# Investigation on GPS Heighting Accuracy with the use of Hong Kong Satellite Positioning Reference Station Network (SatRef)

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**Key words**: GPS heighting, ellipsoidal height, geoid-ellipsoid separation, geoid model.


## SUMMARY

With the establishment of the Hong Kong Satellite Positioning Reference Station Network (SatRef), GPS static data and real-time correctional signals are provided to users to increase the efficiency and improve the accuracy of their GPS surveys. While horizontal positions determined by GPS post-processing and Network-RTK methods are able to achieve millimeter to centimeter-level accuracy and reliable survey results, the performance of GPS heighting is inferior due to the inherent weaker satellite constellation and error induced from atmospheric effect.

In this paper, the accuracy of GPS heighting is first reviewed. The accuracy of GPS-derived height in both static and real-time methods with the use of the SatRef system data is then investigated. The investigation is divided into three parts. The first is to compute the ellipsoidal height differences between pairs of stations over a long period of time to check the performance and assess the accuracy of GPS heighting by static GPS method during different time of a year. Secondly, the height values of a number of test points at different areas of Hong Kong are surveyed by using both single-RTK and Network-RTK methods. The surveyed results are converted to HKPD (Hong Kong Principal Datum) height values for comparison with their corresponding values surveyed by conventional levelling work. Thirdly, the temporal change of the Network-RTK-surveyed height values is examined by continuous measurements taken over a point; and further tests will be conducted on points with different network configurations of Reference Stations. Following the investigation, the method of transforming GPS surveyed ellipsoidal height to HKPD height by interpolation of the geoid-ellipsoid separation within a small local area is introduced. Finally, constraints in applying the method and possible improvements would be discussed.

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## 1. INTRODUCTION

In order to facilitate GPS surveys in Hong Kong, the Survey and Mapping Office (SMO) of Lands Department has established an active GPS reference station network called “Hong Kong Satellite Positioning Reference Station Network” (SatRef) as a land-based augmentation system in support of various kinds of positioning activities. By using GPS static data and real time correctional signals generated from the SatRef system, it is possible to achieve millimeter to centimeter-level of accuracy in position fixation. However, due to inherent weaker satellite constellation and error induced from atmospheric effect, accuracy of the height component of GPS measurements is often lower than that of the horizontal components.

In addition, the heights obtained by GPS surveys are relating to WGS 1984 ellipsoid while our daily used local heights (i.e. HKPD height) are based on an equipotential surface known as geoid surface. The difference between the WGS84 ellipsoidal height and the height from the geoid is the geoid-ellipsoid separation. In order to derive the HKPD height from ellipsoidal height, the geoid-ellipsoid separation of the survey points need to be determined. Thus, to assess the overall accuracy of GPS heighting, the accuracy of the height component in GPS measurements and the geoid-ellipsoid separation should be taken into account.

In the following sessions, the accuracy and limitations of GPS heighting would first be reviewed. In the second part of the paper, three studies on the investigation about the accuracy of GPS-derived height in both static and real-time methods are presented. Furthermore, the method of GPS height transformation by interpolation of the geoiod-ellipsoid separation within a small local area is introduced. Finally, the constraints of the interpolation method and any possible improvements would be discussed.

## 2. ACCURACY AND LIMITATIONS OF GPS HEIGHTING

### 2.1 Errors that Significantly Affect GPS Measurements

The first consideration in GPS heighting is the accuracy of GPS measurements. A number of errors that may affect the quality of GPS measurements should be considered. These errors could be categorized into three areas: i) *Satellite based error* including satellite clock error, satellite orbit error and satellite constellation; ii) *Signal propagation based error* including ionospheric and tropospheric errors; and iii) *Receiver based error* including dilution of precision (DOP), signal noise error, multipath error, receiver clock error and antenna phase centre variation (PCV).

Since most of the errors are spatially-correlated, receivers at adjacent sites experience similar errors. To mitigate the common errors, relative positioning over short baseline is employed instead of single point positioning. In addition, the satellite and receiver clock bias errors can be eliminated by double-difference processing.

For the errors due to site constraint, field reconnaissance is important for assessing the applicability of GPS method for the survey task. A proper GPS planning with the use of up-to-date almanac information would ensure that the GPS survey take places in period of good satellite geometry and with sufficient number of satellites. In static GPS surveys, millimeter relative accuracy level is achievable under favourable conditions. However, as the GPS vertical component is more sensitive to certain influencing factors than the horizontal one, high accuracy GPS-derived height is often difficult to achieve. The influencing factors include the following:-

- **Satellite Constellation:** Due to the fact that no satellites under horizon are available, GPS receivers can only receive signals from satellites above ground. This results in a poor geometry for fixing the height component in GPS measurements. In contrast, the horizontal components are fixed by satellites from different azimuths of the sky. It is therefore the VDOP value is higher than the HDOP value and the accuracy of vertical component is often less precise than horizontal one by 2 to 3 times or even more depending on the satellite geometry.

- **Tropospheric Delay:** The troposphere consists of dry gases and water vapour that affect the propagation delay of the radio frequency signals. The dry gas (primarily $N_2$ and $O_2$) causes zenith excess delay to approximately 2.3 metres and varies with local temperature and atmospheric pressure with a reasonably predictable manner. The dry atmosphere effect varies by less than 1% in a few hours. The water vapour (exists only below 12 km, mostly less than 4km above the sea level) causes delay in a smaller magnitude of 1-80 cm at zenith, i.e. about 1/10 the size of the dry atmosphere delay effect. However, it varies markedly 10-20% in a few hours and is less predictable even with surface humidity measurements (Spilker, 1996).

