Investigating the Effect of Neglecting Parts of the EGD Geodetic Height on the Transformation from Helmert 1906 to WGS84

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Key words: Datum transformation, Molodensky, GPS, Terrestrial networks.

SUMMARY

Missing part of the information obtained from a set of data, is an interesting issue especially when dealing with old geodetic networks under modification. It is beneficial to study the effects of neglecting any unavailable part of the EGD height information, which could be the geoid undulation N^*, the orthometric height H or the geodetic height h altogether, relative to EGD at some or all of the network points, on the derived values of the seven transformation parameters, as well as on the computed values of the EGD Cartesian coordinates (x, y, z), as reflected later on the 3-D transformed coordinates to WGS84, have been theoretically analyzed. However, in this research such effects can be numerically investigated, particularly in case of analyzing the first order control networks. This is done, simply by determining the seven transformation parameters of Molodensky model twice, firstly by using the available geoid undulation, say from ASU2000 geoidal model, and secondly when neglecting such undulations. Then, each one of the two sets of transformation parameters will be used to transform selected check points to WGS84, from which the corresponding coordinate discrepancies can be computed. The required statistical parameters of these discrepancies is evaluated and analyzed. In addition, the same investigation of neglecting the geoid undulation N^*, is repeated once again when neglecting the entire EGD geodetic height h, in the process of estimating the corresponding transformation parameters. Neglecting any unavailable part of the EGD height information, has dual effects, firstly on estimating the transformation parameters of Molodensky model, and secondly on the computations of the 3-D Cartesian coordinates (x, y, z) relative to EGD. In this case, the WGS84 transformed coordinates will be affected by the wrongly estimated transformation parameters and/or the wrongly computed (X, Y, Z) Helmert coordinates. Both effects will be numerically analyzed here. In this context, three cases are studied: firstly the effect of wrong transformation parameters; secondly, the effect of wrong (x, y, z) Helmert only; and thirdly, the combined effect of wrong transformation parameters and wrong (x, y, z) Helmert combined together. The paper ends with conclusions and recommendations with respect to the suitability, accuracy and efficiency of the methods used.

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1. INTRODUCTION

Extension of existing geodetic control using traditional terrestrial techniques has become impractical nowadays, as far as the time and cost are concerned. Instead, the modern space techniques using artificial satellites, particularly the global positioning system GPS, are employed. Therefore, the combination of terrestrial and satellite networks is essential to benefit from the satellite modern measuring technology and accuracy. Since the GPS results are delivered in the satellite global average terrestrial system of WGS84 datum, while the terrestrial geodetic network results are expressed in the geodetic coordinate system of the national local or regional geodetic datum, which is the Egyptian geodetic datum (EGD of Helmert 1906) in our case here of Egypt, which is generally different from the WGS84 datum.

The combination of GPS and terrestrial coordinate systems should logically require a certain mathematical model with appropriate transformation parameters for carrying out the transformation and combination between these different systems of coordinates. The mathematical model used in this paper for datum transformation is the Molodensky seven parameters transformation model (Nassar et al., 1997), which were found to be suitable for this purpose, due to the use of one set of rotations only. However, the main problem is to find out constant or fixed values, for the required transformation parameters that will be reliable enough and valid for the entire region of interest. Of course, such a requirement necessitates the availability of a sufficient well distributed number of common points of known coordinates in the two systems under transformation, which generally cannot be satisfied in practice.

The neglect of any part of the height information such as the geoid undulation N^*, the orthometric height H, or the entire geodetic height h, relative to the terrestrial datum, which from now on will be denoted as h^*, is of great interest. This will affect, of course, the derived Molodensky seven transformation parameters, to a certain extend. In addition, the neglected height information will affect the transformed coordinates from EGD to WGS84 in two ways. The first effect will be due to the corresponding changes in the seven transformation parameters. The second effect will be due to the changes in the EGD computed Cartesian coordinates (X, Y, Z) of any point under transformation. Abd el Motaal and El Tokhey (1997), have investigated the above effect for the neglected geoid undulation N^* only. However, in our case here, these approach will be followed here also, however, with assuming the entire height information h^* to be missing, and hence will be neglected in the transformation process.

