Vertical Reference Frames in Practice
– Time Dependence & Transformations

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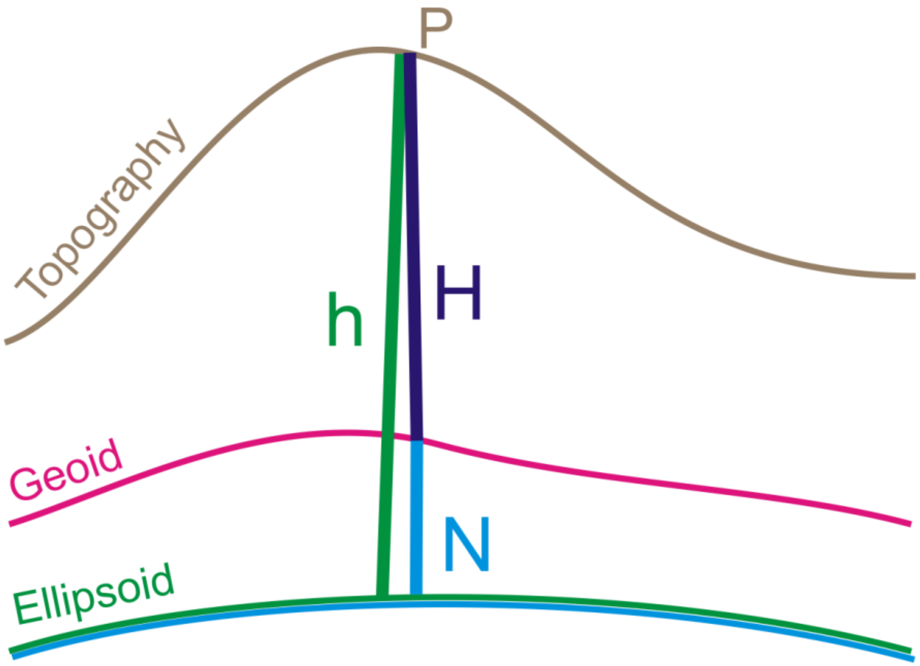
Putting h, H, N together

1) Ellipsoidal heights h and (quasi-)geoid undulations N must be given wrt the **same ellipsoid**:
    - $[X, Y, Z] \Leftrightarrow [\varphi, \lambda, h]$
    - Reference field (surface) for solving the GBVP and for scaling global gravity models (GGM)



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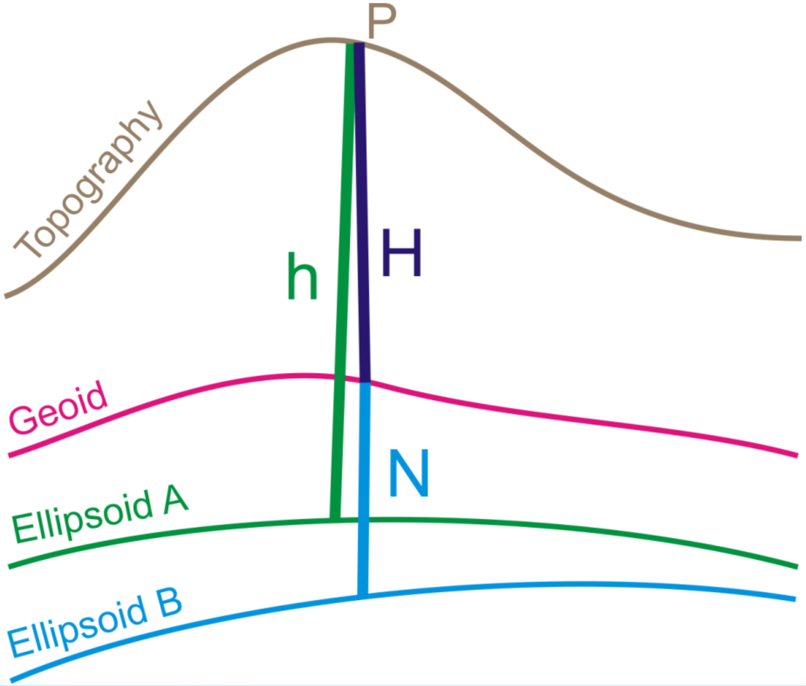
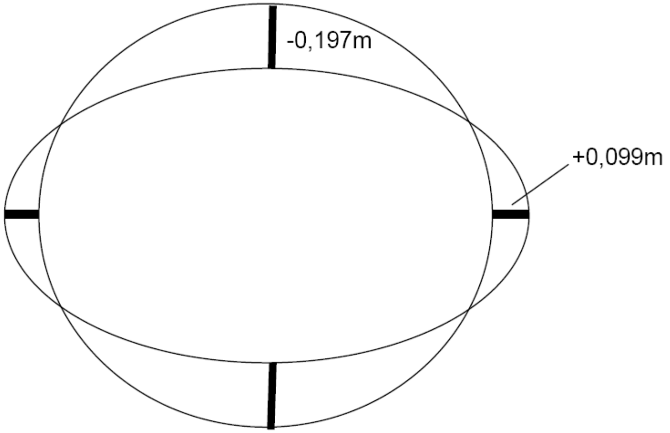
In practice:

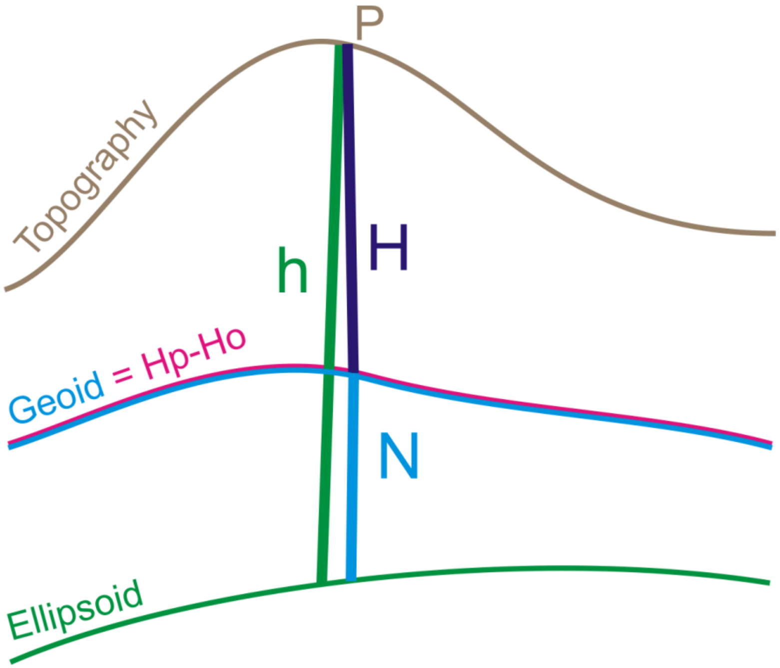
- Different ellipsoid parameters (e.g. a, GM) in geometry and gravity
- Different tide systems for h and N *(see later slides re Tides)*:
    - Oceanography, satellite altimetry, levelling in **mean-tide system**
    - ITRF positions, GRS80, some geoids in **tide-free system**
    - Some geoids, terrestrial gravity data in **zero-tide system**



Differences between mean and zero tide geoids



2) Physical heights H and (quasi-)geoid undulations N must reflect the same **reference surface**:
   - H_p (from levelling) – H_0 (datum point) → geoid from geometry
   - N (from the GBVP) → geoid from gravity



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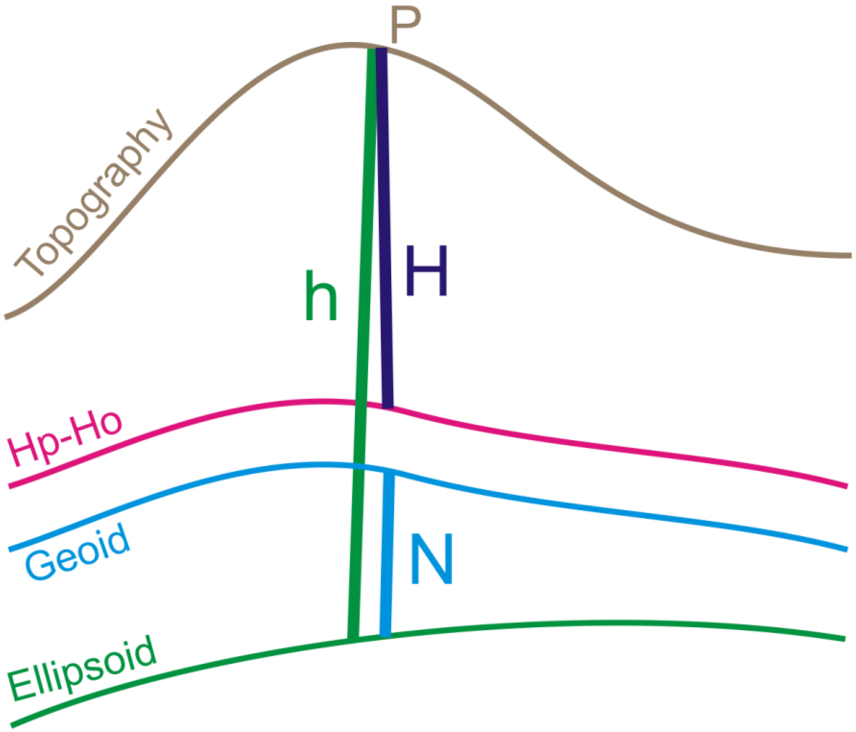
In practice:

- Orthometric heights and geoid from GBVP with different hypotheses
- Different tide systems for H and N
- Systematic errors over long distances in levelling (reliability of H_p-H_0)



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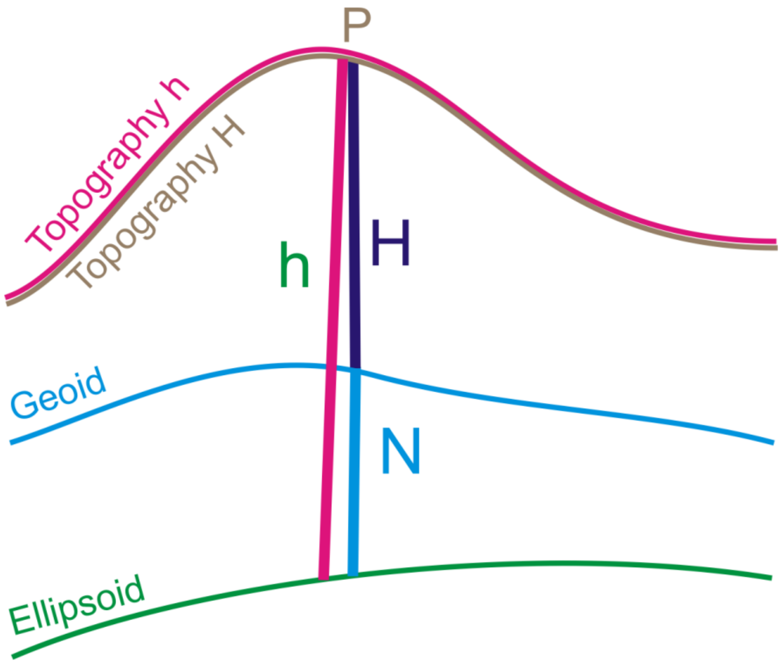
3) Physical heights H and ellipsoidal heights h must represent the **same Earth's surface**



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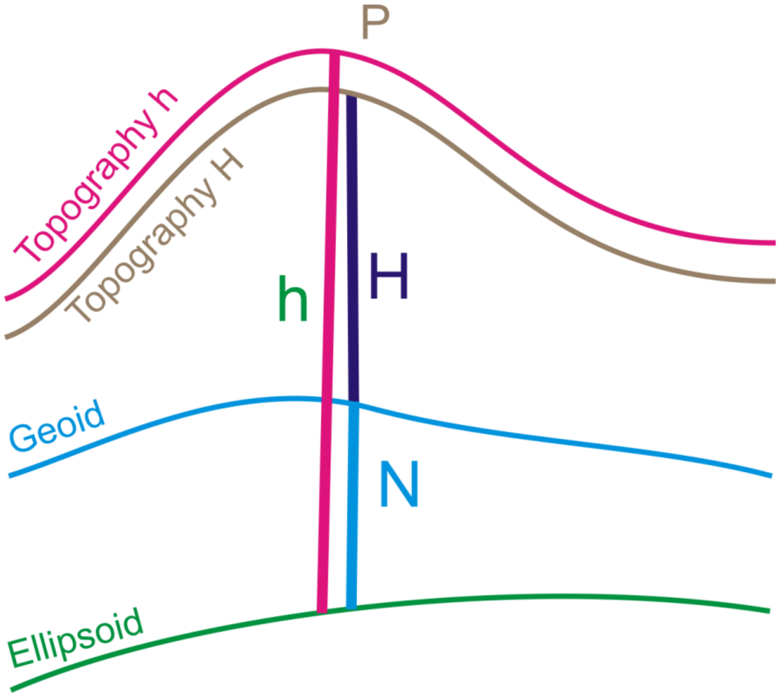
In practice:

- Different reference epochs (with unknown *dH/dt*)
- Different reductions (Earth-, ocean & atmospheric tides, ocean & atmospheric loading, post-glacial rebound, etc.)