  As compared with the horizontal components, the effect of troposphere is particularly significant for height measurements. (Chen et al 2002). In order to mitigate the tropospheric effect, short GPS baselines are observed and processed by double-differencing. However, for steep baselines (height difference between the two stations >100m), the change in signal delay caused by the changing density of water vapour of the atmosphere may not be correctly reflected in standard tropospheric models. The difference in tropospheric conditions at the reference and rover sites may not be cancelled by double-difference processing even for short baseline of 5 km.

- **Phase Centre Variation (PCV) and Offset:** In principle, a GPS baseline should be measured between the electronic phase centres of two antennas which are centering over two survey stations. However, in reality, there are offsets between them. Furthermore, the antenna phase centre is not a fixed point but a varying one. Its position depends on the elevation, azimuth and intensity of the satellite signal and is different for L1 and L2 (in most types of antenna). It is fairly difficult to model the antenna phase centre variation because it is different for each antenna and also for various types (Hoffmann-Wellenhof, et al. 2000 pp. 124-5). The magnitude of biases in observed carrier phase can be of the order of several centimetres. In order to mitigate such bias, GPS manufacturers generate their own PCV calibration files for different antenna models. However, as PCV calibration files from different GPS organizations refer to different reference antennas, mixing use of them can introduce several centimetres error in the height component. Therefore, it is important to use PCV calibration files from same GPS organization for the GPS antennas employed in the GPS survey task. The National Geodetic Survey (NGS) antenna models which are the most commonly used PCV calibration files are available in the NGS website.

- **Multipath:** Multipath is mainly caused by reflecting surfaces near the receiver, though reflections at satellite during signal transmission is also possible. There is no general model of multipath effect because of the arbitrarily different geometric situations. In addition, from the geometry of reflection, signals from low elevation are more susceptible to multipath than signals from high elevation (Hofmann-Wellenhof, 2000: pp.125-6). Multipath is station dependent and may be significant for even short baselines. It has the effect of both contaminating the station coordinates and ambiguities (Hofmann-Wellenhof, et al, 2000, pp. 214). If the atmospheric conditions where the signals travel were the same, the same multipath effect would occur under the same satellite-receiver geometry. It is therefore stacking results of consecutive days of code-carrier range residual observation may trace the occurrence of multipath effect.

### 2.2 Geoid-ellipsoid Separation Accuracy

As mentioned above, GPS heighting is based on ellipsoidal height determination. However, in most survey projects in Hong Kong, the levels are orthometric heights referenced to the Hong Kong Principal Datum (HKPD). Information about geoid-ellipsoid separations is required to convert the GPS derived heights to HKPD heights. Therefore, apart from the height accuracy in GPS measurements, an accurate geoid-ellipsoid separation model is essential for GPS heighting. Such model can be obtained by two approaches: the gravimetric approach and/or the geometric approach. For gravimetric approach, a regional geoid model is developed based on gravity measurements over the territory. On the other hand, geometric approach makes use of GPS/levelling data to model the geoid-ellipsoid separation (Chen et al. 2000).

## 3. INVESTIGATION ON GPS HEIGHT ACCURACY

The following is the investigation about the accuracy of GPS height measurements made by static and real-time GPS methods. Three studies were carried out with the use of the SatRef system, focusing on:

i) the accuracy of static GPS ellipsoidal height,
ii) the RTK-derived HKPD height accuracy, and

iii) the ellipsoidal height accuracy of Network-RTK method.

### 3.1 Study One - The Accuracy of Static GPS Ellipsoidal Height

The objective of this study is to assess the accuracy and precision of GPS ellipsoidal height surveyed by static GPS. A pair of baselines was selected for processing by some commercial GPS software in assessing the precision of static GPS on heighting as such processing method is commonly adopted in survey operations. The two baselines comprise a flat and a steep baseline of similar length, but represent the flat and rough terrains respectively. Figure 1 shows the configuration of the baseline pair, baseline distances and height differences between the unknown and reference stations.

3.1.1 Assessing the Height Precision by Static GPS - processed by commercial software

The station HKKT was regarded as an unknown station. Its ellipsoidal height was fixed by the baseline from each of the stations HKFN and HKST respectively to evaluate the ellipsoidal height accuracy over the flat and steep terrains respectively. Moreover, the precision of the ellipsoidal height of HKKT obtained from such static GPS method was further assessed by comparing the results of using data from a day each month over a period of 18 months from July 2004 to December 2005. For each computation, a day of 24-hour data was divided into 12 two-hour observation sessions for data processing. The data was processed with the parameters as shown in Table 1 below.

