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Specific procedures for monitoring geotectonic recent movements in the Košice Basin, Slovakia

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ABSTRACT

The research study deals with the investigation of deformations due to geotectonic recent movements in the east of Slovakia. Geodetic measurements were carried out using GNSS techniques. Satellite measurements were realized on points of the monitoring station in the Košice Basin. The adjustment with constraints and free adjustment were applied to adjustment processing of all points of the geodetic network in the monitoring station. Transformation of coordinates from WGS-84 into the national geodetic system of Slovakia was realized on the basis of the spatial Helmert transformation. The main objective of deformation measurements was to determine geotectonic recent movements in the city conurbation of the city of Košice. The deformation investigation was carried out in the Košice Basin since 1997. Satellite measurements were periodically realized twice a year, i.e. in spring and autumn. 3D coordinate differences presented by the deformation vectors were subjected to the selected test statistics. The outputs from deformation investigations were provided to the municipalities of the city of Košice and Košice province. For the monitoring period of nineteen years the expected geotectonic recent movements in the monitored area of the Košice Basin exceed the safety limit values were not recorded.

INTRODUCTION AND RESEARCH OVERVIEW

Geodetic and geophysical monitoring the earth surface deformations due to movements of terrestrial tectonic blocks of varying magnitude have been devoted to the past and many scientific studies are still continuing worldwide. For example, the almost historical study by L. Ahorner of 1975 deals with current so-called “seismotectonic” block movements along tectonic faults in central Europe (Ahorner, 1975). A similar study of recent tectonic stresses in central Europe, particularly in the Pannonia region, with modelling movement elements was also devoted to the study of G. Bada and the team of authors from the end of the 20th century (Bada et al., 1998). A. Schlatte and the team of authors present the scientific study results on the recent vertical movements from in the vicinity of the city of Basel, Switzerland obtained from a precise levelling (Schlatte et al., 2005). Description of so-called "tectonic reorganizations" in the Mediterranean due to geotectonic recent movements is a subject of the scientific study by S. Goes and his colleagues (Goes et al., 2004). Some scientific studies aimed at exploring recent geotectonic movements in Slovakia are many and broad-spectrum. In spite of a relatively long research on the recent geotectonic movements in east Slovakia from a geodetic point of view, relatively only a few outputs were published (Sedlák, 2000).

The presented study deals with the investigation of deformations due to the geotectonic recent movements in the east of Slovakia. Deformation measurements were carried out using GPS and GNSS satellite measurement techniques. NAVSTAR GPS (NAVigation System for Timing And Ranging - Global Positioning System, hereinafter referred to as GPS) and later also GNSS (Global Navigation Satellite Systems) measurements were implemented at points of the geodetic network in a frame of the monitoring station in the Košice Basin in the south-eastern region of Slovakia. Adjustment with constraints and free adjustment were applied to the numerical processing (adjustment calculus) of the co-ordinates of all points of the geodetic network.

Transformation of the co-ordinates from WGS-84 (World Geodetic System of 1984) into ŠTS (State Trigonometric Network) and Bpv (Baltic Vertical Datum After Adjustment) is the most often carried out using the seven-element (spatial, 3D) Helmert transformation with using three identical geodetic points (Sedlák, 2014; Späth, 2004; Volk, 2003; Watson, 2006). The geodetic network in the Košice Basin was adjusted by two ways. If the datum parameters of the geodetic points are absolutely accurate then the adjustment with constraints was considered. If the datum parameters of the geodetic points are determined with a specific accuracy (what has also an influence on an accuracy of adjusted parameters except the measured accuracy) then the free adjustment was considered. The aim of these deformation measurements was determining the geotectonic recent movements and landslides especially in the urban agglomeration of the city of Košice. The city is situated in the centre of the Košice

Basin. The deformation investigations are carried out by the extensive geodetic satellite measurements for a long period (since 1997) in the Košice Basin. GPS and GNSS (hereinafter GPS/GNSS) measurements were realized every year since 1997. All points of the geodetic network (reference and object points) were measured by a static method. The object points of the geodetic network were determined by the method of double GPS/GNSS vector always to two reference points. The geodetic network with the object points is stabilized on the territories of the Košice Basin, in which according to the interested geologist the geotectonic recent movements are presupposed. The major tectonic fault in the Košice Basin, according to which two expressive geological blocks of the top part of the earth crust could be each other move, passes in the line of the north-south direction along the Hornád River, which flows through the city of Košice. The secondary tectonic faults of smaller extent are in the direction perpendicular to the major fault. 3D co-ordinate differences (deformation vectors) of the object points of the geodetic network have been subjected to the test statistics hypotheses. 3D modelling deformations in the Košice Basin are applied in GIS space for utilization of the city administration of the city of Košice. The deformation investigations outputs have confirmed the assumption that for the monitored period 1997-2016 in the Košice Basin were not recorded geotectonic recent movements beyond the boundary limit of 25 mm.