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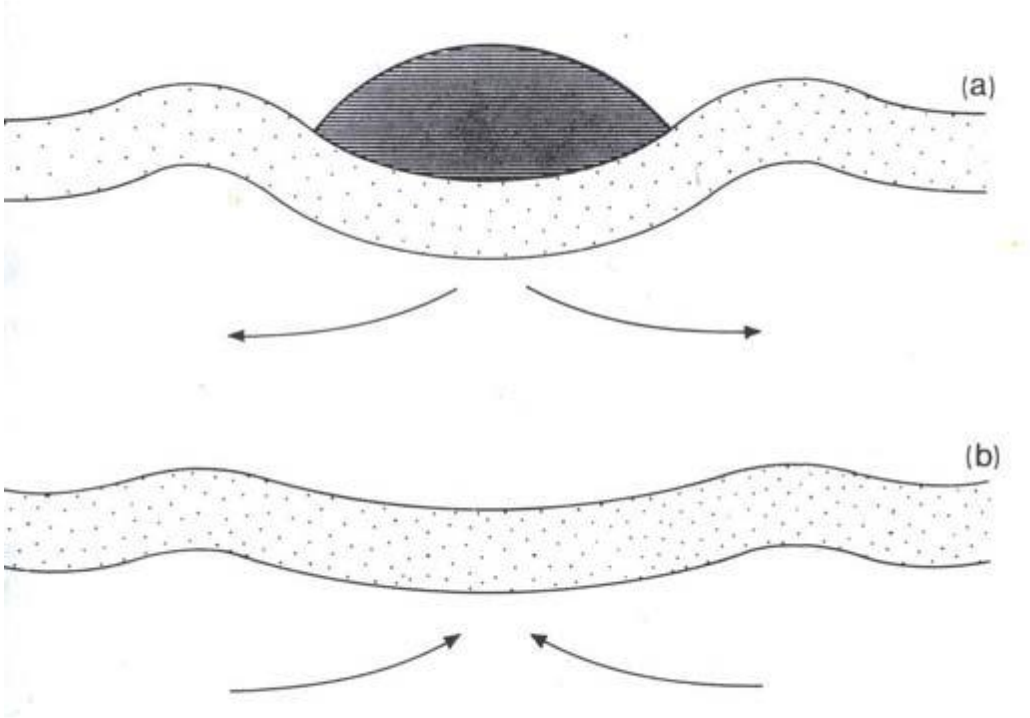
E.g. Glacial Isostatic Adjustment (GIA)



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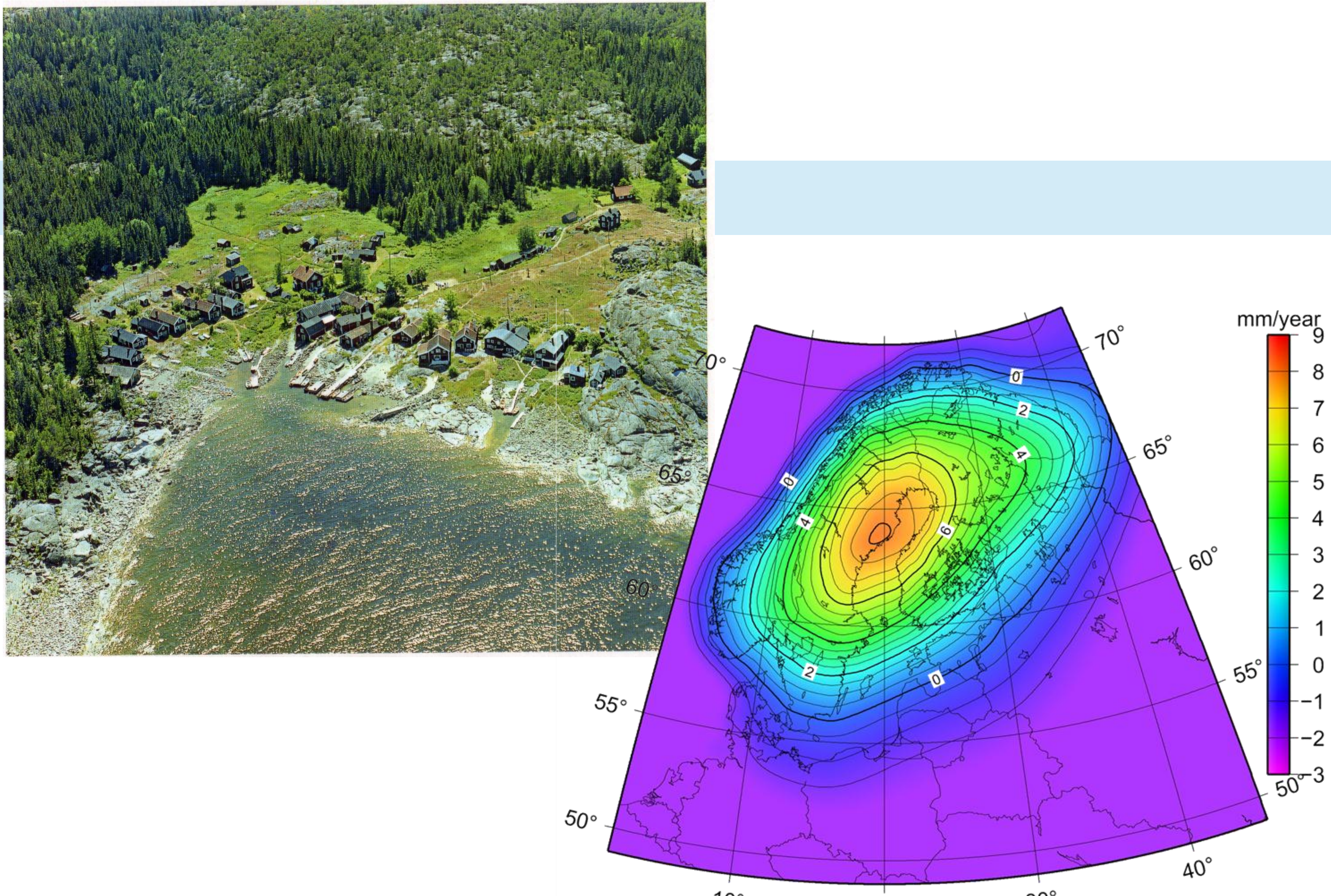
Postglacial Land Uplift in Fennoscandia



Converting Between Physical Height Systems

From Geopotential Numbers to Physical Heights

Dynamic heights:

From potential differences:

$$H^{DYN} = \frac{C}{\gamma_o^{\varphi}}$$

Using the dynamic correction:

$$\Delta H_{AB}^{DYN} = \Delta n_{AB} + k_{AB}^{DYN}$$

$$k_{AB}^{DYN} = \int_A^B \frac{g - \gamma_o^{45}}{\gamma_o^{45}} \delta n = \sum_A^B \frac{g - \gamma_o^{45}}{\gamma_o^{45}} dn$$

γ_o^{φ} Normal gravity for the surface of the level ellipsoid at certain latitude φ, normally 45° .

Normal heights:

From potential differences:

$$H^N = \frac{C}{\gamma_m} \quad ; \quad \gamma_m = \frac{1}{H^N} \int_0^{H^N} \gamma \, dH^N$$

Using the normal correction:

$$\Delta H_{AB}^{N} = \Delta n_{AB} + k_{AB}^{N}$$

$$k_{AB}^{N} = \int_A^B \frac{g - \gamma_o^{45}}{\gamma_o^{45}} \delta n + \frac{\gamma_m^A - \gamma_o^{45}}{\gamma_o^{45}} H_A^N - \frac{\gamma_m^B - \gamma_o^{45}}{\gamma_o^{45}} H_B^N$$

γ_m Mean normal gravity along the normal plumb line between telluroid and ellipsoid (analytically estimable, iterative)

$$\gamma_m = \gamma_o + \frac{1}{2}\left(\frac{\partial \gamma}{\partial H}\right)_o H^N + \frac{1}{2!}\left(\frac{\partial^2 \gamma}{\partial H^2}\right)_o \left(H^N\right)^2 + \ldots = \gamma_o^{\varphi}\left[1 - \left(1 + f + m - 2f\sin^2\varphi\right)\frac{H^N}{a} + \frac{\left(H^N\right)^2}{a^2}\right] \quad [m s^{-2}]$$

a semi-major axis, f flattening, φ latitude of the point, $m = \frac{\omega^2 a^2 b}{GM}$

From Geopotential Numbers to Physical Heights

Orthometric heights:

From potential differences:

$$H^O = \frac{C}{g_m} \quad ; \quad g_m = \frac{1}{H^O}\int_0^{H^O} g\,dH^O$$

Using the orthometric correction:

$$\Delta H_{AB}^O = \Delta n_{AB} + k_{AB}^O$$

$$k_{AB}^O = \int_A^B \frac{g-\gamma_o^{45}}{\gamma_o^{45}}\delta n + \frac{g_m^A-\gamma_o^{45}}{\gamma_o^{45}} H_A^O - \frac{g_m^B-\gamma_o^{45}}{\gamma_o^{45}} H_B^O$$

g_m Mean real gravity along the plumb line between Earth's surface and geoid. It can only be estimated by means of hypotheses about the (unknown) Earth's internal mass distribution and the (unknown) vertical gravity gradient. **Each different hypothesis produces a different type of orthometric height**.

Some examples of orthometric hypotheses:

Helmert:

$$g_m = g_{H/2} = \frac{1}{2}\left(g_p + g_0\right) = g_p + \left(3{,}086 - 0{,}83818\rho_p\right)10^{-6}\,\frac{H_p^O}{2}$$

First method of Ramsayer:

$$g_m = \frac{1}{2}\left(g_p + g_0\right) - \left(g_0 - g_0^{AP}\right)\frac{\hat{H}^O}{H_p^O} \qquad \hat{H}^O = \frac{1}{n}\sum_{i=1}^{n} H_i^O$$

Ledersteger:

$$g_m = \frac{1}{n}\sum_{i=1}^{n}\left(g_i + 3{,}086\,x10^{-6}\,H_i^O\right) - \frac{1}{2}3{,}086\,x10^{-6}\,\frac{H_p^O}{2}$$

Normal Orthometric: use normal gravity instead of observed surface gravity

Units: g_p, g_m, $g_{H/2}$, g_0 → [m s^{-2}] ; ρ_p → [10^{-3} kg m^{-3}] ; H^O → [m]

Physical Heights – Summary Comments (1)