2. BACKGROUND

2.1 Datum definition

2.1.1 The Egyptian terrestrial geodetic datum (EGD) of Helmert 1906

Between 1853 and 1859, a complete survey of Egypt was done, but did not depend on a triangulation scheme (Cole, 1944). Later on, many attempts were made for constructing a geodetic triangulation networks, but they were not of higher order. In 1907, it became possible to begin a new work for establishing a geodetic triangulation frame for Egypt, which is considered to be the first national network to be established in Africa (Moritz, 1981). The main reason for carrying it out was to fix, with the greatest possible accuracy, fundamental control stations on which the cadastral survey and national mapping of Egypt was based (Awad, 1997). The Egyptian (EGD) Datum, the reference ellipsoid was chosen to be Helmert l906 ellipsoid, for which the size and shape defining parameters are the semi major axis a=6387200 and the reciprocal flattening 1/f =298.3. The initial point is “Venus” station on the Moquattam hill (near Cairo on the east side of the Nile).

2.1.2 Global satellite Geodetic Reference System WGS84 of GPS

WGS 84 is an earth fixed global reference frame, which is an average or conventional terrestrial coordinate system, including an earth model, which is defined by a set of primary and secondary parameters. The primary parameters define the size and shape of its geocentric ellipsoid, which are semi major axis a= 6378137 and reciprocal flattening 1/f =298.257.

2.2 Statement of Problem

With the existence of WGS84, nearly all the local and regional terrestrial geodetic datums have been redefined on the basis of satellite geocentric positioning approach. The relation of these local datums with WGS84 should be clearly identified to benefit from the new satellite technology advantages. For historical reasons each country has its own geodetic network and national geodetic reference frame, like EGD in our case here. Most of the national reference frames are not identical with the global WGS 84 reference frame. For practical reasons, they are surveyed and coordinated with respect to the national reference frame. The basic problem arising nowadays is to transform the national coordinates to the WGS 84 and express all coordinates in this global system, which will be accompanied of a number of problems that has to be investigated (El-Habiby, 2002). One of these problems is missing part or parts of the data set used in the transformation process. In this research, the neglect of parts of height information will be investigated, as mentioned before.

2.3 Combined Least Squares Adjustment for Molodensky Coordinate Transformation Model

Molodensky coordinate transformation model describes the relationship between any two different three-dimensional coordinate systems; say one global satellite system and one terrestrial system by seven unknown parameters, which are 3 shift components (X_0, Y_0, Z_0), three rotation parameters (ω_x, ω_y, ω_z) of the terrestrial network as connected to the datum initial point, relative to the geodetic terrestrial system, and one unique scale factor ($1+k$), for the entire geodetic network, to account for the scale error or distortion k between the satellite and terrestrial systems. The mathematical expression of Molodensky coordinate transformation model (e.g. El hoseny, 1990), takes the following form:

$$\vec{X}_{p_{GPS}} = \vec{X}_0 + \vec{x}_{i_{Helmert}} + (1+k)R\Delta\vec{x}_{ip_{Helmert}} \qquad (1)$$

Combined least squares adjustmetn is used in solving the transformation problem between EGD and WGS84. Molodensky model provides three parametric condition equations, for each common point of known Cartesian coordinates (X, Y, Z) in the new satellite WGS84 system and (x, y, z) in the old EGD system. At least 3 common points are needed for having a solution for the involved seven transformation parameters, however, a number of common points, greater than three, should be usually available for a reliable least squares solution, which is the case in this research. In this case, both vectors of observables L and of unknown parameters X, will is defined, as follows:

$$L^T_{1*3m} = (x_1, y_1, z_1), (X_1, Y_1, Z_1), (x_2, y_2, z_2), (X_2, Y_2, Z_2), ..., (x_m, y_m, z_m), (X_m, Y_m, Z_m) \qquad (2)$$

$$X^T_{1*7} = (X_0, Y_0, Z_0, \omega_x, \omega_y, \omega_z, k) \qquad (3)$$

More details about the design matrices of this least squares problem can be found in El-Habiby (2002).

3. EFFECT OF NEGLECTING ANY UNAVAILABLE PART OF HEIGHT INFORMATION RELATIVE TO THE TERRESTRIAL DATUM ON THE DATUM TRANSFORMATION PROCESS (THEORETICAL APPROACH)

The Egyptian geodetic control networks are termed as horizontal geodetic control networks, since they have been computed in two dimensions in terms of (φ, λ) only. In addition, generally, the orthometric height H of the networks points is also available. However, H sometimes is found to be missing for some of the network control points, while in the mean time the interest is concentrated on the 2-D geodetic coordinates (φ, λ). In the other hand, the geoid undulation N^* availability, depends mainly on an existing reliable geoid model for Egypt, such as the ASU geoid 2000 (Nassar et. al., 2000). However, in the past, such geoid undulations where not available, and thus had to be neglected into the underlying transformation process.