At the first GPS and later GNSS measurements are realized at points of the geodetic network of the monitoring station localized in the Košice Basin, east Slovakia (Figure 1). The aim of these measurements was determining recent geotectonic movements in the urban agglomeration of the city of Košice (Sedlák, 2000). Satellite (GPS/GNSS) measurements are periodically realized twice a year (spring and autumn). Altogether, 16 points of the geodetic network in the monitoring station in the Košice Basin are measured by means of using the GPS/GNSS kinematic method. Priority of the chosen kinematic method for our measurements is above all a high accuracy in determining point positions which is conditioned by the period of measurement on a determined point (approx. 5 minutes). The accuracy of the kinematic methods is same like at the static method (Sedlák, 2000). A full paper is required for every accepted abstract, either oral or poster. Authors can submit their contribution as a peer review or non-peer review paper.

The determined geodetic network points are solved by double GPS/GNSS vector technology always regarding two reference points. The main tectonic fault in the Košice Basin according to which two expressive geological faults of the earth ground blocks should move is assumed in the north-south direction along the Hornád River. The secondary tectonic faults of smaller extent are in the direction perpendicular to the Hornád fault, i.e. in the east-west direction. These secondary tectonic faults are mutually parallel (Sedlák, 2000). Three double-frequency GPS/GNSS receivers were used to satellite measurements. Co-ordinates of all points of the geodetic network were transformed from WGS-84 into S-JTSK and Bpv. The non-linear rotary matrix method was applied to the adjustment. After transformation, the co-ordinates were consecutively adjusted by the adjustment with constraints.

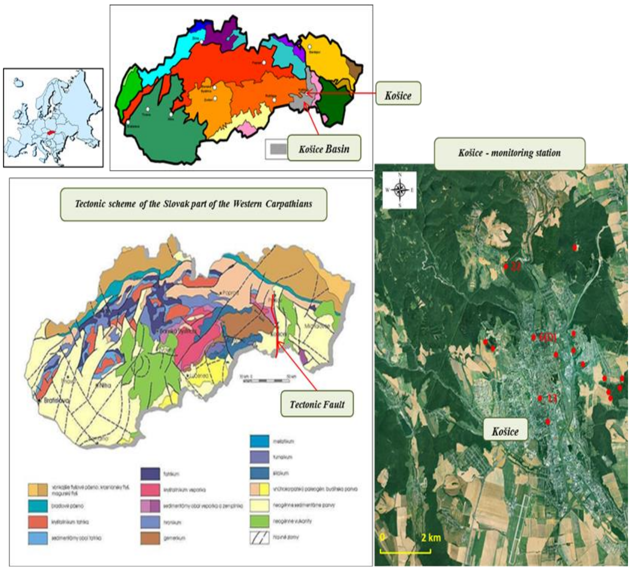

Figure1. Ortho-photo map of the city of Košice and with constellation of the geodetic network points (red) in the Košice Basin monitoring station

The geodetic network of the Košice Basin monitoring station (Figure 1) includes also the reference points (A1, B10) of the geodetic network in the former mining area of the Košice-Bankov magnesite mine (a research project for monitoring mining subsidence) (Sedlák, 2014) as well as reference points (KN1-KN5) of the geodetic network in the Košice city district of Košická Nová Ves (a research project for monitoring landslides). All points of the geodetic network of the Košice Basin monitoring station have a deep stabilization (stone blocks and buried in concrete iron casing pipes of drill holes), deep 2 to 12 m.

INTRODUCTION AND RESEARCH OVERVIEW CO-ORDINATE TRANSFORMATION OF THE GEODETIC NETWORK POINTS FROM WGS-84 INTO S-JTSK AND B PPV

GPS and GNSS are geodetic systems use the global co-ordinate system WGS-84 with the purpose of expressing the position anywhere in the earth and space. The reality that systems GPS/GNSS determine a localization and navigation in the global dimensions is its priority. However, the disadvantage for surveying is its limitation in a plane rectangular system that is the Slovak national geodetic co-ordinate system S-JTSK.