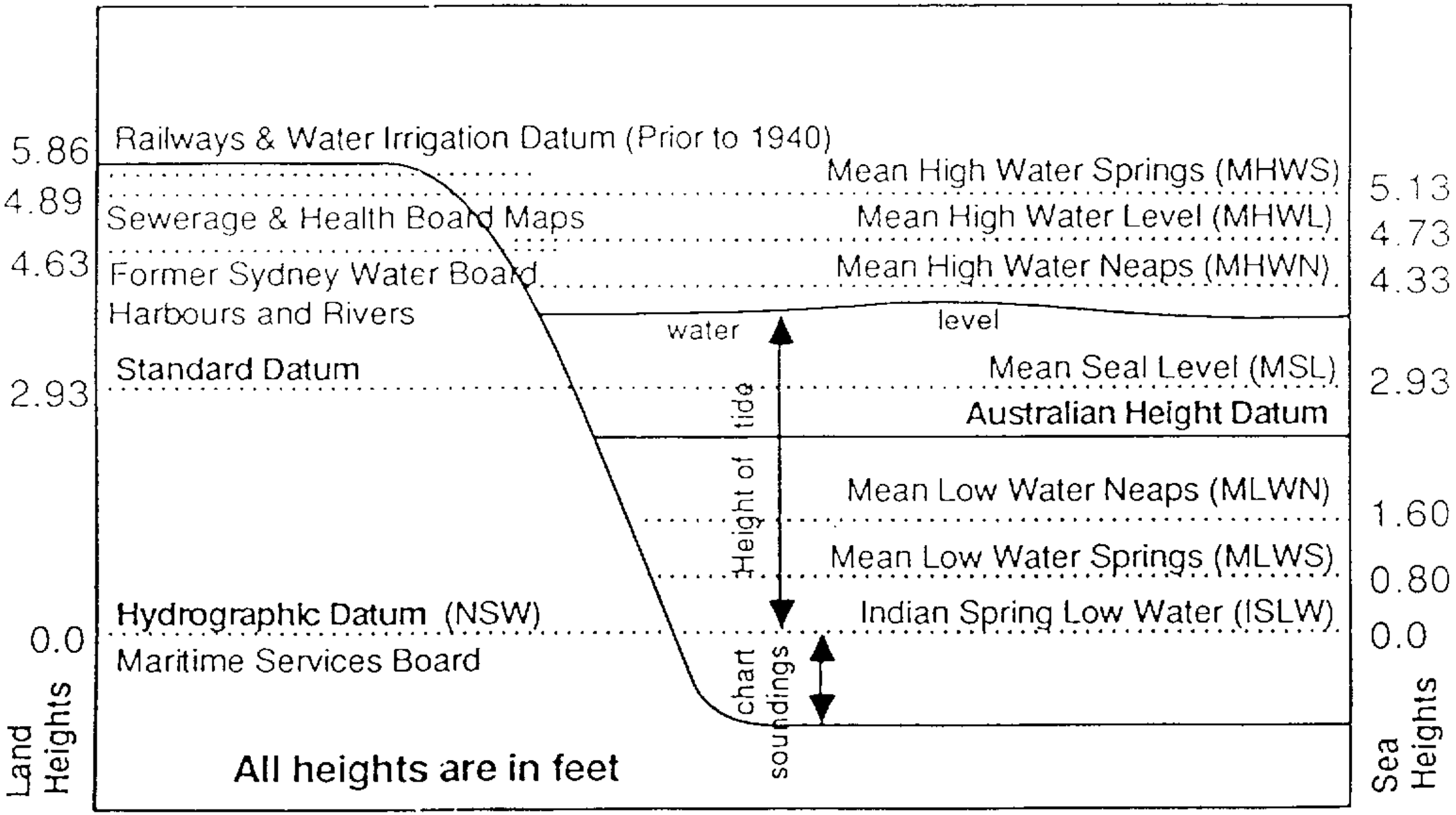
	Dynamic heights	**Orthometric heights**	**Nomal heights**
Definition of $\hat{\mathbf{g}}$	γ_o^{κ}: constant normal gravity value at an arbitrary latitude φ (usually $\varphi = 45°$).	g_m: Mean real gravity value along the plumb line between the geoid and P.	γ_m: Mean normal gravity value along the normal plumb line between the ellipsoid and the telluroid (or between the quasi-geoid and P).
Description	Simple conversion to height units (scaled geopotential numbers) $H^{DYN} = \frac{C}{\gamma_o^{\varphi}}$	Distance, along the plumb line, between the surface point P and the geoid. $H^O = \frac{C}{g_m}$; $g_m = \frac{1}{H^O}\int_0^{H^O} g\,dH^O$	Distance, along the normal plumb line, between the ellipsoid and the telluroid (or between the quasi-geoid and P) $H^N = \frac{C}{\gamma_m}$; $\gamma_m = \frac{1}{H^N}\int_0^{H^N} \gamma\,dH^N$
Correction (for levelling)	Magnitude: < 20 m $\Delta H_{AB}^{DYN} = \Delta n_{AB} + k_{AB}^{DYN}$ $k_{AB}^{DYN} = \int_A^B \frac{g-\gamma_o^{45}}{\gamma_o^{45}}\delta n = \sum_A^B \frac{g-\gamma_o^{45}}{\gamma_o^{45}} dn$	Magnitude: mm ... dm $\Delta H_{AB}^{O} = \Delta n_{AB} + k_{AB}^{O}$ $k_{AB}^{O} = \int_A^B \frac{g-\gamma_o^{45}}{\gamma_o^{45}}\delta n + \frac{g_m^A-\gamma_o^{45}}{\gamma_o^{45}} H_A^O - \frac{g_m^B-\gamma_o^{45}}{\gamma_o^{45}} H_B^O$	Magnitude: mm ... dm $\Delta H_{AB}^{N} = \Delta n_{AB} + k_{AB}^{N}$ $k_{AB}^{N} = \int_A^B \frac{g-\gamma_o^{45}}{\gamma_o^{45}}\delta n + \frac{\gamma_m^A-\gamma_o^{45}}{\gamma_o^{45}} H_A^N - \frac{\gamma_m^B-\gamma_o^{45}}{\gamma_o^{45}} H_B^N$
Remarks	• No geometrical meaning • Points on the same level surface have the same height value • Hypotheses are not required	• Reference surface: the geoid $H^O = h - N$ h: ellipsoidal height, N: geoid undulation • Heights of points on the same level surface differ in the same manner as the g_m gravity values • Hypotheses about mass density and distribution as well as about the gravity vertical gradient $(\partial g/\partial H)$ are necessary. The value of H^O depends on the adopted hypotheses. • g_m cannot be estimated univocally, only approximately.	• Reference surface: the quasi-geoid (close to the geoid but not a level surface) $H^N = h - \zeta$ h: ellipsoidal height, ζ: height anomaly • Points on the same level surface and at the same latitude have the same normal heights. In other cases, heights differ in the same manner as γ_m varies with the latitude. • Hypotheses are not required • γ_m is estimable univocally.

Physical Heights – Summary Comments (2)

Characteristics	Height type		
	Dynamic	Orthometric	Normal
Uniqueness Heights values shall be univocally determinable, i.e. they shall not depend on the levelling path.	☺	☺	☺
Zero-height surface with physical meaning and independent of the heights (i.e. the zero-height surface shall not change if heights change).	☹	☹	☹
Geometric meaning Heights shall represent the vertical distance between two points (one on the Earth's surface and one on the reference surface)	☹	☺	☺
Units of length Heights shall be given en units of length (or distance), i.e. in metres.	☺	☺	☺
The same height value on the same equipotential If water does not flow between two points, they shall have the same height value.	☺	☹	☹
Use of hypotheses The use of hypotheses shall be avoided. If hypotheses are improved, the height system must be changed totally.	☺	☹	☺
Connection with geometrical heights Physical heights shall be able to be combined with ellipsoidal heights.	☹	☺	☺
Small gravity corrections To be avoided in practical applications of local extension.	☹	☺	☺

Challenges Defining the Zero Reference Level

Arbitrary Vertical Datums, e.g. Sydney



- **Sea surface height** (**SSH**): vertical distance of sea surface wrt ellipsoid (geometric height from satellite altimetry of GNSS):

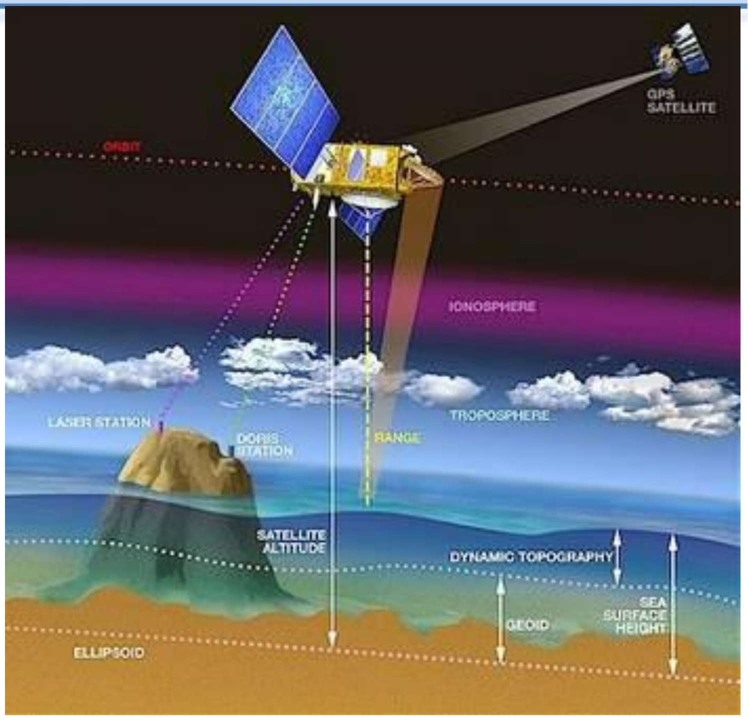
$$SSH = h_s - r_j$$
$$SSH = N + DT$$

- **Dynamic topography** (**DT**): difference between sea surface and geoid (physical height):

$$DT = h_s - r_j - N$$

Mainly caused by tides, currents, winds, Earth rotation, seasonal effects, temperature, salinity, etc.
Determined using ocean (dynamic) models based on hydrostatic equilibrium laws.

Source: http://www.aviso.oceanobs.com



- **Mean sea surface** (**MSS**) : long-term average of sea surface heights:

$$MSS = \frac{1}{y}\sum_{y} MSH$$

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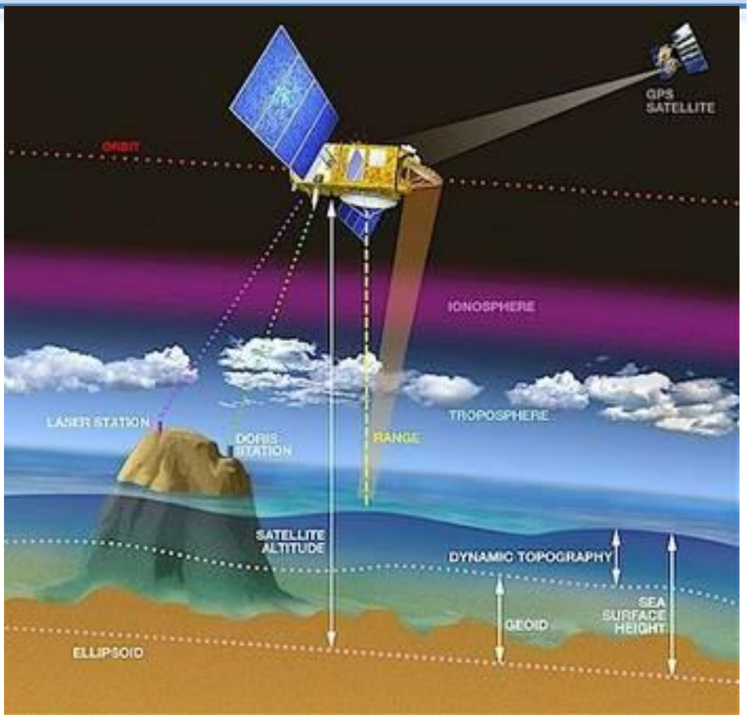
- DT is separated in **mean dynamic topography MDT** (considered semi-stationary) and **dynamic ocean topography DOT** (time-variable part of DT):

$$DT = MDT + DOT$$

- The **MDT** is the “oceanic relief”, mainly caused by geostrophic currents. Also referred to as **Sea Surface Topography** (**SSTop**):

$$MDT = SSTop = MSS - N$$

Source: http://www.aviso.oceanobs.com



- **DOT** contains contributions from wind and other high frequency effects. Usually inter-annual, or other short-term, variations from MDT