Let one starts with analyzing the effect of neglecting the height information h^* on the transformation parameters. In fact, since equation 1 represents three equations into seven transformation parameters, the required analysis will not be possible from the theoretical point of view. Its purpose can be sufficiently achieved, if one concentrates on the special case of considering three shift parameters only. In this case, equation 1 becomes.

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix}_{GPS} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_{GPS} + \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}_{Helmert} \qquad (4)$$

Keeping in mind that $(X, Y, Z)_{GPS}$ are known at each common point, which are usually given with high accuracy relative to the terrestrial coordinates, they can be considered as fixed quantities in the present analysis. Hence, by taking the total differential of equation 4, the effect of neglecting the height information on the derived translation parameters denoted here by δ can be written as:

$$\begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} = -\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \qquad (5)$$

The above results indicate that the effect of neglecting the height information on the three shift parameters, will be the same effect on the computed (*x*, *y*, *z*) geodetic coordinates relative to the old terrestrial datum, however, with a negative sign.

The effect of neglecting the height information on the computed (*x*, *y*, *z*) coordinates relative to the terrestrial datum, can be simply obtained, by evaluating the following equation twice:

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} (\nu_p + h_p)\cos\phi_p \cos\lambda_p \\ (\nu_p + h_p)\cos\phi_p \sin\lambda_p \\ (\nu_p(1-e^2) + h_p)\sin\phi_p \end{bmatrix} \qquad (6)$$

where *e* is the first eccentricity of Helmert ellipsoid.
Firstly, when taking the height information *h** into consideration; and secondly, when neglecting *h*, and then, taking the difference between the two computations, and one gets

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}_{Helmert} = h^* \begin{bmatrix} \cos\phi_i \cos\lambda_i \\ \cos\phi_i \sin\lambda_i \\ \sin\phi_i \end{bmatrix}_{Helmert} \qquad (7)$$

from equation 5and 7, one can write:

$$\begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} = -h^* \begin{bmatrix} \cos\phi \cos\lambda \\ \cos\phi \sin\lambda \\ \sin\phi \end{bmatrix}_{Helmert} \qquad (8)$$

Equation 8 shows that, the effect of neglecting the height information *h** on the transformation parameters (shift components in our case here), will be practically significant, in the order of a significant percentage of the height *h*.

At his point, one can start analyzing the effect of neglecting the height information *h** on the transformed curvilinear coordinates (φ, λ, *h*) relative to the new datum. In this case equation 6 becomes:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_{GPStransformed} = C \begin{bmatrix} \delta\phi \\ \delta\lambda \\ \delta h \end{bmatrix}_{GPStransformed} \qquad (9)$$

, in which C is 3*3 matrix, containing the partial derivatives of the right hand side of equation 6, with respect to (φ, λ, h) respectively, which takes the following form, that is

$$C = \begin{bmatrix} -(\rho + h^*)\sin\phi\cos\lambda & -(\nu + h^*)\cos\phi\sin\lambda & \cos\phi\cos\lambda \\ -(\rho + h^*)\sin\phi\sin\lambda & (\nu + h^*)\cos\phi\cos\lambda & \cos\phi\sin\lambda \\ (\rho + h^*)\cos\phi & 0 & \sin\phi \end{bmatrix} \qquad (10)$$

in which ρ is the meridional radius of curvature of Helmert ellipsoid, and ν is the prime vertical radius of curvature of Helmert ellipsoid.

Note here that, the elements of the C matrix can be numerically evaluated using the known (φ, λ) of the point relative to the Helmert ellipsoid. Then inversing equation 10, it yields the following

$$\begin{bmatrix} \delta\phi \\ \delta\lambda \\ \delta h \end{bmatrix}_{GPS} = C^{-1} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_{GPS} \qquad (11)$$

where C^{-1} is given by

$$C^{-1} = \begin{bmatrix} -\dfrac{\sin\phi\cos\lambda}{\rho + h^*} & -\dfrac{\sin\phi\sin\lambda}{\rho + h^*} & \dfrac{\cos\phi}{\rho + h^*} \\ -\dfrac{\sin\lambda}{(\nu + h^*)\cos\phi} & \dfrac{\cos\lambda}{(\nu + h^*)\cos\phi} & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \qquad (12)$$

the effect of neglecting the height information on the transformed GPS Cartesian coordinates, that is ($\delta X \ \ \delta Y \ \ \delta Z$), due to the corresponding effect of the translation components only, can be obtained by differentiating equation 4, and one finds that