Transformation of co-ordinates from WGS-84 into local topocentric horizontal system

The co-ordinate axes $(X,Y,Z)_{WGS\text{-}84}$ with the origin in the centre of ellipsoid create the system $S_{WGS\text{-}84}$ (Figure 2). The co-ordinate axes $(X'',Y'',Z'')_{S''}$ create the local topocentric horizontal co-ordinate system *S´´*. Its origin lies in the point *D*. The point *D* is one of the points of the geodetic network of the monitoring station in the Košice Basin. This point is situated approximately in the geodetic network centre. To assume that the geodetic horizon in the point *D* is a parallel plane to the system S-JTSK is only possible in a case of the local geodetic network with a small dimension (if distances between network points are not longer than a few kilometres). Table 1 presents co-ordinates of the point *D* in WGS-84 and in the system S-JTSK. Axes *X´´* and *Y´´* lie in a geodetic horizon of the point *D*. The axis *X´´* is oriented into the south branch of a meridian. The axis *+Z´´* lies in a normal line and is directed into the geodetic zenith and the axis *+Y´´* creates with the mentioned axes a left-hand system.

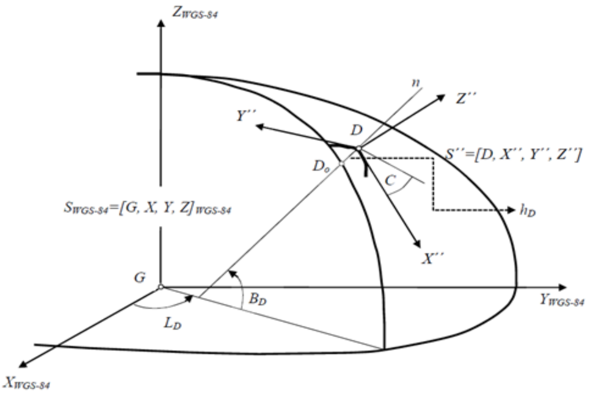

Figure 2. 3D co-ordinate systems SWGS84 and S´´

Table 1. 3D co-ordinates of the point D in WGS-84 and S-JTSK before any adjustment

3D-axis	***WGS-84*** [m]	***S-JTSK*** [m]
X	3 927 761.772	1 237 997.588
Y	1 528 741.401	262 066.547
Z	4 771 351.232	212.074
Ellipsoid latitude: B_D = 48° 44' 7.12" Ellipsoid longitude: L_D = 21° 16' 21.22" Meridian convergence: C = 2° 39' 42.89" *Notice: The point D as the origin of the local topocentric co-ordinate system S´´ is the point No. 6 of the geodetic network in the Košice Basin.*		

The co-ordinates transformation of from the system $S_{WGS\text{-}84}$ into the system *S´´* is possible according to (index *D* means that the tangent point is considered)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S''} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \boldsymbol{R}_Y(90^\circ - -B_D) . \boldsymbol{R}_Z(L_D) . \begin{bmatrix} X - X_D \\ Y - Y_D \\ Z - Z_D \end{bmatrix} \qquad (1)$$

where B_D,(L_D) = ellipsoidal geocentric latitude (longitude) of point *D*
$\boldsymbol{R}_Y(90^\circ - B_D)$, $\boldsymbol{R}_Z(L_D)$ = non-linear rotation matrices

The co-ordinates of the geodetic network points in WGS-84 are obtained by a convenient adjustment of measurements, which were realised by GPS/GNSS. The right-hand system is changed into left-hand which is preferred in geodesy. This change can be reached by multiplying a diagonal matrix with the diagonal (1,-1,1). The point *D* has the co-ordinates $(X,Y,Z)^T$ = $(0,0,0)^T$ in the system *S´´*.