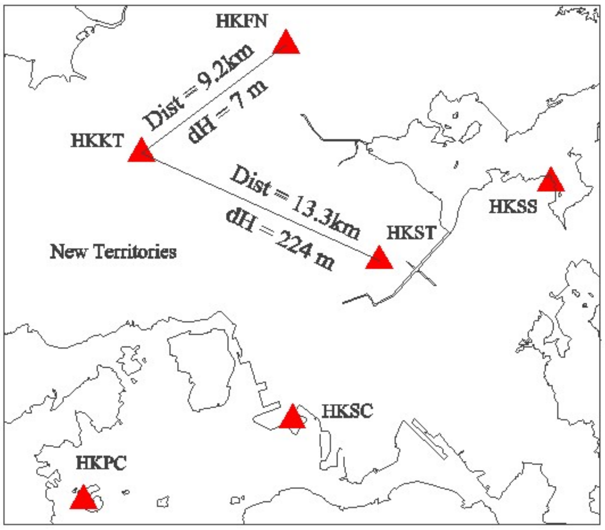


**Figure 1.** Configuration of the baseline pair (HKFN-HKKT and HKST–HKKT)

| | |
|---|---|
| **Cut-off Angle:** 15 degrees | **Frequency:** L1+L2 Iono free frequency |
| **Ephemeris Type:** Broadcast | **Tropospheric Model:** Hopfield |

**Table 1.** GPS Baseline Processing Parameters used

Outliers of GPS data were detected and removed by examining the GDOP values and results of repeated baselines.

In addition to the 24-hour solution, two sets of 6-hour GPS data on each day: Set I (8:00 to 12:00 + 14:00-16:00) that represents the normal working hours and Set II (22:00- 04:00) that represents the time of the most favourable atmospheric condition for GPS measurements were selected and processed for each baseline for investigating the effect of observation duration on the accuracy. The ellipsoidal height of HKKT published by Lands Department was used for comparison of the computed results.

Figures 2 and 3 show the height differences between the computed ellipsoidal heights and the published ellipsoidal height over 18 months for the scenarios of flat baseline and steep baseline.

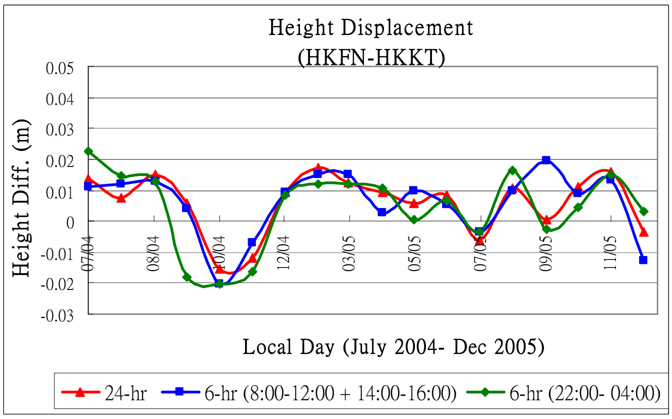


**Figure 2.** Variation of height difference by Static GPS (commercial software) over 18 months – flat baseline

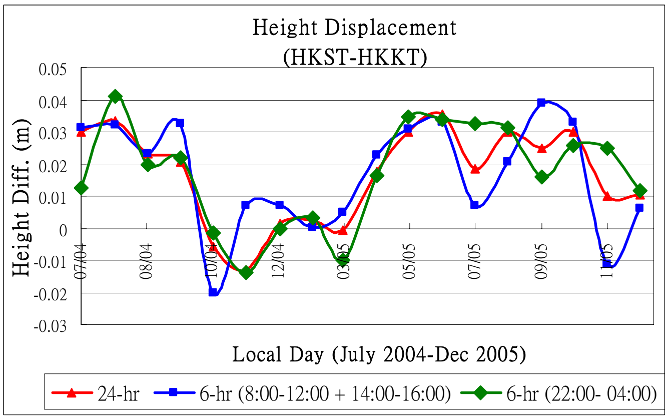


**Figure 3.** Variation of height difference by Static GPS (commercial software) over 18 months– steep baseline

The static GPS method with **flat baseline** achieves the results that all height displacements from the published ellipsoidal height are less than +/- 2cm for both 24-hour and 6-hour solutions. The maximum height differences for the two solutions are 4 cm (+2.0 cm and -2.0 cm respectively. The two sets of 6 hours static GPS, at day-time and night-time respectively do not show significant difference, except two instances – Sept/2004 and Sept/2005.

For the **steep baseline**, the spreads of the results over 18 months are 4.2 cm for the 24-hour solution and 5.9 cm for the 6-hour solutions. Unlike the results of the flat baseline, the two sets of 6-hour solutions for steep baseline deviate from each other by a maximum magnitude of 2-3 cm. The results of the night-time observation follow the trend of the 24-hour observation while the day-time observations show different results.

The above results show that static GPS processing by using commercial software has a better result on flat baseline as compared to steep baseline. Some seasonal trend can be observed from the results and the precision of steep baseline is in the range of 6 cm over 18 months for 6-hours static GPS. Such uncertainty may be due to inaccurate atmospheric model by the software used in GPS data processing or other unknown factors changing over the year.

### 3.1.2 Assessing the Height Precision by Static GPS – processed by scientific software

To find out whether the results could be improved by using some other scientific GPS software, another test was conducted by choosing another two baselines of similar length and test on the results of flat and steep baseline processing.

The station HKSC was adopted as reference station. The station was held fixed to compute the ellipsoidal heights of stations HKPC and HKST respectively. The stations HKSC and HKPC are of similar altitudes and this baseline represents flat area. The height difference between the HKSC and HKST is over 230 m and this baseline represents the steep terrain. Figure 4 shows the configuration of the baseline pair, baseline distances and height differences between the unknown and reference stations.