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_{transformed} = \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix}_{GPS} \qquad (13)$$

Substituting from equations 8, 12, and 13 into equation 11, the final expression for the required effect in the transformed curvilinear coordinates, can be written as:

$$\begin{bmatrix} \delta\phi \\ \delta\lambda \\ \delta h \end{bmatrix} = C^{-1} \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} = -C^{-1} h^* \begin{bmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -h^* \end{bmatrix} \qquad (14)$$

From the last equation, one can see that, the effect of neglecting the height information on the 3-D transformed curvilinear coordinates is practically negligible on both (φ, λ) coordinates, in case of neglecting the respective part of the height information, when computing the three shift components only as the transformation parameters However, such effect on the geodetic height h will be in the order of the negligible h^* itself, but with negative sign.

Of course, the above simple and trivial case of considering three shift components only will not be the case in practice, since it is introduced here for the sake of demonstration only, and the Molodensky seven parameter model is generally used. In this case, the effect of neglecting the height information on the transformed (X, Y, Z) coordinates, due to the corresponding effect on the wrongly computed seven transformation parameters, can be simply obtained by differentiating equation 1, and one gets

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = \begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + A_\omega \begin{bmatrix} \delta\omega_x \\ \delta\omega_y \\ \delta\omega_z \end{bmatrix} + B_k \delta k \qquad (15)$$

where

$$A_\omega = (1+k)\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}, \quad B_k = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

substituting from equation 15 into 11, After matrix multiplication and algebraic manipulation the following expression will be found for the required $(\delta\phi\ \delta\lambda\ \delta h)$ GPS

$$\underbrace{\begin{bmatrix} \delta\phi \\ \delta\lambda \\ \delta h \end{bmatrix}_{GPS}}_{wrong\ transformation\ paramters} = C^{-1}\begin{bmatrix} \delta X_0 \\ \delta Y_0 \\ \delta Z_0 \end{bmatrix} + D_\omega \begin{bmatrix} \delta\omega_x \\ \delta\omega_y \\ \delta\omega_z \end{bmatrix} + E_k \delta k \qquad (16)$$

in which C^{-1} is given by equation 12, and D_ω and E_k will be as follows

$$D_\omega = C^{-1}A_\omega = (1+k)\begin{bmatrix} -\nu\left(1-e^2\sin^2\phi\right)+h\dfrac{\sin\lambda}{\rho+h^*} & \nu\left(1-e^2\sin^2\phi\right)+h\dfrac{\cos\lambda}{\rho+h^*} & 0 \\ \left(1-\dfrac{e^2\nu}{\nu+h^*}\right)\tan\phi\cos\lambda & \left(1-\dfrac{e^2\nu}{\nu+h^*}\right)\tan\phi\sin\lambda & -1 \\ -e^2\nu\sin\phi\cos\phi\sin\lambda & e^2\nu\sin\phi\cos\phi\cos\lambda & 0 \end{bmatrix} \qquad (17)$$

$$E_k = C^{-1}B_K = \begin{bmatrix} -\dfrac{e^2\nu\sin\phi\cos\phi}{\rho+h^*} \\ 0 \\ \nu\left(1-e^2\sin^2\phi\right)+h^* \end{bmatrix} \qquad (18)$$

The careful examination of equation 17, reveal that, the magnitude of the first two rows in the involved three matrices C^{-1}, D_ω, E_k are drastically small than the corresponding magnitude of the third row of these matrices. This indicates that, the final effect of neglecting the height information on the transformed (φ, λ) coordinates will be much smaller in magnitude, than the corresponding effect on the height component. This result confirms with the pervious result, of analyzing the simple case of three shift transformation parameters only.

Similarly, the effect of neglecting any part of the height information, when computing the $(x, y, z)_{EGD}$ cartesian coordinates, on the EGD transformed WGS84 coordinates, can be simply obtained by merging equations 7 and 11, and one gets

$$\begin{bmatrix} \delta\phi \\ \delta\lambda \\ \delta h \end{bmatrix}_{GPS} = C^{-1} h * \begin{bmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{bmatrix} \qquad (19)$$

Of course, the combined effect of wrongly transformation parameters, as well as wrongly computed $(x, y, z)_{EGD}$ due to the neglect of any part of the height information *h*, on the transformed WGS84 curvilinear coordinates (φ, λ, *h*), will be nothing else but, the algebraic addition of both equations, for the wrongly computed transformation parameters and equation 19, for the wrongly computed EGD coordinates. If one accepts that, the former effect has a negative sign according to equation 14, and the latter effect, has a positive sign according to equation 19, one should expect that, the algebraic sign of the combined effect, depend upon which individual effect is larger than the other.