The system *S´´* is also possible to obtain by using the following equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S''} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \boldsymbol{R}_Y(90^\circ - -B_D) .\ \boldsymbol{R}_Z(L_D) . \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS-84} + \begin{bmatrix} -\Delta X \\ 0 \\ -\Delta Z \end{bmatrix} \qquad (2)$$

where ΔX =normal line distance from ellipsoid centre in direction of axis $X_{WGS\text{-}84}$ of system $S_{WGS\text{-}84}$
ΔZ = displacement (distance) of plane $(XY)_{WGS\text{-}84}$ (in point *D*) in normal line direction

In the sense of Figure 3, ΔX and ΔZ values are calculated using the following equations

$$\begin{aligned} \Delta X &= N(B_D) e^2 \, sinB_D \, cosB_D \\ \Delta Z &= N(B_D) - \Delta X tgB_D + h_D \end{aligned} \qquad (3)$$

where $N(B_D)$ = transverse radius of curvature in point *D*
e = numerical eccentricity
h_D = ellipsoid height of point *D*

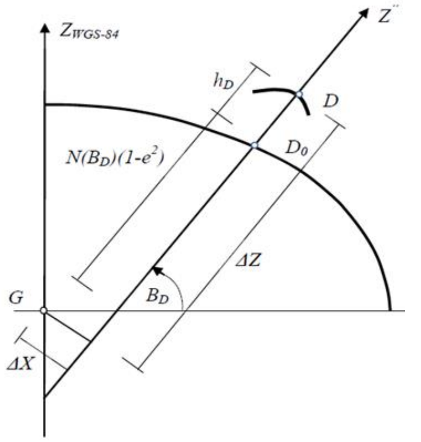

Figure 3. Transformation from the geocentric into topocentric system

Transformation between the local *S´* and the commonly used national system S-JTSK should be the simplest in their contact point for a purpose of using the state trigonometric network. It means that the co-ordinates of the local geodetic network should not much differ from S-JTSK. It can be reached by turning the system S'' in the point D about the meridian convergence C (Fig. 3) of the co-ordinate system S-JTSK and by displacement of the origin of the system S'' round the rectangular co-ordinates $(X_D, Y_D)_{S\text{-}JTSK}$ in S-JTSK. This mentioned transformation is expressed by

$$\begin{bmatrix}X\\Y\\Z\end{bmatrix}_{S'} = \boldsymbol{R}_M(-C).\begin{bmatrix}X\\Y\\Z\end{bmatrix}_{S''} + \begin{bmatrix}X_D\\Y_D\\H_D\end{bmatrix}_{S-JTSK,Bpv} \tag{4}$$

where $\boldsymbol{R}_M(-C)$ = rotation matrix
H_D = altitude of point D in Bpv

In this way we obtained the topocentric horizontal co-ordinate system which the co-ordinate axes $(X', Y')_{S'}$ lie in the geocentric horizon of the point of a normal intersection of the point D with used geoid (the Besell ellipsoid). Because the point D has the co-ordinates $(0,0,0)^T$ in the system S'', then this point will obtain identical co-ordinates with the co-ordinates in S-JTSK by adding the vector $(X_D, Y_D, Z_D)^T_{S-JTSK}$.

Transformation of co-ordinates into the local co-ordinate system

It is not possible to calculate directly the values of elements which would harmonise with the values measured in a terrain using the co-ordinates of points $(X', Y', Z')_{S'}$ in the system S'. These co-ordinates are influenced by the Earth curvature and also by a relative difference in elevation of point over the horizon plane. Regarding to a network dimension, we can substitute the ellipsoid by the reference sphere whose radius is equal to the mean radius of the Earth curvature R in the point D according to

$$R = \sqrt{M(B_D)N(B_D)} \tag{5}$$

where M (N) = meridian (transversal) radius of curvature of the Bessel ellipsoid

An influence of the difference of elevation is possible to eliminate if the co-ordinates X' and Y' are reduced into the intersection of the normal with the tangent plane, or with the basic plane. The reduction from a relative difference of elevation is possible to influence significantly by moving the geodetic horizon into the basic plane of the geodetic network in the height h_D. This height equals to approximate mean elevation value in which geodetic measurements are realised. The reduction of the co-ordinates X' and Y' of the geodetic network points in the system S' into the intersection of the normal with the basic plane equals to a gnomic projection which regarding to the network dimension is considered a conform projection.

The presented method has several priorities. Above all, it is the fact that a relative high accuracy in determining point positions by means of using GPS/GNSS technology is not less than at a classic terrestrial surveying. Similarly a measurement on one identical point is only enough instead of three identical points, by that a transmission of some errors at the transformation can be reduced. Reductions from elevation and cartographic distortion are needed in S-JTSK where only reduction from a relative height or elevation is considered in some local system. This reduction is also minimised by a convenient choice of transformation parameters.