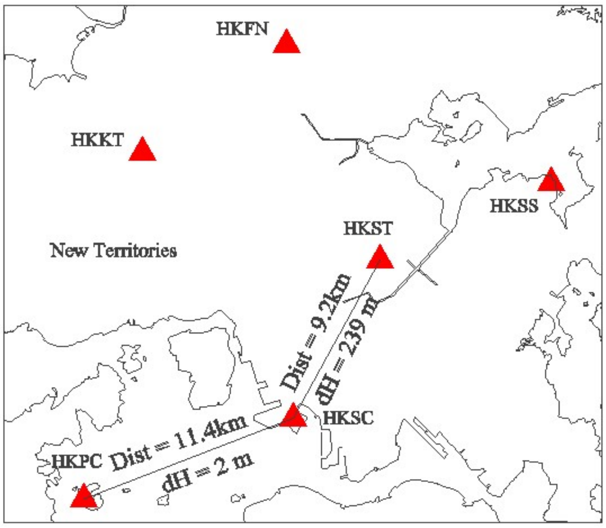


**Figure 4.** Configuration of baseline pair (HKSC-HKPC and HKSC–HKST)

The ellipsoidal heights of stations HKPC and HKSC were computed daily from April 2005 to December 2006. 24-hour GPS data was used for each ellipsoidal height computation. The processing parameters used were same as the parameters listed in Table 2 except the troposphere estimation approach. Apart from standard tropospheric model, the scientific GPS software bases on GPS observations to evaluate additional parameters for modelling the tropospheric error due to large height differences between stations. The height differences between the computed ellipsoidal heights and the 'known' value for the flat and steep baselines are presented in Figures 5 and 6. However, as the 'known' value was derived from GPS commercial software, it is therefore this evaluation only focuses on the spread of variation in the height difference of the baseline rather than the magnitude of displacement from the published value.

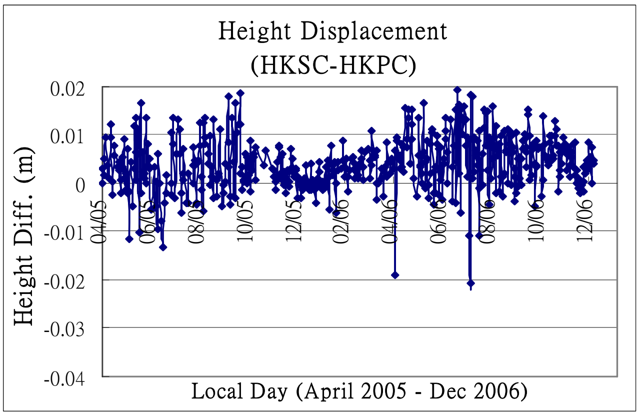


**Figure 5.** Variation of height difference by Static GPS (scientific software) over 18 months– flat baseline

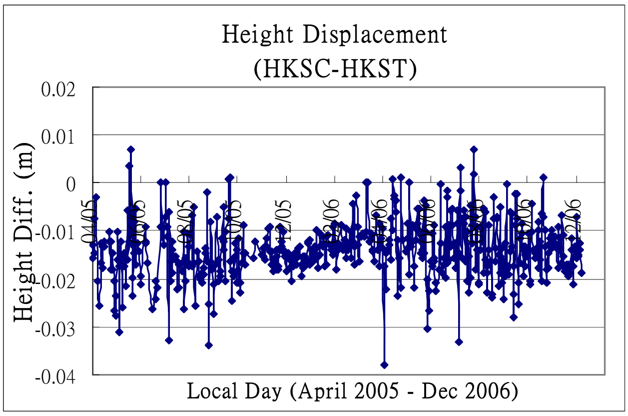


**Figure 6.** Variation of height difference by Static GPS (scientific software) over 18 months– steep baseline

The results show that the ellipsoidal height accuracy is improved when processed with the scientific GPS software. In general, the range of height variation is within 2 cm for flat baseline and 3 cm for steep baseline. Furthermore, seasonal effect on the GPS height measurements is observed. The ellipsoidal height accuracy in winter (October 2005 to April

2006) is more accurate and the height variations are halved as compared with the results in summer. Also, it is noted that all relatively large height differences occurred in summer.

From the results of this study, it indicates that the commercial GPS software may not effectively model the tropospheric effect of steep baseline. Thus, the altitudes of rover and reference stations should be similar to minimize the effect of tropospheric error. Alternatively, some scientific GPS software which has a more rigorous troposphere estimation approach for mitigation of tropospheric errors should be adopted. In addition, the lesser variation in GPS heighting during the winter period should also be noted in precise measurements.

### 3.2 Study Two - The RTK-derived HKPD Height Accuracy

In the summer of 2005, a comprehensive GPS survey was carried out to check the height values of test points across different areas of Hong Kong. The objective of the survey was to study the achievable accuracy of RTK-derived HKPD heights. Both single-RTK and Network-RTK correctional signals from the SatRef system were tested. The system provides three types of Network RTK solution, including Flachen-Korrektur-Parameter (FKP), Virtual Reference Station (VRS) and the Pseudo-Reference Station (PRS) for user selection. In this study, the RTK solution in FKP mode was chosen. The rover GPS receiver first sent its approximate position to the Network RTK software. The atmospheric-corrected and distance-independent RTK corrections were then computed based on the rover position and real-time data from all SatRef stations. Finally, the Network RTK correction message in FKP mode would be transmitted to the rover station. For the single RTK method, the SatRef station nearest to the test point would be chosen as the base station.