The above theoretical analysis, will be verified again, however, by using actual data. In this context, the effect of neglecting a part of the height information only, namely the geoid undulation N*, will be investigated first. Then, the effect of neglecting the entire height information h, in the transformation 3-D curvilinear coordinates, will be studied.

4. METHODOLOGY OF INVESTIGATING THE EFFECT OF NEGLECTING ANY PART OF THE EGD GEODETIC HEIGHT INFORMATION *H** ON THE TRANSFORMED COORDINATES TO WGS84

The effects of neglecting any unavailable part of the EGD height information, is done, simply by determining the seven transformation parameters of Molodensky model twice, firstly by using the available geoid undulation, say from ASU2000 geoidal model, and secondly when neglecting such undulations. Then, each one of the two sets of transformation parameters will be used to transform a number of selected check points to WGS84, from which the corresponding coordinate discrepancies can be computed, by the following equations:

$$\begin{aligned} \delta\varphi &= (\varphi_{known} - \varphi_{transformed})_{WGS84} \\ \delta\lambda &= (\lambda_{known} - \lambda_{transformed})_{WGS84} \\ \delta h &= (h_{known} - h_{transformed})_{WGS84} \end{aligned} \qquad (20)$$

The above discrepancies are represented in meter units by using the following relationship:

$$\begin{aligned} \delta\phi_m &= \delta\phi_{radians} \times \rho \\ \delta\lambda_m &= \delta\lambda_{radians} \times \upsilon\cos\phi \end{aligned} \qquad (21)$$

The required statistical parameters of these discrepancies is evaluated and analyzed for neglecting the geoid undulation *N**, and the entire EGD geodetic height *h*.

On the other hand, neglecting any unavailable part of the EGD height information, has dual effects, firstly on estimating the transformation parameters of Molodensky model, and secondly on the computations of the 3-D Cartesian coordinates (*x*, *y*, *z*) relative to EGD. In this case, the WGS84 transformed coordinates will be affected by the wrongly estimated transformation parameters and/or the wrongly computed (*X*, *Y*, *Z*) Helmert coordinates. Although, the first effect only has been theoretically analyzed, as mentioned above, both effects will be numerically analyzed here. In this context, there will be three cases needed to be studied: firstly the effect of wrong transformation parameters; secondly, the effect of wrong (x, y, z) Helmert only; and thirdly, the combined effect of wrong transformation parameters and wrong (x, y, z) Helmert combined together.

5. ANALYSIS AND RESULTS

As has been stated before, the 3-D coordinates of the terrestrial geodetic network point positions, are usually given in two different groups. The first group is the horizontal coordinates (φ, λ). The second group includes the orthometric height H, which can be obtained preferably from sprit leveling measurements or if not possible, from trigonometric leveling measurements. Therefore, in order to get the EGD ellipsoidal height h one needs a reliable local or regional geoidal model, such as ASU93 or ASU2000 models, to get the corresponding geoid undulation N^*, which should be added to the orthometric height H to get h. However, in some occasions, such reliable geoid is not available, particularly over the desert areas, where no gravity measurements or geodetic control points exist. In other words, over these areas, the only available height information is the orthometric height H only.

In this case the 3-D coordinate transformation, using for instance, the Molodensky seven parameter model, from EGD to WGS84, will be performed on the 3-D coordinates (φ, λ, H), instead of the appropriate (φ, λ, h), after being, of course, transformed to their corresponding Cartesian coordinates $(X, Y, Z)_{\text{Helmert}}$, of course, one should expect some effects or changes in the transformed WGS84 coordinates, due to the unavailability, or hence, the neglect of the corresponding geoid undulations, into the performed transformation process. This effect can be looked as dual effect, as mentioned before.

According to the theoretical studies performed by El Tokhey and Abd El Motal (1997), as mentioned before, neglecting the geoid undulation at the EGD common points only, will have significant effects on the corresponding derived seven transformation parameters of Molodensky model. However, the corresponding effects on the transformed WGS84 coordinates, using these parameters, will be in the order few decimeters in both latitude and longitude coordinates, and may reach an error in the transformed ellipsoidal height h of the same order of the same magnitude of the geoid undulation itself, with a different algebraic sign. However, the effect of neglecting the geoid undulation N^* when computing the (x, y, z) terrestrial coordinates, on the transformed WGS84 coordinates, was not studied or reported by such investigation.