2D geodetic network adjustment with constraints

Many geodetic networks can be adjusted by two ways. If we consider datum parameters as absolutely accurate and we do not include them into an adjustment process, the adjustment with constraints is considered in this case. In case that datum parameters are also determined with a concrete accuracy that has an influence on an accuracy of adjustment parameters except for measurement accuracy. In this case a network can be adjusted by a free adjustment with consideration of datum parameters. Regarding the applied confinement adjustment in the geodetic network in the Košice Basin, the theoretic procedure of this adjustment is presented, which is the most convenient for the Slovak national co-ordinate system S-JTSK.

The Least Mean Square Method (LMSM) is chosen as an estimate principle, and the inverse solution is chosen as a mathematical principle, which is a standard procedure in an adjustment of the geodetic network. After adjustment the position and form of the geodetic network are changed but the datum point positions are not changed (datum points are considered as absolutely accurate). This fact is presented so that the configuration matrix $\boldsymbol{A}$ and also matrix $\boldsymbol{N}$ of the geodetic network will be regular; the rank of matrices $h(\boldsymbol{A}) = k$ and $h(\boldsymbol{N}) = k$ (k is number of determined parameters). For the adjustment the following four vectors and matrices are necessary to be: $\boldsymbol{C}^o_{k,1}$ is the vector of approximate co-ordinates of the network points; $\boldsymbol{l}^o(n,1)$ is the vector of approximate values of measured observations and where n is a number of measurements; $\boldsymbol{A}(n,k)$ is the configuration matrix of the geodetic network. Terms of this matrix are determined by partial derivatives of the model equations L according to the studied parameters. For a check it is possible to spread this matrix for the datum (object) points too, by this way we can get a global configuration matrix. A sum of the terms in a row of the constructed matrix must be equalled to zero. However, we only consider a sub-matrix containing the determined points at calculations; $\boldsymbol{Q}_l(n,1)$ is the cofactor matrix of the measured quantities. It is the matrix in which the cofactors q_{l_i} of the measured quantities are occurred.

Solving equations of the estimate statistic model by means of using LMSM we will get the following the linear equation

$$\boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.\boldsymbol{A}.d\widehat{\boldsymbol{C}} - \boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.d\boldsymbol{l} = 0 \tag{6}$$

where $d\boldsymbol{l} = \boldsymbol{l} - \boldsymbol{l}^o$ = vector of reduced observations
$\boldsymbol{l}$ = vector of observed quantities
$\boldsymbol{l}^o$= vector of approximate values of measured quantities

If we indicate $\boldsymbol{N} = \boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.\boldsymbol{A}$ and $n = \boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.d\boldsymbol{l}$, we will get the following equation for the vector of the adjusted co-ordinate complements $d\widehat{\boldsymbol{C}}$

$$d\widehat{\boldsymbol{C}} = \boldsymbol{N}^{-1}.n \tag{7}$$

$$d\widehat{\boldsymbol{C}} = (\boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.\boldsymbol{A})^{-1}.\boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.d\boldsymbol{l} \tag{8}$$

After adding $d\widehat{\boldsymbol{C}}$ to the vector of the approximate co-ordinates of points we will obtain the adjusted co-ordinates $\widehat{\boldsymbol{C}}$ of points according to

$$\widehat{\boldsymbol{C}} = \boldsymbol{C}^o + d\widehat{\boldsymbol{C}} \tag{9}$$

The quality of the adjusted network is universally characterised by two matrices: the cofactor matrix of the co-ordinates estimates $\boldsymbol{Q}_{\hat{C}}$ of the co-ordinates

$$\boldsymbol{Q}_{\hat{C}} = (\boldsymbol{A}^T.\boldsymbol{Q}_l^{-1}.\boldsymbol{A})^{-1} = \boldsymbol{N}^{-1} \tag{10}$$

and the covariance matrix of the estimates $\Sigma_{\hat{C}}$ of the co-ordinates

$$\Sigma_{\hat{C}} = s_0^2.\boldsymbol{Q}_{\hat{C}} \tag{11}$$

The vector of corrections $\boldsymbol{v}$ is determined by

$$\boldsymbol{v} = \boldsymbol{A}.d\widehat{\boldsymbol{C}} - d\boldsymbol{l} \tag{12}$$

The covariance estimates of the co-ordinates are situated on a diagonal of the covariance matrix in a direction of individual axes. The adjusted values of the measured terms $\hat{\boldsymbol{l}} = \boldsymbol{l} + \boldsymbol{v}$ are also determined in a frame of an adjustment.