A total number of 180 test points (excluded the points at Hong Kong Island and Lantau Island) and 242 test points (for the whole territory) were taken for single-RTK and Network-RTK approaches respectively (Figure 7). They were selected because of their known HKPD values surveyed by precise levelling. The test points were measured twice for each mode of RTK methods and each measurement comprised 15 observation epochs. The integer ambiguity of the RTK measurements was fixed independently to ensure uncorrelated survey results for analysis.

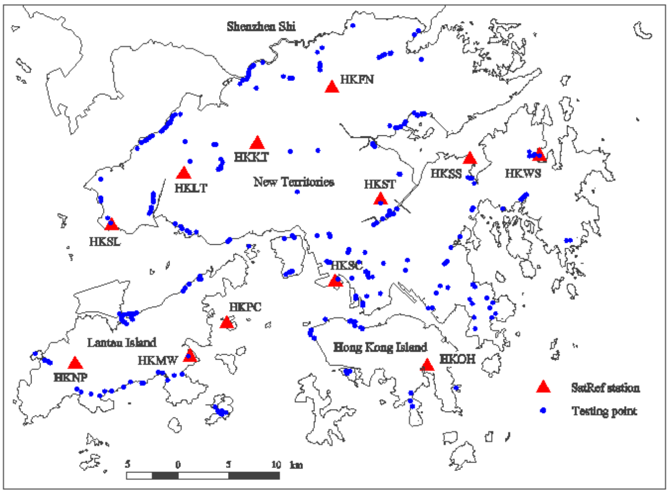


**Figure 7.** Plan showing the distribution of SatRef stations and testing points

During the survey, the Coordinate Quality (CQ) values shown in the GPS receivers were recorded. The CQ values represent the estimated accuracy of observations at the time of survey based on the satellite geometry, measurement noises and environmental conditions. The measurement which did not meet the accuracy requirement of Plan CQ < 0.02 and Hgt.

CQ < 0.03 was considered as a failed measurement. 312 Network-RTK and 172 single RTK measurements were used for subsequent data analysis. The measurement summary is presented in Table 2 below.

| | **Network RTK survey** | **Single RTK survey** |
|---|---|---|
| **No. of failed measurement** (Plan CQ>0.02 or Hgt. CQ>0.03) | 172 / 35.5 % | 188 / 52.2 % |
| **No. of qualified measurement** (Plan CQ<0.02 and Hgt. CQ<0.03) | 312 / 64.5 % | 172 / 47.8 % |
| **Total no. of measurement** | 484 / 100 % | 360 / 100 % |

**Table 2.** Summary of Network RTK and Single RTK measurements

As shown in Table 2, 64.5% of the Network-RTK measurements achieved a qualified CQ (Plan CQ<0.02 and Hgt. CQ<0.03) while only 47.8% was found in Single-RTK. In summary, Network-RTK is much easier to obtain a qualified CQ value for the survey as compared to Single-RTK.

Furthermore, in order to convert the heights of GPS surveyed test points to HKPD height for comparison, a set of control points with both GPS and levelling values are used for the conversion. The height differences between the RTK-derived HKPD heights and their corresponding known heights are computed and presented in Table 3 and Figure 8. As shown in Figure 8, heights surveyed by using Network-RTK are closer to the truth and more normally distributed as compared to that of the single RTK solution. Network-RTK is more readily to achieve high accuracy GPS heighting (51 % of the measurements of Network-RTK are within 3 cm from the 'truth'). In addition, three large height discrepancies (maximum 58 cm) occurred in single RTK measurements are not plotted in Figure 8. For lower accuracy GPS heighting, the performance of single-RTK and Network-RTK (FKP mode) are more or less the same (80 % of the results within 6 cm height accuracy and 92-93% of the results within 9 cm height accuracy).

| | **No. of measurement** | |
|---|---|---|
| **Ht. Diff (cm)** | **Network-RTK** | **Single-RTK** |
| **0-3** | 160 / 51.3% | 73 / 42.4% |
| **3-6** | 91 / 29.2% | 65 / 37.8% |
| **6-9** | 35 / 11.2% | 22 / 12.8% |
| **9-12** | 12 / 3.8% | 3 / 1.7% |
| **>12** | 14 / 4.5% | 9 / 5.2% |

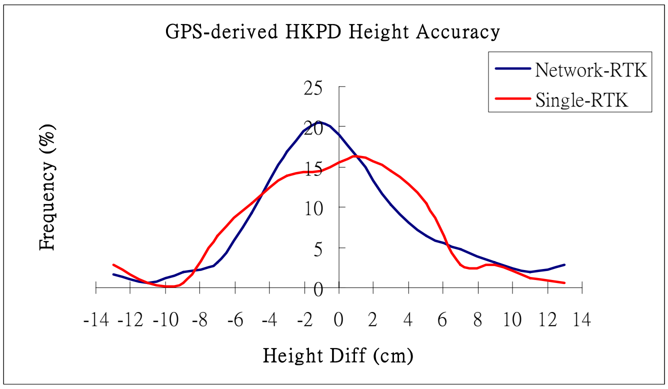


**Table 3 and Figure 8.** Accuracy of GPS-derived HKPD Height by using single RTK and Network-RTK(FKP mode) methods

The results indicate that RTK-derived HKPD heights can generally be done at 6cm accuracy. The Network-RTK method is considered to be more reliable and accurate in achieving high accuracy in ellipsoidal height at different areas of Hong Kong under different site conditions.