On the other hand, the latter, and hence combined effect of neglecting the geoid undulation, in computing the transformation parameters as well as in computing the terrestrial (x, y, z) coordinates, on the final transformed 3-D geodetic coordinates to WGS84, have been manipulated theoretically in section 3. Furthermore, such theoretical analysis was based on a generalized manner. In other words, such investigation assumes theoretically, the case in which, not only the geoid undulation N^*, but also the entire EGD height information is not available.

Consequently, the purpose of this research is to investigate such effect of neglecting the geoid undulation or the geodetic height, from the practical point of view. For this purpose, the 46 first order network points, with known coordinates relative to ESA, and WGS84 will be used (El-Habiby, 2002). More explicitly about 50% of these points will be used as data points for estimating the required seven transformation parameters, after neglecting the geoid undulation (or the entire geodetic height), and the remaining points will be used as the check points, at which the effect of neglecting any part of the height information on the transformed 3-D coordinates, will be investigated.

In this context, there will be two basic items, to be investigated, as far as their effect on the transformation process is concerned. The first item is the neglect of the geoid undulation N^* only, while the second item is the neglect of the total height information h only, relative to EGD terrestrial datum. Note here that, the neglect of the EGD geodetic height h all together will be equivalent to neglecting the orthometric H, as far as the final decision and conclusion is concerned, since for the Egyptian situation, the orthometric height H of the geodetic control points, is generally greater than the corresponding geoid undulation N^*. This is the case, since all first order control points are usually established at the more respected higher locations. Thus, considering the neglect of the EGD height h, in our case her, will represent the extreme situation.

In addition, and based on the above discussion, one can find that, the investigation of each one of the two items, will involve three different cases. The first case is evolved, when neglecting the missing part of the height information h^* in the determination of Molodensky seven transformation parameters only, which is at the common points only. The second case, will be arising, when neglecting the missing part of the height information h^* in the step of computing the EGD Cartesian coordinates (x, y, z), of course, of the points under transformation. Finally, the third case, will be found when neglecting the missing part of the height information h^*, during the entire transformation process, that is working with wrongly computed transformation parameters and the wrongly computed EGD Cartesian coordinates. Accordingly, this means that, six cases of study will be investigated. The three cases, corresponding to each one of the stated two items, will be handled below in two subsection.

5.1 Effect of Neglecting the Geoid Undulation N^* Only on the Transformation Process

In this subsection, the effect of neglecting the unavailable geoid undulation information N^*, on the transformation process from EGD to WGS84, will be investigated. The transformation process requires two groups of information to be known, namely: the adopted set of transformation parameters of Molodensky model, and 3-D Cartesian coordinates (x, y, z) of the points under transformation relative to EGD system, of the points to be transformed. The geoid undulation N^* is inevitably contained within the mathematical models of computing both the transformation parameters, using Molodensky model (equation 1), as well as for computing $(x, y, z)_{EGD}$, through the corresponding known (φ, λ, h). Thus neglecting the geoid undulation, in anyone of these two steps, or in both steps together will lead, of course, to some sort of effect on the final transformed coordinates from EGD to WGS84.

As stipulated above, there are three cases of the effect of neglecting the geoid undulation *N**, on the transformation process from EGD to WGS84. The first case is neglecting the geoid undulation when computing the transformation parameters only. The second case is neglecting the geoid undulation, when computing the EGD (*x, y, z*) cartesian coordinates only. The third case is when neglecting the geoid undulation during both steps of determining the transformation parameters and of computing the EGD coordinates together, which is nothing else but the integration of the first two mentioned cases together. The analysis of the three cases, from the point of view of their effect on the final transformed WGS84 coordinates, of the first order geodetic network of Egypt, will be given below.

The obtained results of the wrongly computed transformation parameters, due to the neglect of the geoid undulation only, along with their computed correct values (that is without neglecting *N**), are presented in table 1.