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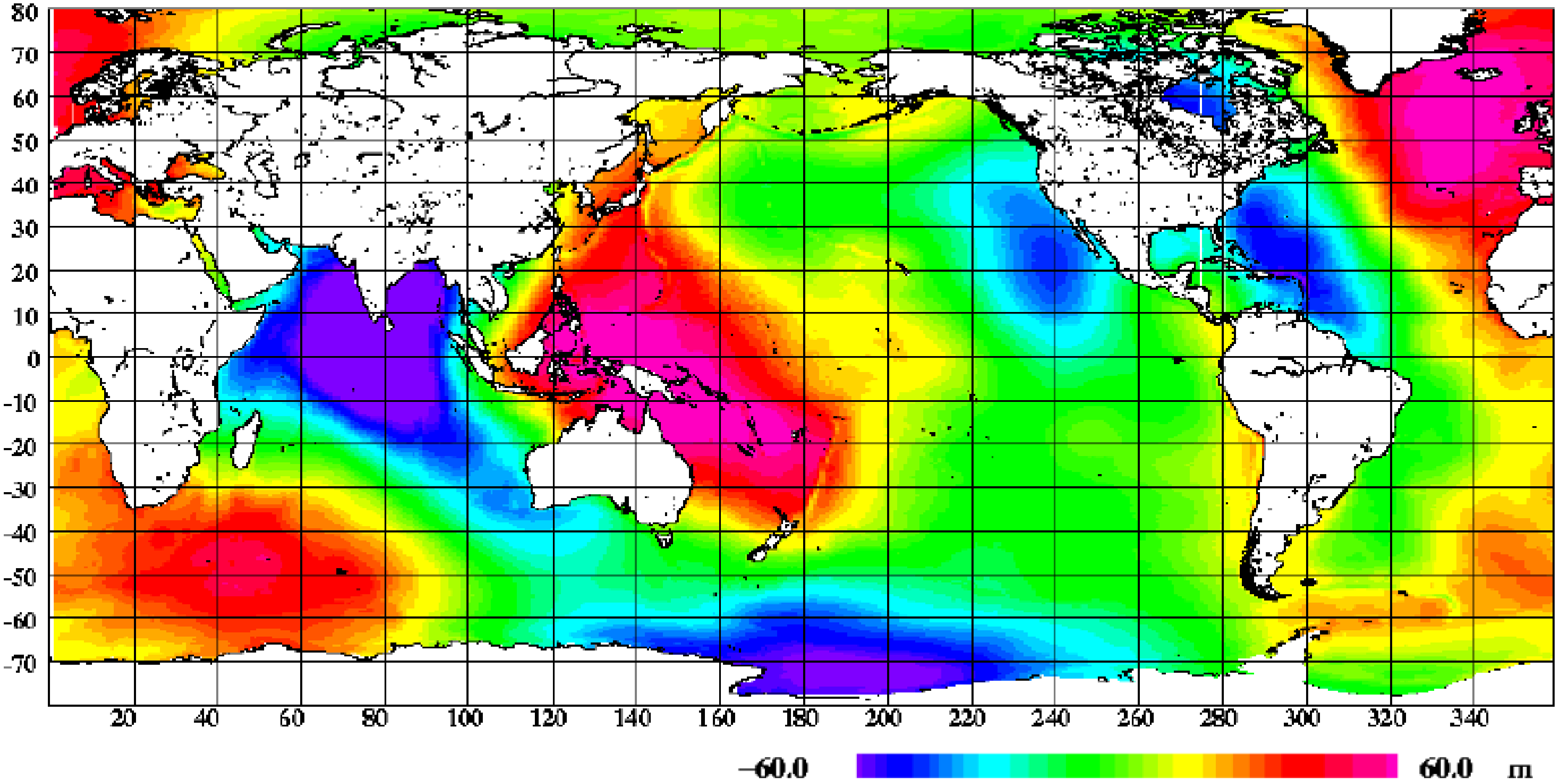
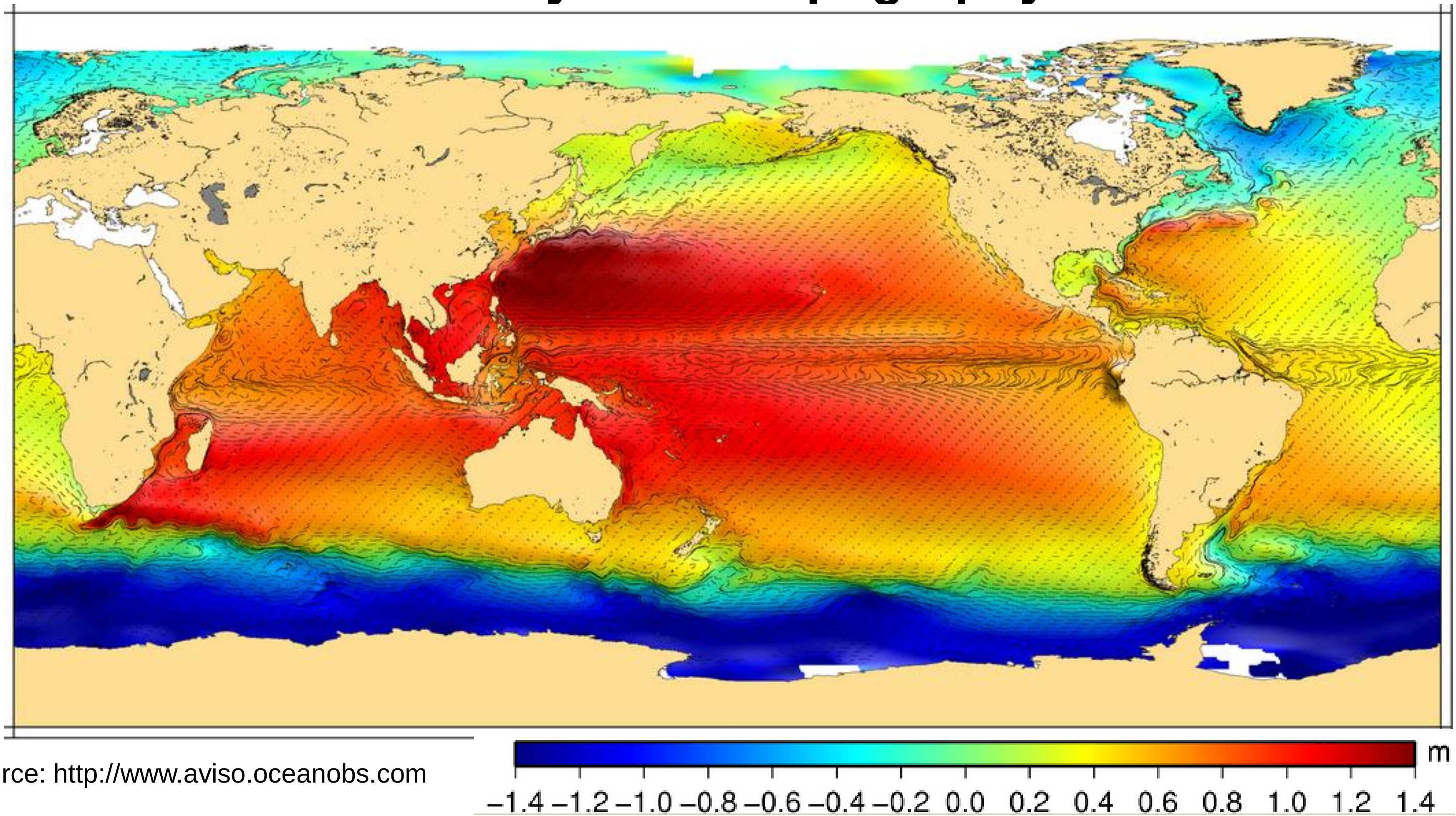
DTU10 Mean Sea Surface Model

Source: http://www.space.dtu.dk/



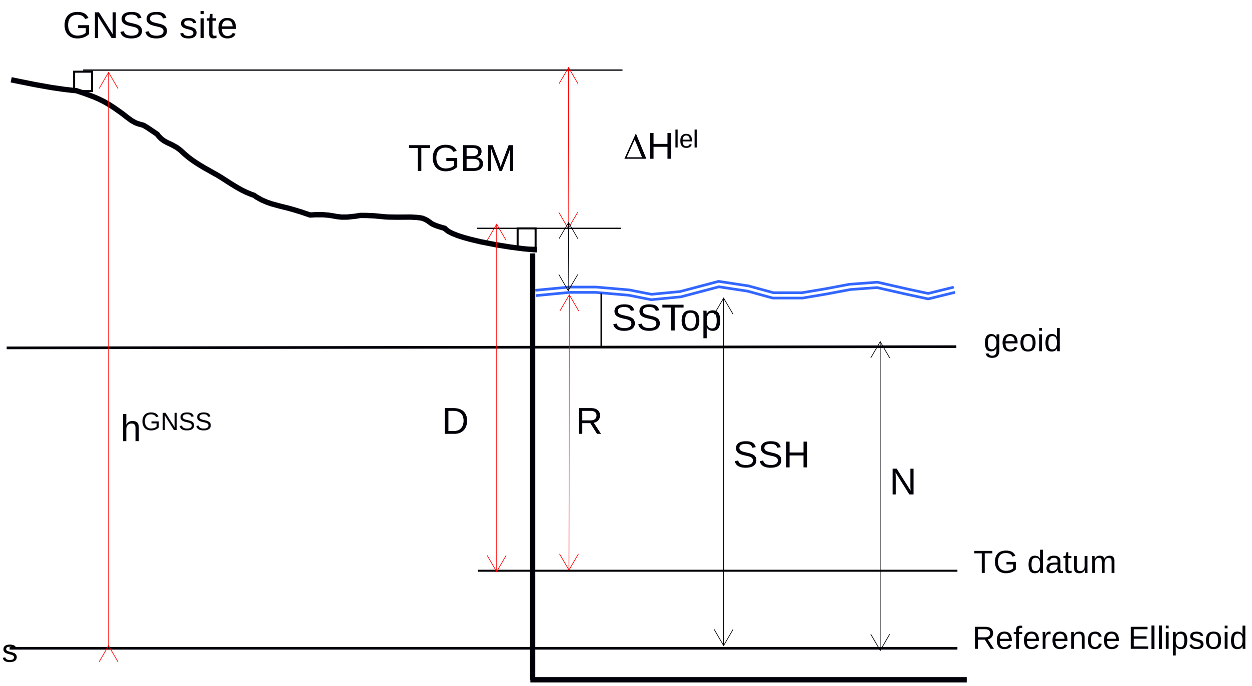
CNES/CLS2012 Mean Dynamic Topography



Source: http://www.aviso.oceanobs.com

$$SSH = h_s = N + SSTop = N + H_s$$
$$= h^{GNSS} - DH^{lev} - D - R$$
$$SSTop = H_s = SSH - N = h_s - N$$
$$= h^{GNSS} - DH^{lev} - D - R - N$$

Where is the zero level defined? How is it realised or propagated across a network?



Courtesy: M. Sideris

Today's vertical reference systems are deficient, e.g.:

- use reference levels determined using different tide gauges averaged over different time periods
- use different types of height coordinate types, different permanent tide systems, etc.
- use different &/or arbitrary zero reference levels
- not been corrected for vertical displacements, at vertical datum points, etc.
- do not take MDT/SSTop into account
- *Therefore do not support accurate combination of H, h and N*

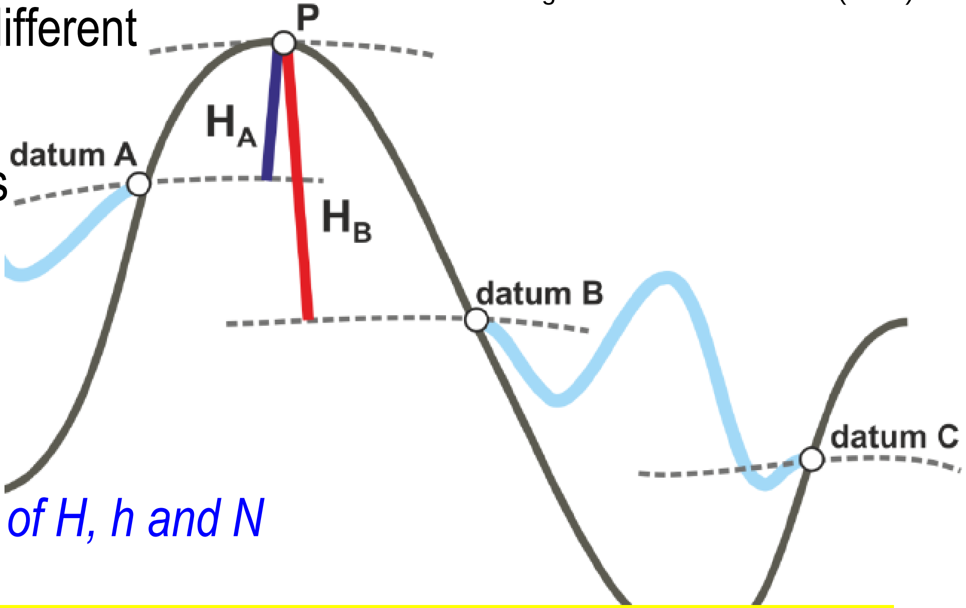


Figure from Sanchez et al. (2012)

There are >100 vertical datums... discrepancies between zero levels range from several dms up to 2m in extreme cases

Vertical Datums, Tide Gauges & Sea Level

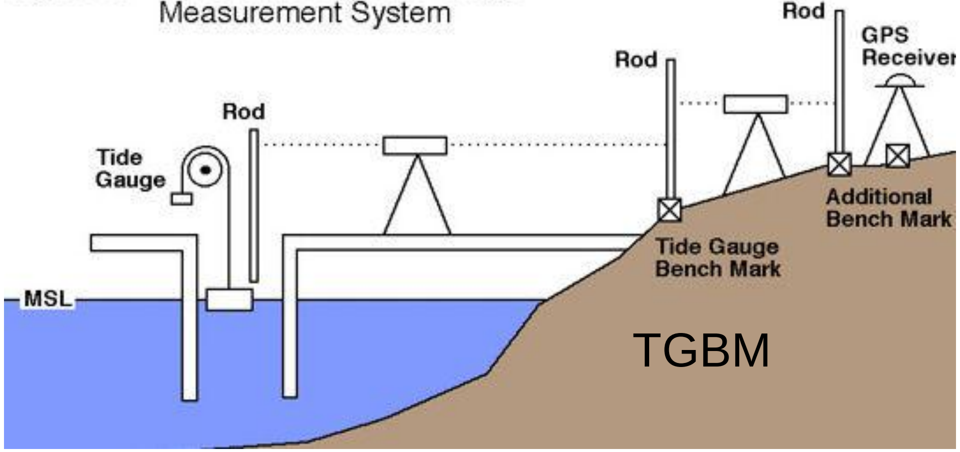
Height Datums, Geoid and MSL: Recap

- Orthometric height is *height above the geoid, related to the gravity field of the Earth*
- A good working definition for the geoid is:

  "… that surface which approximates Mean Sea Level …"
- Height datum *is realised by heights of level bench marks, which may be related to one (or more) TGBMs (MSL-based)*
- Most survey and mapping requirements accept a height datum defined arbitrarily, *but sea level is the most convenient zero height for engineering applications*
- For many purposes MSL and the geoid can be considered synonymous, but there may be a local "offset" which doesn't affect *slopes* (or height differences), *but slopes can be incorporated into "geoid correction surfaces"*
- Tide gauges are important for height datum definition, except for land-locked countries

Tide Gauge Datums

Figure 1: Schematic of a Tide Gauge Measurement System



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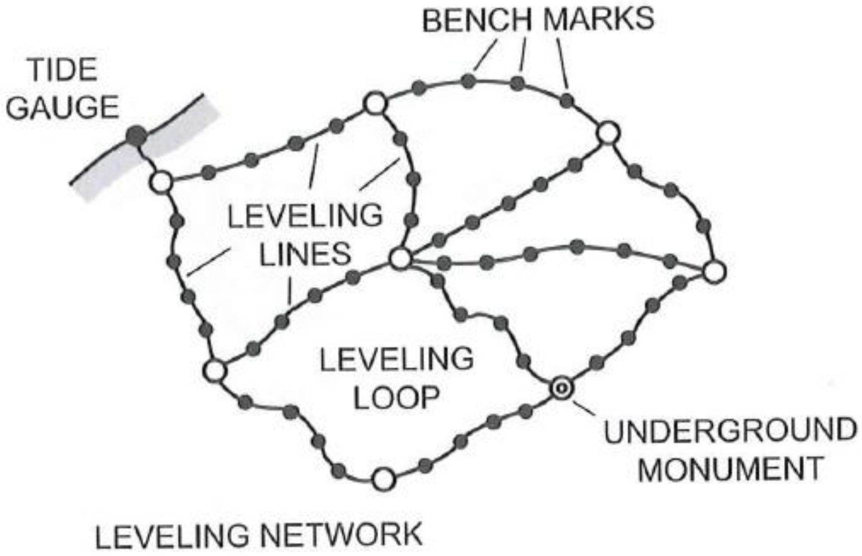
- Tide gauges continuously record height of water level relative to a local bench mark
- The sea level varies both in time and space
- The time variation is dependent on:
  - Ocean tides
  - Meteorological factors (atmospheric pressure, winds, etc.)
  - Oceanographic factors (currents, changes in density due to temperature and salinity, etc.)
  - Variable river/harbour influx
  - Geodynamic (land) movements
- Averaging over long time periods eliminates most of the time variation, e.g. 18.6 years (the lunar nutation cycle)