Furthermore, the ellipsoidal height accuracy should be monitored by checking the quality indicator during the survey.

### 3.3 Study Three - The Ellipsoidal Height Accuracy of Network-RTK Method

In this study, three points under different network configurations of SatRef stations were surveyed continuously by Network-RTK method (Figure 9). The purposes of the study are to assess the Network RTK accuracy and investigate the temporal change of RTK measurements under different satellite geometry and network configuration of reference stations. The three selected testing sites represented different scenarios of network geometry, a good geometry one (Ng Tung Chai), a weak geometry one (Ying Pun) and the weakest geometry one (Sha Tau Kok). The three sites are all with clear sky window, good GSM network coverage and no multipath from nearby structures to minimize disturbing factors on the study.

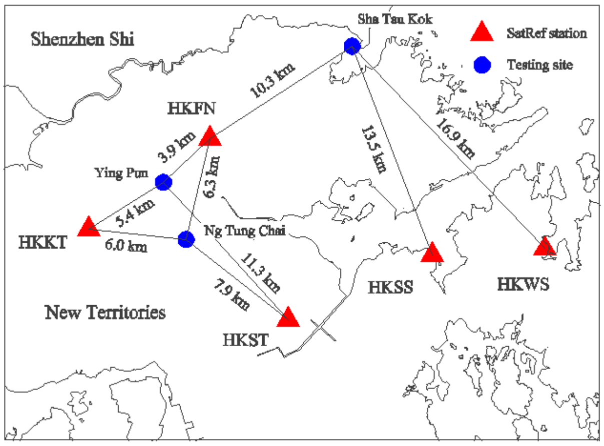


**Figure 9.** Plan showing the geometry between the test sites and their corresponding GPS Reference Stations

For each testing site, the field survey lasted for three days and the Network-RTK surveyed heights were recorded at least six hours on each day (from 9:30 to 15:30). The data logging rate was set to one second with elevation mask angle of 15 degrees. GSM data link was established to receive the Network RTK corrections. Apart from the Network-RTK measurements, 6-hour static survey was performed to measure the ellipsoidal heights of the survey stations as control for comparing the RTK measurements. The adjacent SatRef stations were adopted as base stations. The measurement summary is presented in Table 4.

| **Testing Site** | **Day of Survey** | **No. of all data** | **No. of RTK fixed** (Plan CQ<0.02 and Ht CQ<0.03) |
|---|---|---|---|
| Ng Tung Chai | 31/07-01/08/2006 | 75137 | 68372 / 91.0% |
| Ying Pun | 08-10/01/2007 | 67661 | 66717 / 98.6% |
| Sha Tau Kok | 11-12, 15/1/2007 | 71427 | 70227 / 98.3% |
| | **Total:** | **214225** | 205316 / 95.8% |

**Table 4.** Summary of measurements collected in the test

Over 95 % of the RTK measurements fulfilled the accuracy of plan CQ <0.02 and Hgt CQ < 0.03 and they were used for subsequent analysis. The data sampling rate for plotting is set at 10 seconds. The height differences between the Network-RTK heights and the known values are computed. The height differences, corresponding Hgt. CQ values and the largest height discrepancies for each testing site are presented in Figures 10 to 12.

As indicated from Figures 10-12, the good geometry case of Ng Tung Chai achieves the best result of 3cm displacement from the known values for most epochs while the other two cases achieve an accuracy of 5-7cm. However, for the case of Sha Tau Kok which is more than 10km away from all reference stations still achieve a height accuracy of 5 cm for most of the time. It is even better than the Ying Pun case. This may be due to similar altitudes of Sha Tau Kok and the reference station HKFN. It is therefore the correlation between control network configuration and the Network-RTK accuracy is low. Regardless of the control network configuration, a 5-7cm GPS vertical accuracy can generally be achieved with the Network-RTK method.