The deformation vector d was estimated by a simple way, i.e. algebraic calculation in rectangular triangles in the plane of S-JTSK. The position deformation vector presents deformations in a plane of X and Y axes and height deformation vector presents deformations (subsidence) in the height system Bpv as a difference between the heights on the geodetic network points.

Accuracy and quality assessment of geodetic network

2D and 3D accuracy of the geodetic network points in the Košice Basin in was appreciated by the global and the local indices. The global indices were used for the accuracy consideration of whole network and these indices were numerically expressed. The following global indices were applied: $tr\ (\Sigma_{\hat{C}})$, i.e. the track of the covariance matrix $\Sigma_{\hat{C}}$, the global indices volume and $det\ (\Sigma_{\hat{C}})$, i.e. the determinant of the covariance matrix $\Sigma_{\hat{C}}$ was used. The local indices (3D mean error, mean co-ordinate error, confidence absolute ellipses) were applied separately for each point and these indices can characterize the reliability of whole geodetic network points.

Table 2 presents values of the spatial (3D) deformation vectors presented by the co-ordinate differences $d\hat{\widehat{\boldsymbol{C}}}$ of the final adjusted co-ordinates of the geodetic network points (year 1997/year 2016).

3D deformation vector values confirm possibility in the deformation vectors valuation according to the above presented theory. However, the deformation vector values need not mean any displacement of the points. In spite of the fact that the network points were adjusted according to the conventional manner by the Gauss-Markov model, the deformation vector values can be burdened by the accumulation of surveying errors. Therefore, for their prominence testing it is required to carry out testing the deformation vector by the global and localization test of the congruence (see the chapter: Global geodetic point congruence test). In the last GNSS measurement during the autumn 2016 the deformation vectors on the tested points (No.: 6, 10, 29, 8H, A1, B10, C21, D7, 11V and KN1-KN5) of the monitoring station in the Košice-Basin were ranged from 9.7 mm (point No. 10, year 2016) to 24.1 mm (point No. 11V, year 2016) (Table 2, Figure 4).

Table 2. Adjusted values - deformation vector $d\hat{\widehat{\boldsymbol{C}}}$ (years 1997/2016)

Point	$d\hat{\widehat{\boldsymbol{C}}}$ [mm]
6	16.4 / 17.6
10	9.8 / 9.7
29	23.9 / 23.0
8H	24.0 / 23.8
A1	18.9 / 17.8
B10	22.8 / 21.7
C21	21.7 / 22.0
D7	20.7 / 18.9
11V	21.9 / 24.1
KN1	22.9 / 23.0
KN2	16.7 / 15.8
KN3	20.2 / 23.3
KN4	21.4 / 19.9
KN5	16.5 / 17.1

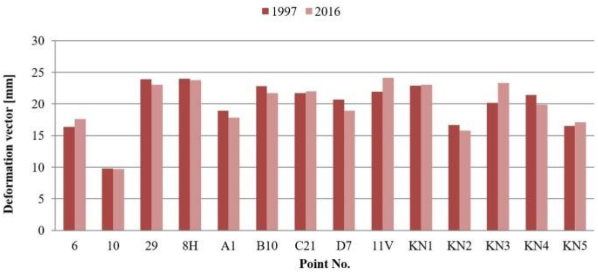

Figure 4. Graphic representation of 3D deformation vectors on the tested monitoring station points; years: 1997, 2016

CONCLUSIONS

The results of measurements in the geodetic network of the Košice Basin by GPS/GNSS technology confirmed a typical event of using these satellite measurements in the geodetic network with extended applications in geodesy and surveying. The applied GPS/GNSS kinematic method at all measurements shows on a high accuracy of satellite measurements. The reached results of the presented transformation procedures refer to the adaptability of transformations from WGS-84 into the national geodetic grid S-JTSK and Bpv. The chosen confinement adjustment by means of using the Gauss-Markov model is demonstrated as the most suitable mathematical model in an adjustment of the geodetic network in the Košice Basin locality. From Table 2 it can be deduced that the values of the deformation vectors (presented by the co-ordinate differences of the final adjusted co-ordinates of the geodetic network points of the monitoring station in the Košice Basin) did not exceed the value of 25 mm. This value (25 mm) was recommended by geotechnical experts as a maximum safety and limit value for any earth surface movements in the Košice Basin site. Because all deformation vectors are in this limit value, we did not presuppose any recent geotectonic movements in the Košice Basin. The research results of the recent geotectonic movements in the Kosice Basin and city of Košice were delivered annually to the municipalities of the Košice province and city of Košice.

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