National Levelling Networks

- **National levelling networks are traditionally separate from the horizontal networks (and also from the modern 3D GNSS networks)**
- **Level surveying is mainly by "spirit levelling".** If required, hydrostatic levelling or other special techniques are applied for short water crossings (less than a few km)... *some trigonometrical levelling in mountainous areas*
- **Observed (or interpolated) surface gravity is used to convert to geopotential numbers; *but normal gravity is often used for Othometric Corrections to levelled height differences***
- All levelling sections (between bench marks) are typically observed in both the forward and backward directions (double run levelling). *Motorised levelling techniques may be used*
- First order levelling in loops of 100-400km using precise levelling with a standard deviation of around 0.3-1.0 mm/km$^{1/2}$... *great care to avoid systematic errors*
- Some large countries (e.g. Australia) use lower order levelling standards



- The 1D adjustment of the levelling network is made using the fact that the loop misclosures of the geopotential numbers (or orthometric heights) should be zero
- Hence corrections need to be applied
- Heights finally computed for the type of height required (orthometric or normal, etc.)
- Reference surface or point(s) define "zero height"

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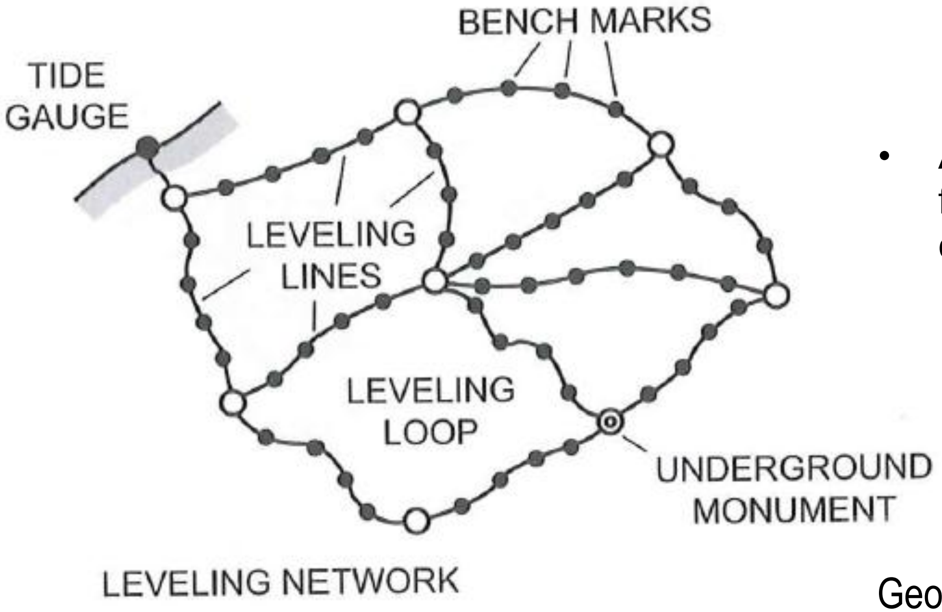
National Vertical Datums

- Traditionally, the vertical datum is defined by MSL as derived at one or more tide gauges
- If more than single tide gauge is used, need to make assumption regarding relationship between geoid (or other) surface and multiple MSL (@ tide gauge) estimates
- Zero height surface may be arbitrary, or historical bench mark, or tide gauge
- At a theoretical level, zero height reference surface may depend on the type of heights (geoid for orthometric, quasi-geoid for normal)
- Long term stability (in a vertical sense) of points that realise the vertical datum must be monitored



- The permanent tide system is usually chosen
- Corrections applied:
  - Levelling errors
  - earth tides (for the permanent tide system in question)
  - geodynamic effects
- Adjustment... adjusted geopotential number finally converted to the chosen height type, e.g.
  - Helmert Orthometric
  - Normal Heights
  - Normal Orthometric

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Tides

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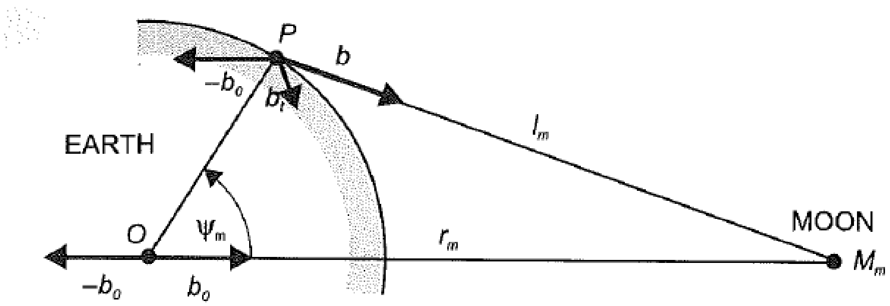
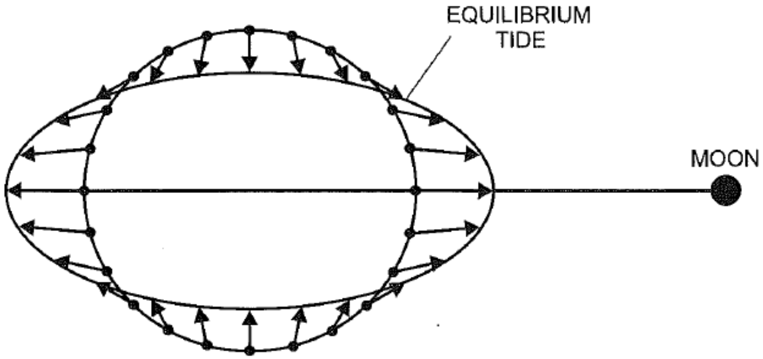
Tidal Acceleration

- Tidal acceleration is caused by the difference between the gravitation caused by the moon/sun and the orbital accelerations generated by the motion of the Earth around the respective barycentre (centrifugal accelerations)
- For a rigid Earth, the tidal accelerations can be directly determined from Newton's Law of Gravitation and the ephemerides of the sun/moon





- The tidal acceleration is given by $\mathbf{b}_t = \mathbf{b} - \mathbf{b}_0$

where **b** is the gravitational acceleration of the sun/moon and $\mathbf{b}_0$ is a constant that is equal to **b** at the Earth's centre, which gives (for the moon, *m*, and equally for the sun, *s*)

$$\mathbf{b}_\mathbf{t} = \frac{GM_m}{l_m^2}\frac{\mathbf{l}_m}{l_m} - \frac{GM_M}{r_m^2}\frac{\mathbf{r}_m}{r_m}$$

Tidal Potential

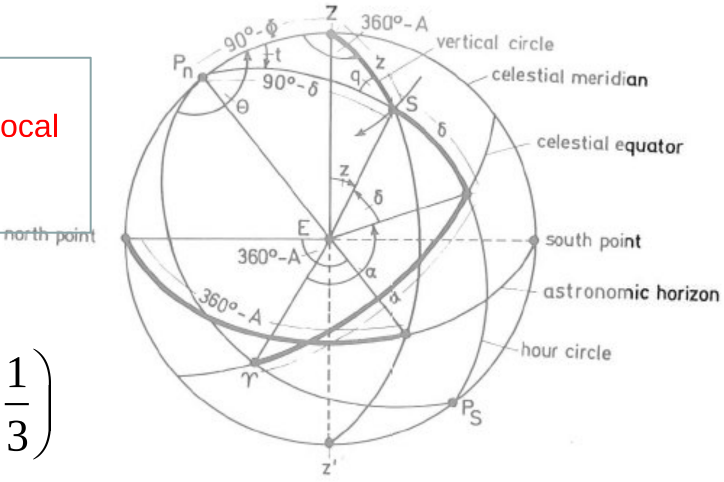
- The tidal potential satisfies

$$\mathbf{b_t} = \mathrm{grad}\ V_t = grad\left(V_m - V_0\right)$$

is given by (again for the moon)

Limiting the Legendre expansion of the reciprocal distance to degree 2.
($r_m >> r$)

$$V_t = \frac{GM_m}{l_m} - \frac{GM_m}{r_m} - \frac{GM_m}{r_m^2} r\cos\psi_m \approx \frac{3}{4} GM_m \frac{r^2}{r_m^3}\left(\cos 2\psi_m + \frac{1}{3}\right)$$



- If this is expressed using spherical coordinates of the observation points and right ascension/declination for the moon/sun, we get Laplace tidal equation for the moon (and of course a similar one for the sun)

$$V_t = \frac{3}{4} GM_m \frac{r^2}{r_m^3}\left\{\left(\frac{1}{3} - \sin^2\bar{\phi}\right)\left(1 - 3\sin^2\delta_m\right) + \sin 2\bar{\phi}\sin 2\delta_m \cos h_m + \cos^2\bar{\phi}\cos^2\delta_m \cos 2h_m\right\}$$

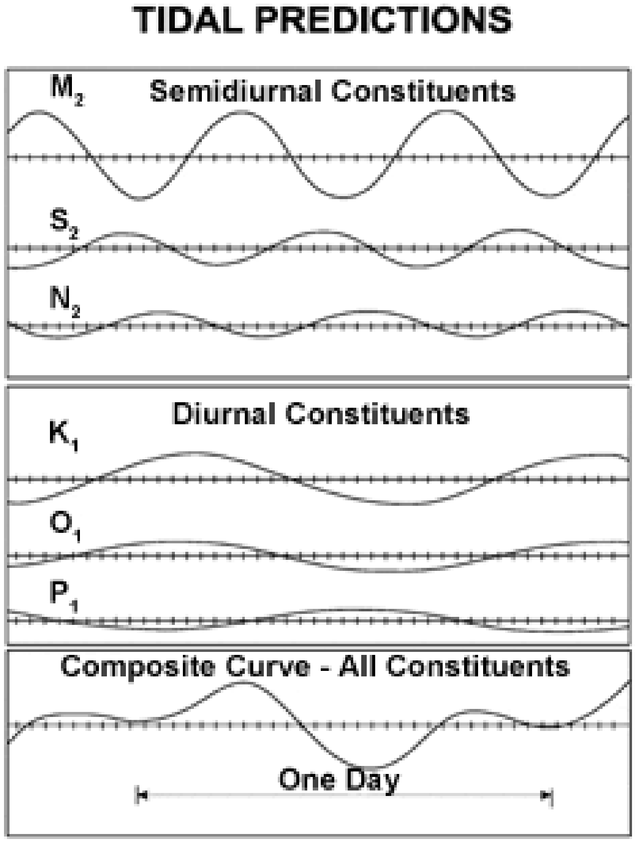
$$h_m = \lambda + GAST - \alpha_m$$

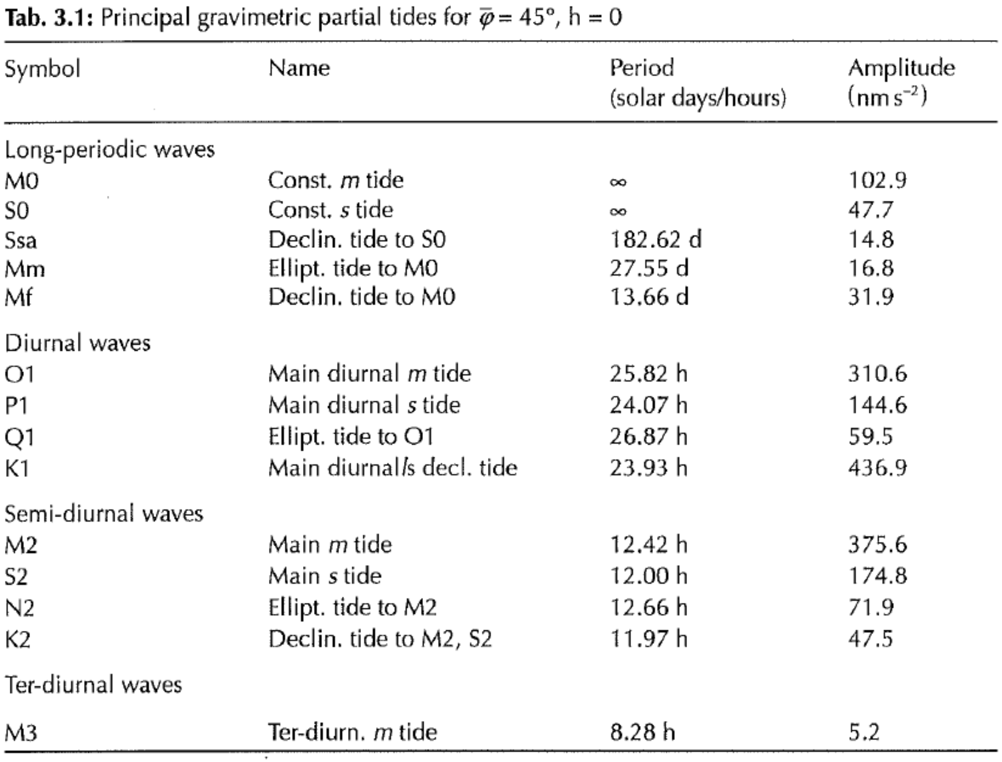
- The expression before the parenthesis is Doodson's tidal coefficient, which have the following values for the moon and sun:

D_{moon} = 2.628 m^{2}s^2, D_{sun}=1.208 m^{2}s^2.