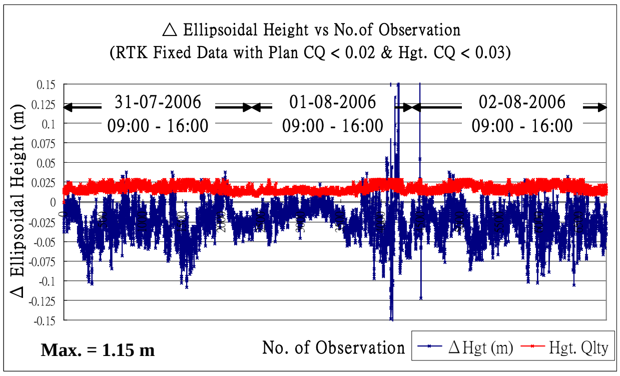


**Figure 10.** Testing site: Ng Tung Chai (good geometry)

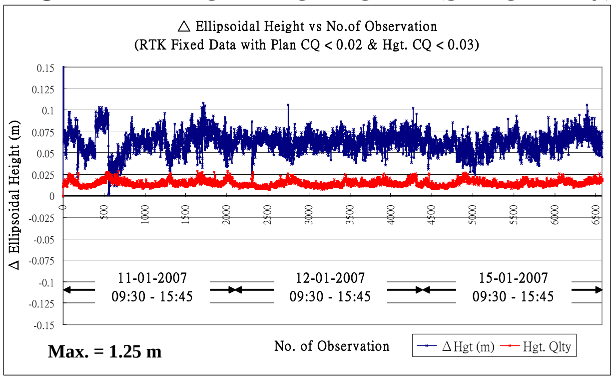


**Figure 11.** Testing site: Ying Pun (weak geometry )

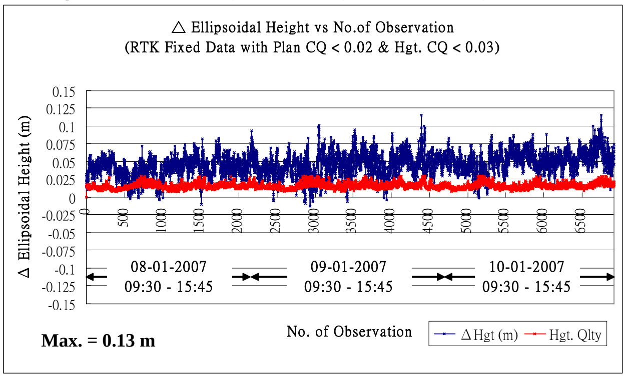


**Figure 12.** Testing site: Sha Tau Kok (weakest geometry)

In addition, two significant height errors were found during the surveys in Ng Tung Chai and Ying Pun. The measured ellipsoidal heights varied from the known height value by over 1 m and the false measurements lasted about 1.5 minutes. Such significant height errors could not be identified on site by examining the CQ values. Also, in most cases, the poor measurements could be detected and eliminated if the plan CQ and height CQ values were taken into account.

However, a few number of gross errors in magnitude of more than one metre occurred during the measurements even with CQ values being checked within the tolerance. In order to identify such erroneous measurements, double shots or independent checking should be conducted throughout the survey.

## 4. GPS HEIGHT TRANSFORMATION BY INTERPOLATION OF GEOID-ELLIPSOID SEPARATION WITHIN A SMALL LOCAL AREA

For most land surveying and engineering applications, WGS 84 geodetic coordinates are required to be transformed into coordinates of local grid system such as HK1980 Grid. Serving this purpose, the Lands Department has prepared a set of 7-parameter datum transformation parameters (7p_ITRF96_HK80_V 1.0) and local map projection parameters for the territory.

While the horizontal transformation can achieve 1-2 cm accuracy, the height obtained by this method may not support precise height accuracy requirement due to the following reasons:- The 7-parameter datum transformation works between the WGS84 ellipsoid and the local ellipsoid (International Hayford 1910 Ellipsoid in the case of Hong Kong). However, the Hayford ellipsoid may not best fit the orthometric height levelling surface with reference to the Hong Kong Principal Datum (HKPD). Moreover, the 7-parameter datum transformation set was derived from using 12 trig stations on hilltops with trigonometric heights rather than precise levels. It is therefore the transformation set was not purposely designed for conversion between GPS surveyed ellipsoidal height and local height.

In order to obtain higher accuracy in the transformation of GPS heighting to local height values, the horizontal and height transformations should be handled separately. Most commercial GPS software programs contain function of separate handling of horizontal and height transformations. The GPS measured coordinates in WGS84 ellipsoid are transformed into the coordinates of a local ellipsoid with the use of 7-parameter transformation. The resulting horizontal component (latitude/longitude coordinates) is adopted while the height component is considered not accurate enough due the above-mentioned reasons. For the height transformation, a flat plane (or contour surface) derived from some local control points with both WGS84 ellipsoidal height and local heights (HKPD) can be used to interpolate the exact value of geoid-ellipsoid separation at a specific position within the interpolation area.

In Hong Kong, a total number of 74 control points with both WGS84 ellipsoidal heights and HKPD heights are used to form the Height Model of Hong Kong. The ellipsoidal heights of these points are determined by static GPS observations and their HKPD heights are surveyed by either precise levelling (Group 1 points) or combination of precise levelling and precise trigonometric heighting (Group 2 points). With the control point data of Hong Kong Height Model, users may use the interpolation method to convert the GPS surveyed ellipsoidal heights to HKPD heights.

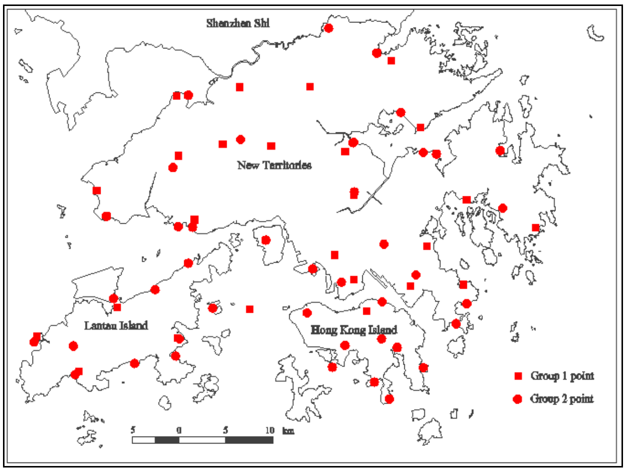


**Figure 13.** Location of control points with both WGS84-ellipsoidal height & HKPD height values

## 5. FURTHER IMPROVEMENTS

Despite the convenient use of the GPS height transformation by interpolation method, some limitations on the accuracy of such method should be noted. Firstly, the HKPD heights of some control points of the Hong Kong Height Model were only derived from trigonometric heighting method. Secondly, the WGS84 ellipsoidal heights of the control points were determined from a few hours of static GPS observations processed by commercial GPS software.

