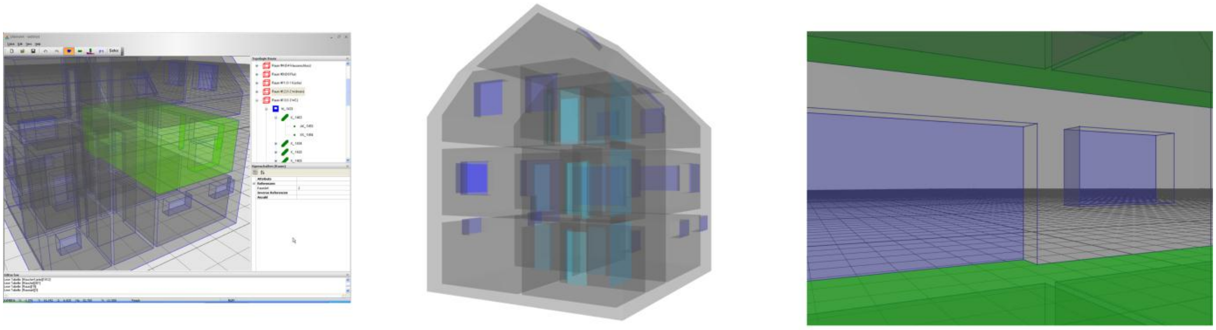


Fig. 7 Plane based parameterization with a residential house [Clemen 2007]

7. OUTLOOK

This article shows in detail, how topological and geometrical normalization reduces the amount of redundantly stored information. [Clemen 2007] showed how practical applications, i.e. data capture tools, can benefit from this data model by applying adjustment techniques. Further research will show which degree of normalization is suitable to which application or problem domain. Furthermore curved surfaces will be introduced to the model. The extended data model and its impact on topological and geometrical validation routines are a subject to further work. [Huhnt 2006] showed that this conceptual model does not only fit to geo-information tasks but also to diverse aspects of civil engineering design.

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