Table 1: Estimated transformation parameters with their estimated standard deviations for Molodensky model with seven transformation parameters (from EGD to WGS84) using ESA adjusted coordinates with and without neglecting the geoid undulation of the EGD

model\transformation		X_0(m)	Y_0(m)	z_0(m)	w_x(sec)	w_y(sec)	w_z(sec)	k(ppm)
Molodensky (without geoid undulation)	value	-126.5107	111.9381	-12.4565	2.39100	0.04488	-4.60511	7.64037
	sd	0.7131	0.7076	0.7088	0.41408	0.57062	0.64410	1.84983
Molodensky (without geodetic height)	value	153.1093	281.4888	150.8952	-79.01936	-27.21416	154.14177	59.51380
	sd	36.5269	36.2439	36.3090	21.20963	29.22110	32.99665	94.75567
Molodensky (correct)	value	-125.0421	112.8374	-11.6511	0.56309	0.90956	-2.47480	7.77760
	sd	0.8396	0.8331	0.8346	0.48755	0.67186	0.75839	2.17804

From this table, it is evident that neglecting geoid undulation *N**, during the determination of the seven transformation parameters, will yield slight variation in the linear transformation parameters, however, with a significant corresponding variation in all the three rotations. Table 2 indicates the effect of using such wrongly computed transformation parameters, on the final transformed WGS84 coordinates. From this table, the effect of this case on the transformed latitude and longitude coordinates, is found to be within few decimeters, while the corresponding effect on the transformed height is, generally, in the order of the neglected geoid undulation itself, however, with reversed sign. These results confirm very well with the theoretical analysis of this case given before in section 3.

Table 2 also illustrates the effect of the second case, that neglecting the geoid undulation *N**, when computing $(x, y, z)_{EGD}$ Cartesian coordinates only, that is using, at the same time the correctly computed set of transformation parameters which are shown at the end of table 1. These results indicate that the effect of this case on both the transformed latitude and longitude coordinates is, in the order of fraction of a millimeter, while the corresponding effect of the transformed geodetic height is nearly in the order of the neglected geoid undulation itself, with the same algebraic sign. On the other hand, when comparing the effects of the above two investigated cases together, one can see an interesting result, which is, the effect of neglecting the geoid undulation in the computation of the transformation parameters only, have a reversed algebraic sign, when compared with the case of neglecting the geoid undulation in the computation of the EGD Cartesian coordinates only.

Table 2: the effect of neglecting the geoid undulation in the different three cases accured during the transformation process from EGD to WGS84 on the final transformed 3-D curvilinear coordinates

Statistical elements 23 points	(Trans. para. Only)			(ESA EGD (x, y, z) Only)			(Trans. and EGD both)		
	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$
range	0.454982	0.35536	16.94333	0.0000	0.0004	28.4402	0.4548	0.3554	11.4969
max.	0.25132	0.09102	9.14756	0.0000	0.0002	12.9701	0.2512	0.0910	5.1743
mean	0.002006	-0.0093	0.997696	0.0000	0.0000	-1.3200	0.0020	-0.0093	-0.3223
min	-0.20366	-0.2643	-7.79577	0.0000	-0.0001	-15.4701	-0.2036	-0.2643	-6.3226
sd	0.12123	0.10304	4.139258	0.0000	0.0001	6.0165	0.1212	0.1030	2.3561
RMS	0.1186	0.1012	4.1694	0.0000	0.0001	6.0305	0.1186	0.1012	2.3268

According to the above findings, one should expect that the effect of the third case in which the geoid undulation is neglected in the whole process of transformation, will be simply the resultant of the addition of the results of the first two cases together, as can be seen from the last three columns of table 2. Again, all these results confirm our theoretical expectation stated in section 3.

5.2 Effect of Neglecting the Geodetic Height *h* on the Transformation Process

In this section, all the included presentation of results, as well as the associated analysis and remarks will be identical to what has been done in the previous section, for the three cases of neglecting the geoid undulation N^*, however, this time, as applied to the three corresponding cases of neglecting the entire EGD geodetic height h.

The first case here, will be the neglect of the geodetic height h, when computing the corresponding transformation parameters only. The corresponding wrongly computed seven transformation parameters are illustrated in the middle of table 1 as compared again to the correctly computed transformation parameters given at the end of the same table. From this table, it is not difficult to see that all the derived transformation parameters, based on the neglect of the EGD geodetic height h, of course, of used 23 common points, produce completely undesirable parameters, in such a way that, they are far from reality, in both their magnitude and estimated standard deviation, as computed to the corresponding correctly computed real values. This can be further verified, from the examination of the corresponding results of the transformed WGS84 coordinates, when using such wrongly computed transformation parameters. Such results are depicted in the first part of table 3.