Principal Tidal Waves

- Each of the different parts varies in complicated ways



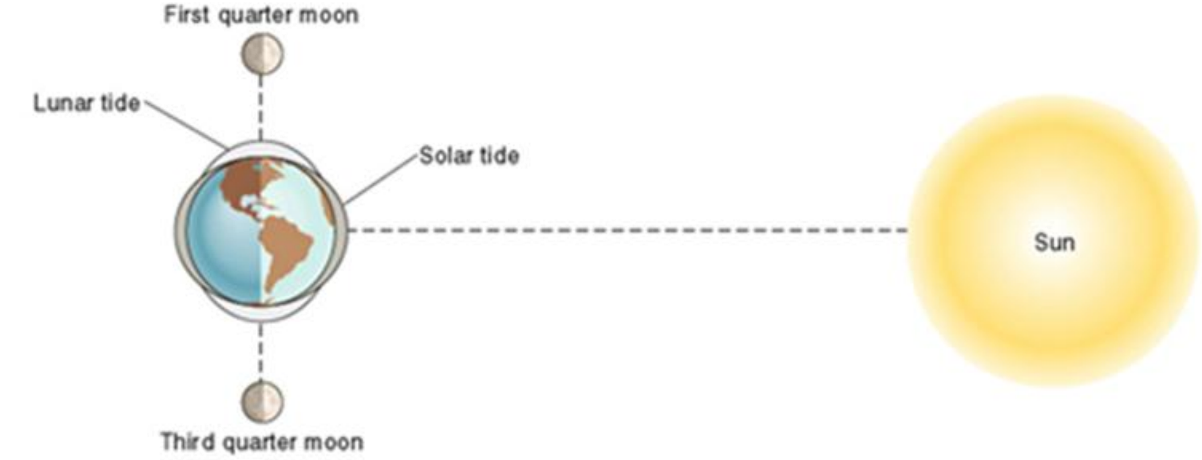


Tab. 3.1: Principal gravimetric partial tides for $\bar{\varphi} = 45°$, h = 0

Symbol	Name	Period (solar days/hours)	Amplitude (nm s^{-2})
Long-periodic waves			
M0	Const. *m* tide	∞	102.9
S0	Const. *s* tide	∞	47.7
Ssa	Declin. tide to S0	182.62 d	14.8
Mm	Ellipt. tide to M0	27.55 d	16.8
Mf	Declin. tide to M0	13.66 d	31.9
Diurnal waves			
O1	Main diurnal *m* tide	25.82 h	310.6
P1	Main diurnal *s* tide	24.07 h	144.6
Q1	Ellipt. tide to O1	26.87 h	59.5
K1	Main diurnal/*s* decl. tide	23.93 h	436.9
Semi-diurnal waves			
M2	Main *m* tide	12.42 h	375.6
S2	Main *s* tide	12.00 h	174.8
N2	Ellipt. tide to M2	12.66 h	71.9
K2	Declin. tide to M2, S2	11.97 h	47.5
Ter-diurnal waves			
M3	Ter-diurn. *m* tide	8.28 h	5.2

Interaction of the Moon and Sun

a Spring tides



b Neap tides

Vertical Datums, Tide Gauges & Sea Level: Examples

North American Vertical Datum 1929

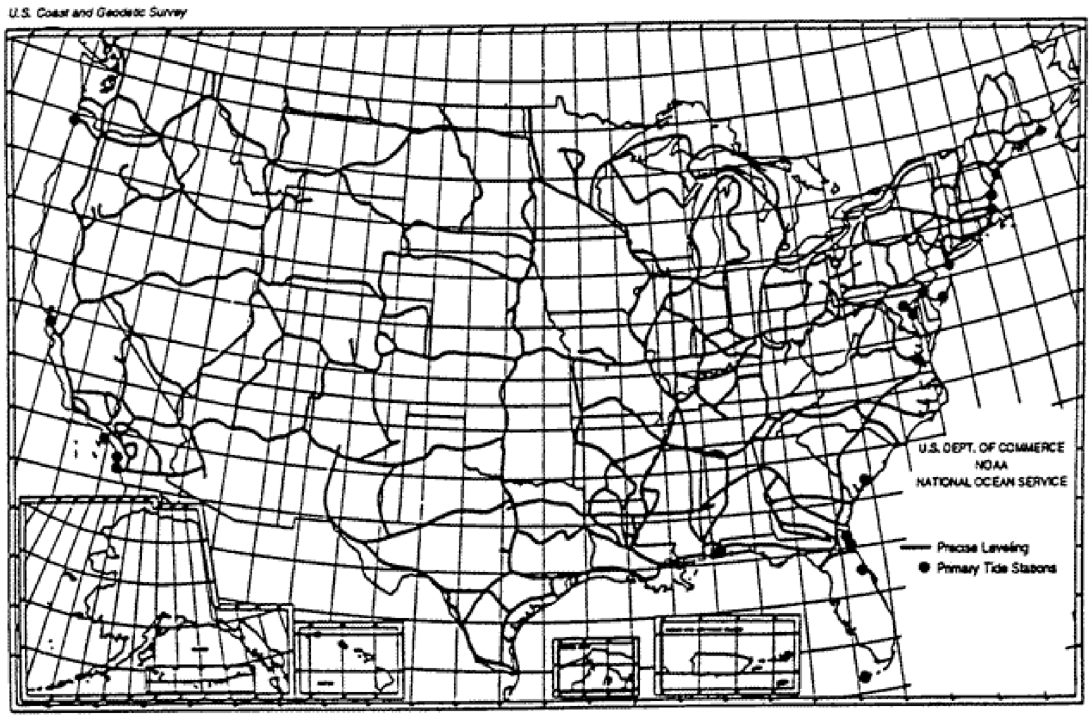


Figure 1. First-order vertical control used in 1929 adjustment.

- ~100000 benchmarks
- 75000km of levelling data in US, 31000km in Canada
- Constrained to 26 tide gauges around the coasts, but tidal epochs differ
- Normal orthometric heights (the zero height surface is not an equipotential surface)
- Thus conversion to geopotential numbers using normal gravity
- Corrections:
  - rod scale and temperature
- Permanent tide system: mean (no earth tide corrections?)

North American Vertical Datum 1988

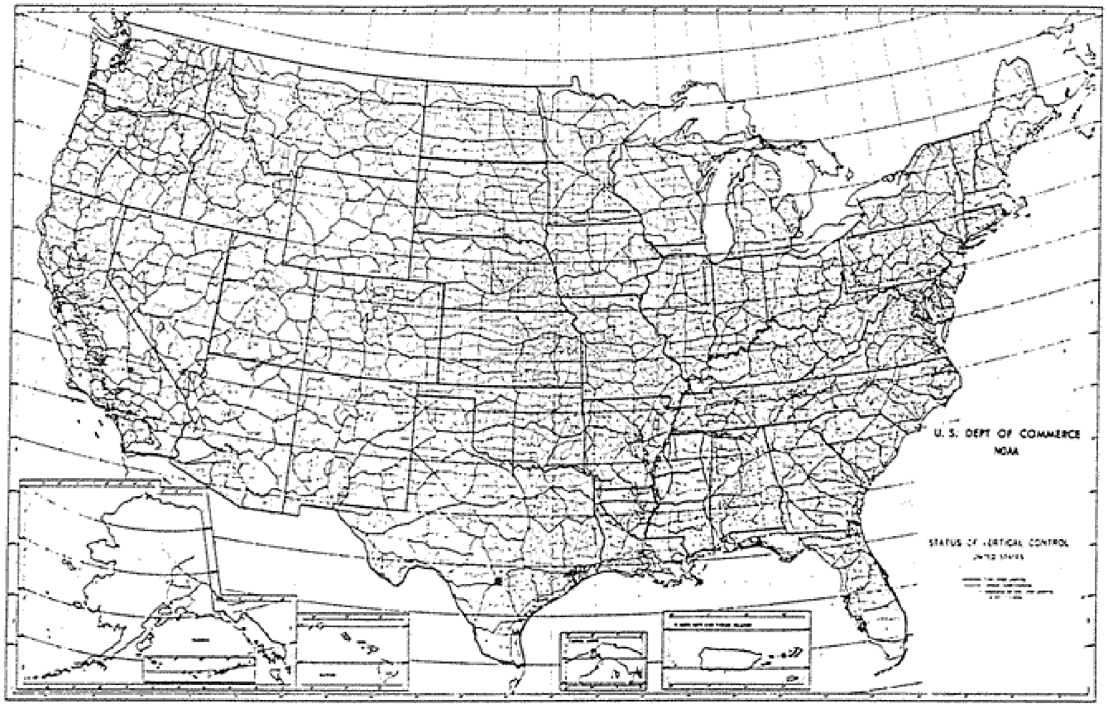
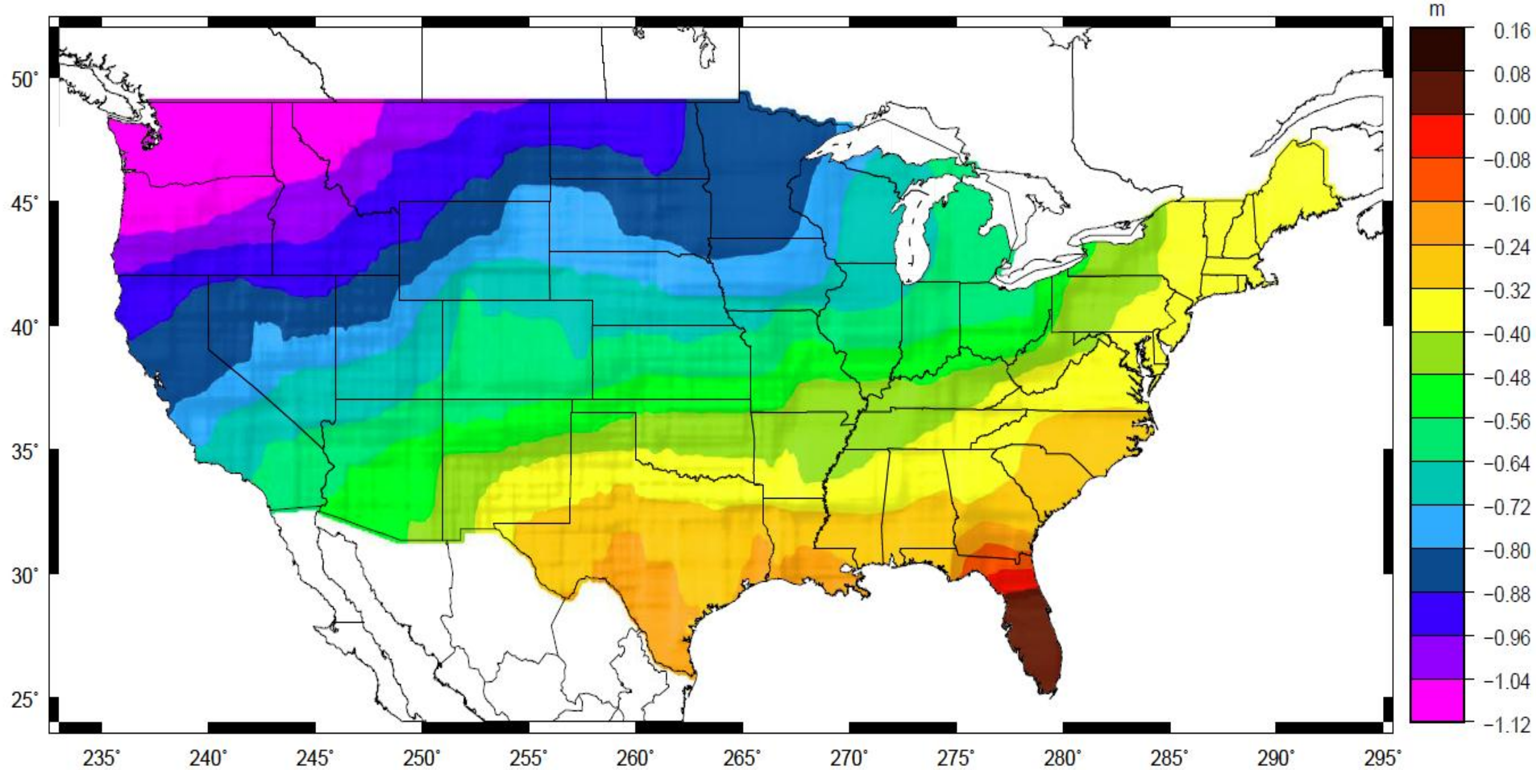


Figure 3. Vertical control used in 1988 adjustment.