Apart from the inherent errors of the Hong Kong Height Model data, the accuracy of the height transformation by interpolation method is affected by the number and distribution of control points used as well as the terrain between the unknown and control points. A minimum of 4 control points scattered in good network geometry is recommended. The unknown points with levels to be converted should be inside the geometry of the control points to avoid extrapolation. For height transformation by interpolation in area with all control points at much higher altitude than the unknown points, the GPS observation error (due to tropospheric effect or others) would not be reflected in the statistical test result of the computation.

To improve the accuracy of the Hong Kong Height Model, re-observations and computations of GPS data taken at the control points (processed by using scientific GPS software) to increase the accuracy of WGS84 ellipsoidal heights of the points, and supply of precise levelling to the control points may be considered. Some new control points scattered at different altitudes may be added where the network geometry is weak to strengthen the height model control network of the territory.

Furthermore, it should also be noted that the height accuracy of GPS measurements is subjected to the influencing factors mentioned in the earlier part of this paper. Improvement to accuracy of the GPS measurement itself is most fundamental to the height accuracy obtained by GPS heighting. Derivation of some local tropospheric model with real-time data filtering would definitely help the GPS measurement accuracy, especially for RTK surveys.

Finally, a geoid model for Hong Kong should be created in the long run to provide a territory-wide solution of geoid-ellipsoid separation instead of individual solutions for small local areas. *Chen et al* (2001) has reviewed on the geometric, gravitational and hybrid approaches on creating the geoid model for Hong Kong. Some feasible studies may be considered for assessing the approaches and derive the method and procedures for implementation of the geoid model with an accuracy sufficient to support engineering works by GPS heighting.

## 6. CONCLUSION

The first part of this paper reviewed on the factors that significantly affect the accuracy of GPS heighting. The inherent weaker satellite geometry for heighting and tropospheric delay are the two major sources of errors that lead to less accurate ellipsoidal height measurement as compared to horizontal components. In addition, appropriate application of PCV calibration files in GPS data processing is critical for precise GPS heighting. Signals from low elevation are more susceptible to multipath than signals from high elevation and should also be noted in observations.

The second part of the paper has investigated the accuracy of GPS-derived height in both static and real-time methods with the use of the SatRef system data of Hong Kong. Height accuracy by static GPS would be dependent on the gradient of the baseline and the processing algorithm of GPS software. Higher accuracy on GPS heighting could be obtained with static GPS on flat baseline with stations at similar altitudes. The WGS84 ellipsoidal height values computed for such baseline are in the range of +/- 2cm throughout the test period of 18 months. For steep baseline, the spread of heighting by static GPS over 18 months are 4.2 cm for the 24-hour solution and 5.9 cm for the 6-hour solution with the use of the commercial GPS software. The results of the night-time observation (6-hour data) for steep baseline follow the trend of the 24-hour observation while the day-time observations (6-hour data) show different results. Such spread of heighting may be due to inaccurate atmospheric model by the GPS processing software or other unknown factors changing over the year. By using scientific GPS software with more rigorous troposphere estimation approach for mitigation of errors, it achieves a range of height variation within 2cm for flat baseline and 3cm for steep baseline.

For the real-time method, Network-RTK was found much easier to obtain a qualified CQ value for the survey as compared to Single-RTK. In addition, Network-RTK is more readily to achieve high accuracy GPS heighting of less than 3cm. For lower accuracy GPS heighting, the performance of single-RTK and Network-RTK (FKP mode) are similar, 80 % of the results of both methods are within 6 cm height accuracy and 92-93% of the results within 9 cm height accuracy. Furthermore, the temporal effects of the Network-RTK solutions under different network geometries are tested. The results indicate that Network-RTK under good geometry of reference stations achieves the best result of 3cm displacement from the known values for most epochs while the other cases of weak geometry achieve an accuracy of 5-7cm. However, for the case of Sha Tau Kok which is more than 10km away from all reference stations still achieve a height accuracy of 5 cm for most of the time, even better than the Ying Pun case. This is may be due to the similar altitude of Sha Tau Kok and the reference station HKFN rather than the correlation between height accuracy and geometry between the reference stations and unknown point. Furthermore, to ensure good quality of RTK measurements and eliminate gross errors, it is important to check the quality indicator and apply independent measurements during the survey.

Lastly, the height transformation by interpolation method of using known control points with both WGS84 ellipsoid height and HKPD height is introduced. Its limitations and possible improvements for the Hong Kong Height Model are discussed. Further feasibility study should be considered for the generation of a geoid model for Hong Kong in the long run to provide a territory-wide solution of geoid-ellipsoid separation instead of individual solutions for small local areas.

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## BIOGRAPHICAL NOTES

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