Table 3: the effect of neglecting the EGD geodetic height in the different three cases occurred during the transformation process from EGD to WGS84 on the final transformed 3-D curvilinear coordinates

Statistical elements 23 points	(trans. para. Only)			(ESA EGD (x, y, z) Only)			(Trans. and EGD both)		
	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$	$\delta\phi(m)$	$\delta\lambda(m)$	$\delta h(m)$
range	32.6974	17.0300	818.9167	0.0011	0.0110	837.4865	32.7197	17.1489	947.8205
max.	11.9593	8.2971	96.9830	0.0008	-0.0006	878.9668	11.9866	8.2007	267.3693
mean	-0.5782	0.5542	-341.6091	0.0000	-0.0025	198.8985	-0.5605	0.3811	-142.7003
min	-20.7381	-8.7328	-721.9337	-0.0003	-0.0115	41.4803	-20.7331	-8.9481	-680.4512
sd	8.8736	3.7595	189.9188	0.0002	0.0025	188.2340	8.8750	3.7401	223.6313
RMS	8.6978	3.7184	388.8415	0.0002	0.0035	271.0206	8.6980	3.6777	261.1512

From this table one can visualize that, the effect of the first case on the transformed (φ, λ) coordinates can reach few tens of meters, while the effect on the transformed geodetic height can reach few hundred of meters (certain % of the neglected height itself), which is significantly different from its original EGD values, with a reverse algebraic sign. Of course, such case cannot be accepted in practice by any mean, and hence, such results should be rejected all together.

The above results are expected, since it deals with the projection of the terrain common points, on the surface of the EGD ellipsoid the is h_{EGD}=0, and not the original terrain point itself, whose 3-D correct coordinates should be correctly known in the two systems, for meaningful 3-D transformation process.

In other words, for 3-D coordinate transformation the adopted transformation parameters must be determined, using the correct height information h^* (that is including both H, N^*) of all involved common points between terrestrial and satellite systems, if one is looking for precise and reliable transformation process. On the other hand the user could derive the transformation parameters by using the orthometric height H only, that is neglecting the geoid undulation N^* only, when ever not available, for the use of (φ, λ) coordinates for mapping purposes only. In this case, the transformation height information will be useless. Finally, it is not allowed by any means, to derive the transformation parameters, by neglecting the entire height information, particularly the orthometric height.

On the other hand, the results obtained from the second case of neglecting the EGD geodetic height h when computing the EGD (x, y, z) Cartesian coordinates only, that is using, at the same time, the correctly derived set of transformation parameters, are given in the middle of table 3. These results indicate that, the effect of this case on the transformed (φ, λ) coordinates, is in the order of few millimeters, while the corresponding effect on the transformed geodetic height h is nearly the same as its original EGD value, with same algebraic sign.

Concerning the effect of the third case of neglecting the EGD geodetic height, when computing both transformation parameters and EGD Cartesian coordinates; the obtained results are shown in the last part of table 3. This case, can be, of course, considered as a trivial case, since the main influence factor will be the first case, of neglecting the EGD geodetic height when computing the transformation parameters, which is unpractical to consider. However, the corresponding results of the third case, are included just for completeness, although its results are known beforehand. Consequently, both first and third cases should not be implemented in practice at all; sine they will lead to extremely wrong transformation coordinates of WGS84. On the other hand, the second case, which is neglecting the EGD geodetic height, when computing the EGD Cartesian coordinates only, and using correctly computed transformation parameters, can be safely implemented in practice, in case of concentrating on the 2-D (φ, λ) coordinates only for mapping purposes, and no interest exist whatsoever in the resulted height information, after transformation to WGS84.

6. SUMMARY AND CONCLUSIONS

The effect of neglecting any part of the geodetic height information, relative to both EGD and WGS84 datums, during the above mentioned 3-D coordinate transformation process of geodetic control networks will have corresponding effect on the transformed WGS84 coordinate values. Such effect could be practically very significant, significant, or insignificant. For instance, and knowing that $h=H+N^*$ for EGD, if N^* is neglected during the steps of computing the $(x, y, z)_{EGD}$ Cartesian coordinates only, or during the step of computing the Molodensky seven transformation parameters only, or during the entire transformation process, it will produce an error in the order of 20 cm in the transformed (φ, λ) coordinates, and an error in the transformed height in the order of the neglected geoid undulation with positive sign, in the former step and with negative sign in the latter step, which is could be practically insignificant for some practical applications. The same results are obtained when neglecting the orthometric height H or the EGD geodetic height h, during the step of computing the EGD (x, y, z) cartesian coordinates, a far as the transformed (φ, λ) are concerned, however, the error in the transformed height will be in the order of the neglected height itself, again with the same sign. Also, if H_{EGD} or h_{EGD} is neglected during the step of computing the transformation parameters only, it will yield very large errors, in the order of tens and hundreds of meters, in the transformed coordinates and hence such case should be practically rejected.

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