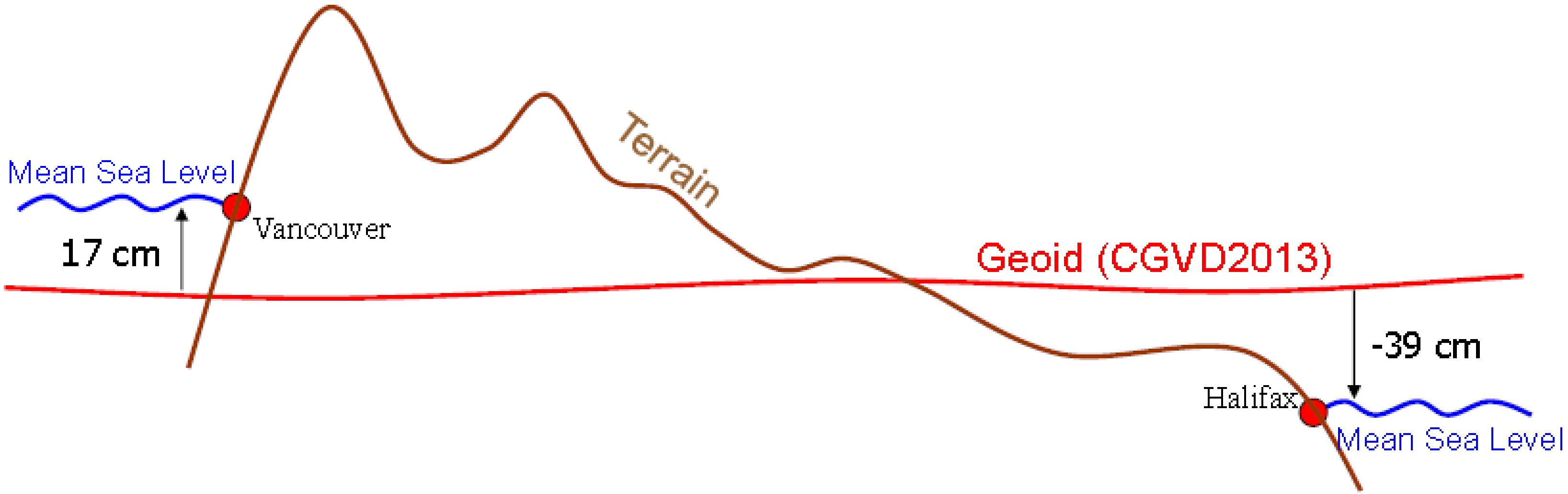
- ~1000000km levelling
- Constrained to 1 tide gauge (Father Point, Rimouski), tidal epoch 1960-1978
- Helmert orthometric heights
- Conversion to geopotential numbers using observed gravity
- Corrections:
  - rod scale and temperature
  - Earth tide
  - Magnetic
  - Refraction
- Permanent tide system: probably non-tidal? (but could be zero)

Why isn't NAVD 88 good enough anymore?



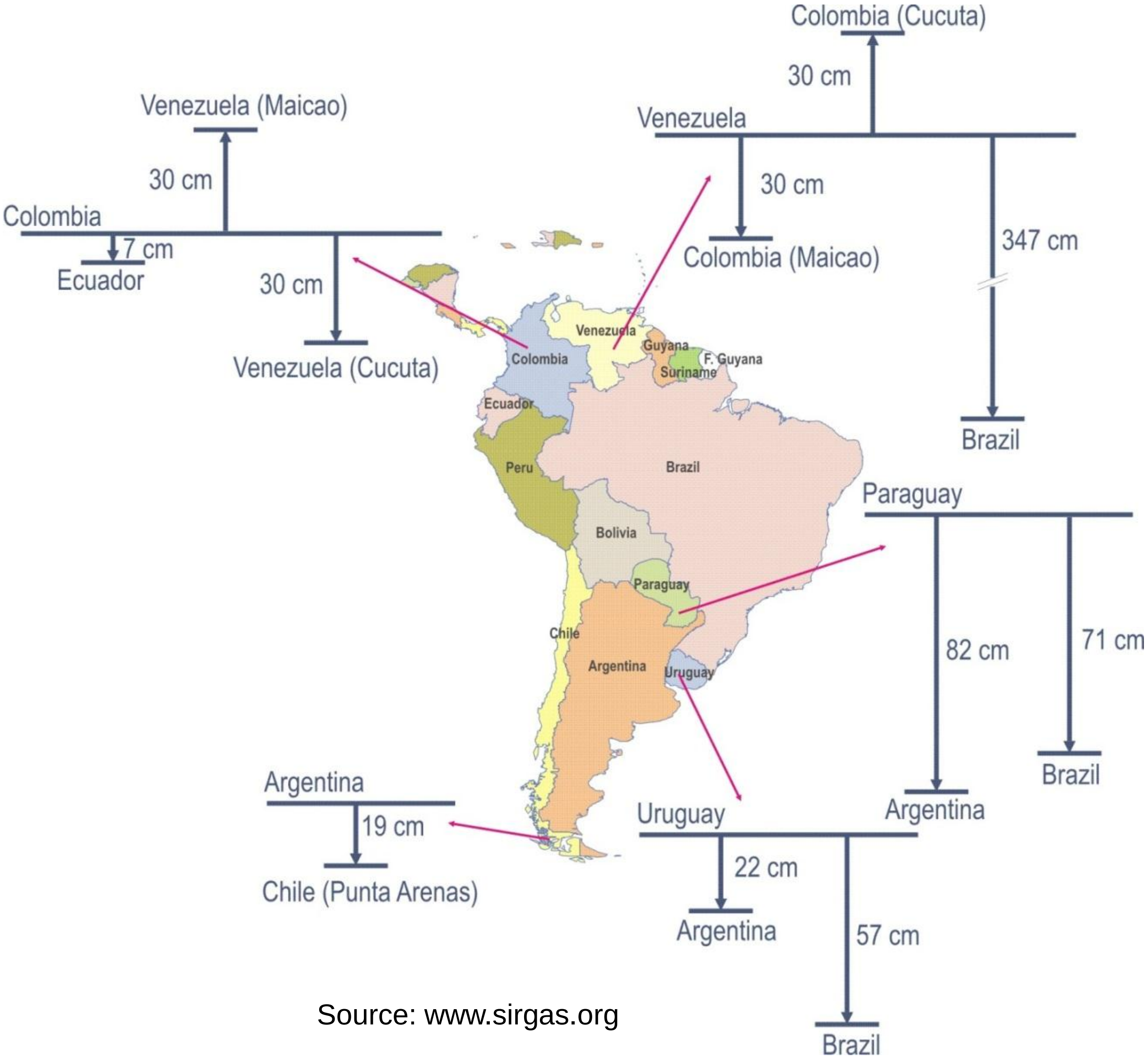
Approximate level of error known to exist in the NAVD 88 zero elevation surface

What is the Difference Between GGVD2013 and MSL?



Véronneau and Huang (2014)

Vertical Datum Discrepancies in South America

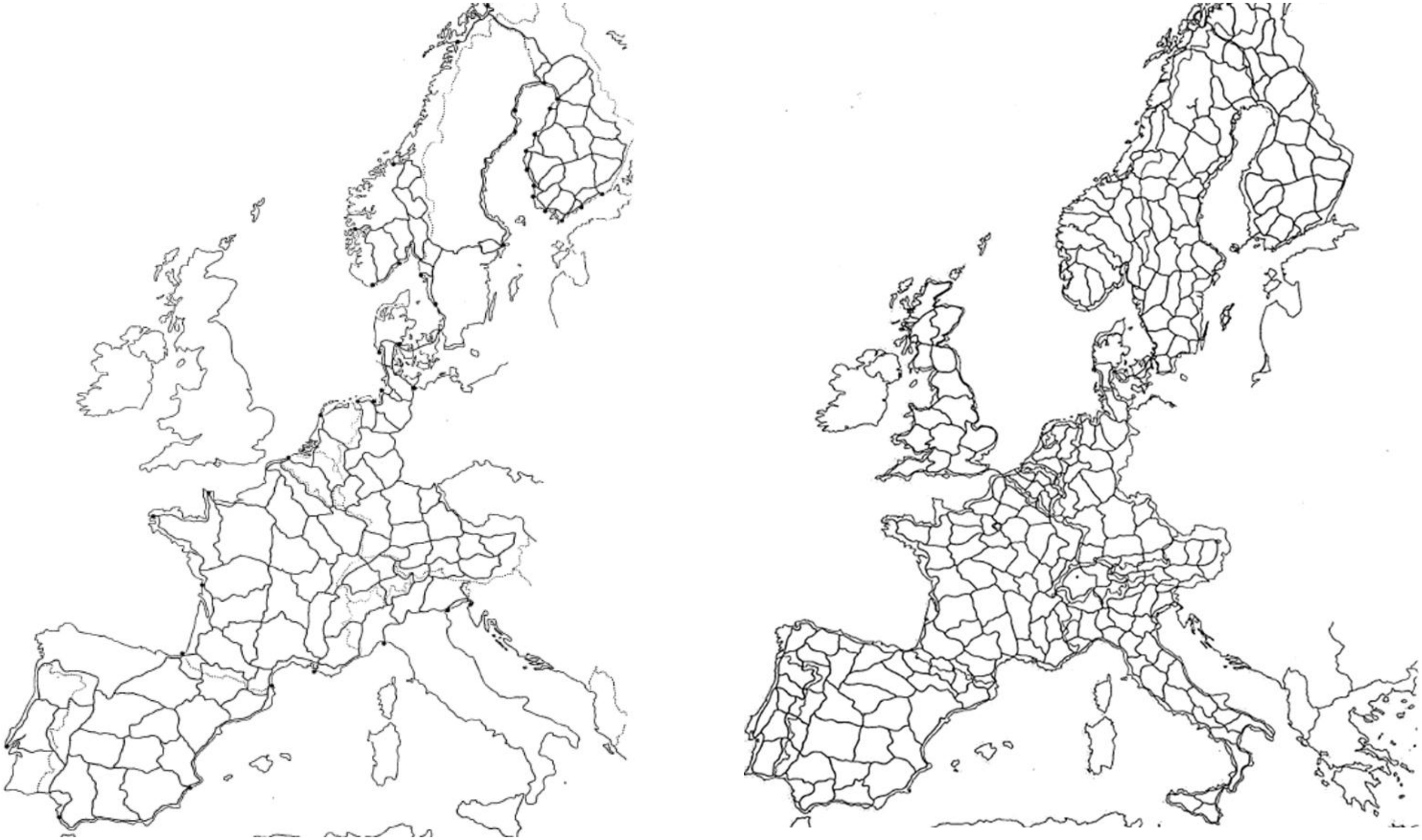


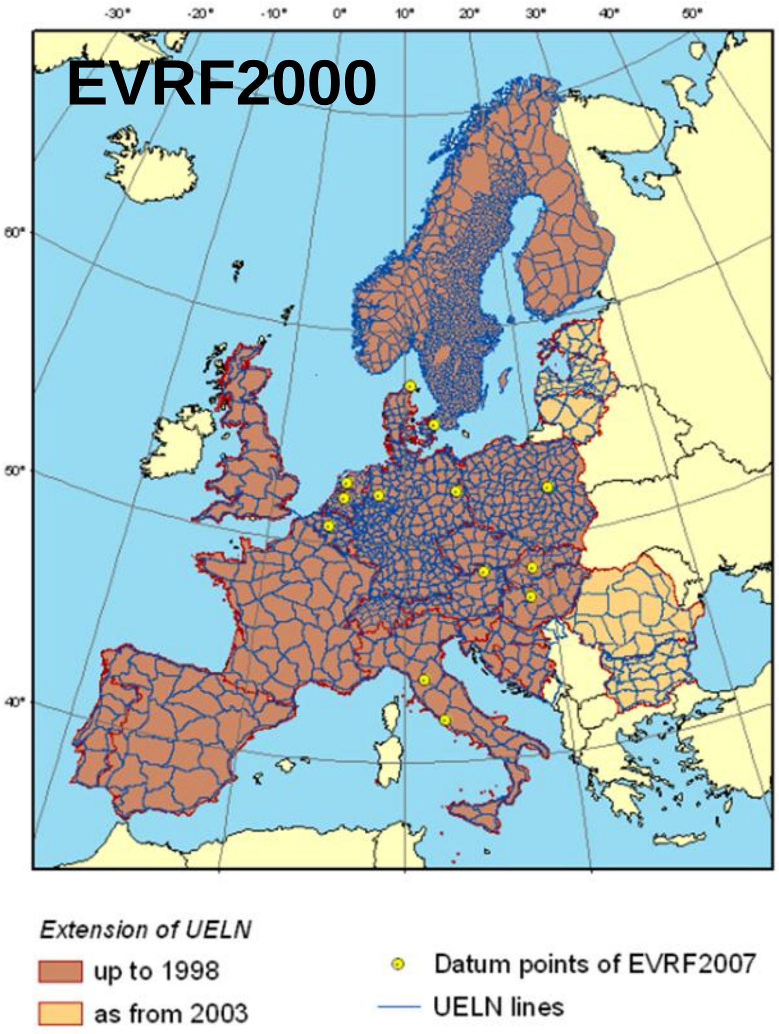
Source: www.sirgas.org

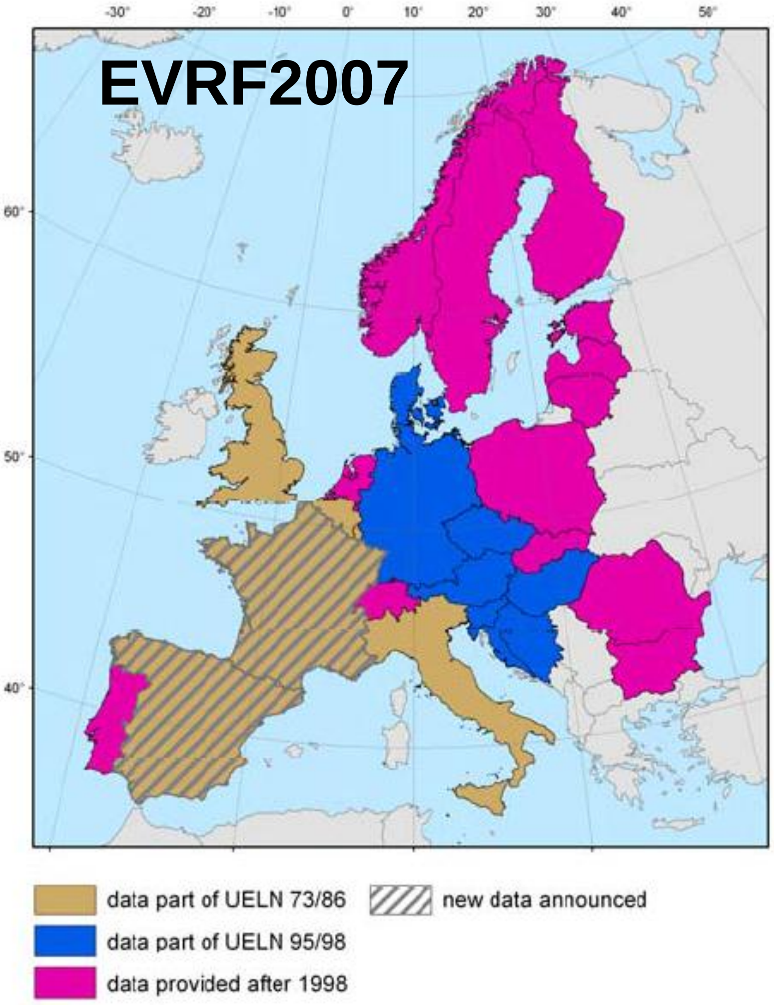
European Levelling Networks & Datums

- During the years, several common adjustments have been made of the national levelling networks in (Western) Europe... main purpose is to relate the national vertical datums (vertical reference frames) to each other
- "Reseau Europeen Unifie de Nivellement" REUN, changed to UELN 55; UELN = United European Levelling Network
- UELN 73/86 finalised in 1986
- UELN 95/98 new adjustment including also parts of Eastern Europe
- New terminology:
  - EVRS (European Vertical Reference System)
  - European realisation is EVRFXXXX (European Vertical Reference Frame)
  - UELN is now used to denote only the network
- EVRF2000 = new name for frame resulting from adjustment of UELN 95/98
- EVRF2007 is the last realisation of EVRS

UELN 55 & UELN 73







From Sacher et al. (2008)

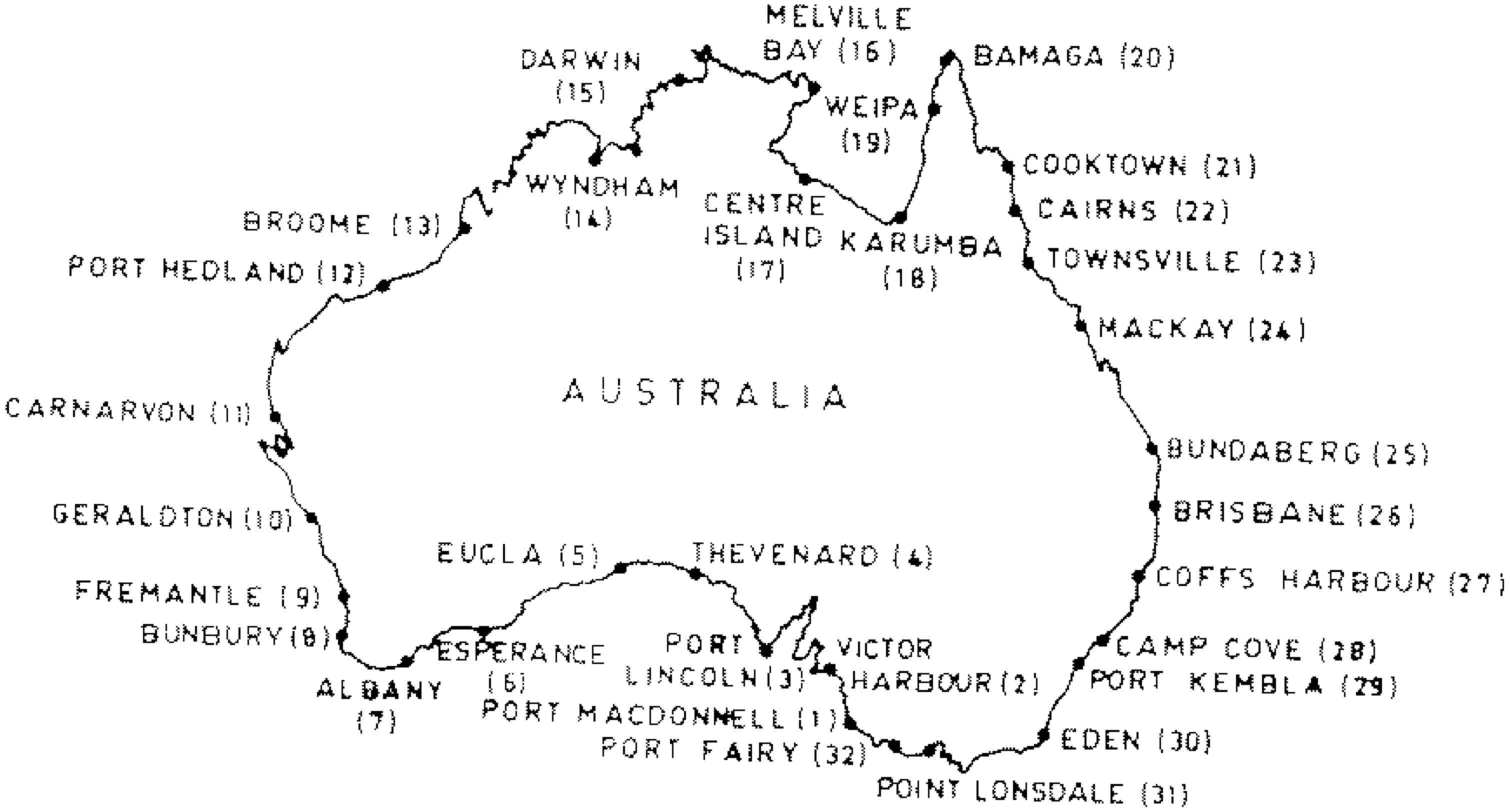
Australian Height Datum 1971 (AHD71)

- Primary levelling (97320km) adjusted: ***adjustment 1*** *Johnston Origin fixed*, ***adjustment 2*** *all 32 tide gauges fixed to zero*
- Much of the levelling network surveyed to 3^rd^ order standard
- **Australian Height Datum** based on *adjustment 2*, 5th May 1971
- Zero height surface is not a geopotential (or geoid) surface (SSTop is ±1-2m and is ignored)
- **AHD is not a true orthometric height system**, as it is *not strictly based on the geoid, nor is observed gravity used for spirit levelling reductions*
- When GNSS is used in Australia, **Ausgeoid98** converts GNSS-derived ellipsoidal heights to orthometric height
- When GNSS is used in Australia, **Ausgeoid09** converts GNSS-derived ellipsoidal heights to AHD heights... *it is a "geoid correction model"*

http://www.ga.gov.au/earth-monitoring/geodesy/geodetic-datums/australian-height-datum-ahd.html

Tide Gauge Stations for AHD71

http://www.ga.gov.au/earth-monitoring/geodesy/geodetic-datums/australian-height-datum-ahd.html



The MSL at these tide gauges were assumed to have zero orthometric height

Sponsors:

Geoid Corrections for Vertical Datums

Geoid Correction Model for GNSS

Assuming that a levelling-based height system is available, then **GNSS-levelling (quasi-)geoid heights** can be determined by making GNSS-derived ellipsoidal heights on levelled bench marks:

$$N_{GNSS/levelling} = h_{GNSS} - H_{levelling}$$

This information can be used to create a **correction model (or corrector surface)** to account for any bias between geoid surface and national zero height surface, *allowing direct transformation from GNSS-derived ellipsoidal heights to national levelled heights:*

$$\hat{H}_{GNSS} = \hat{h}_{GNSS} - N_{correction\ model}$$

Defining a *Corrector Surface*

1) The quality of the *corrector surface* depends on the number and quality of the included points with co-located data (h, H, N)
2) The more points the better the *corrector surface*
3) The better the geographic distribution of co-located data the better the *corrector surface*
4) There are different types of models, e.g. parametric surfaces, look-up grid, contour surface

Parametric Models

- Usually
  - a constant shift (1 parameter)

$$\mathbf{c}^{\mathrm{T}}\mathbf{x} = x_1$$

  - a constant shift and a tilt (3 parameters)

$$\mathbf{c}^{\mathrm{T}}\mathbf{x} = x_1 + \phi \cdot x_2 + \lambda \cos\phi \cdot x_3$$

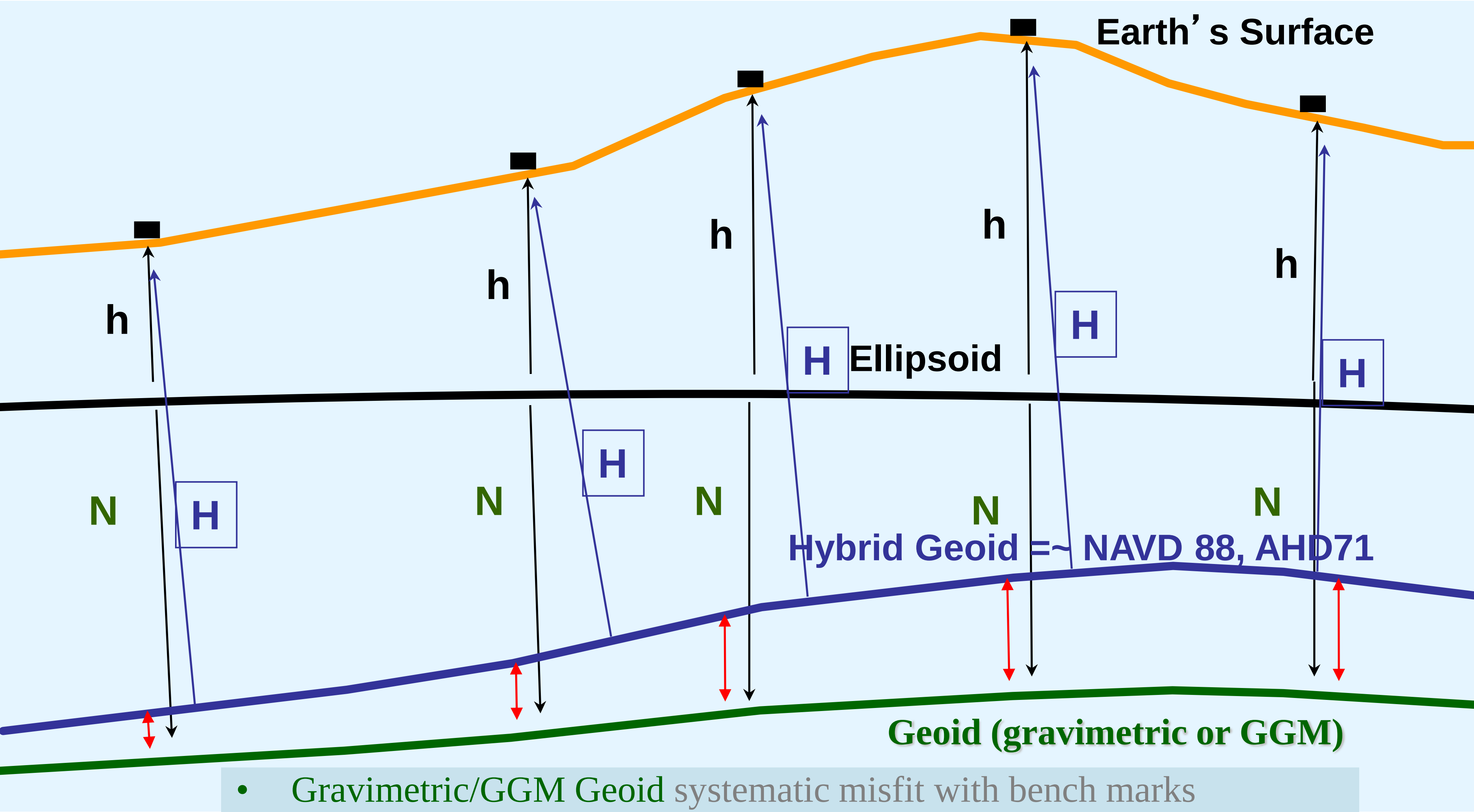
  - the zero and first degree effects (4-parameters). This is one constant shift plus a motion of the mass centre.

$$\mathbf{c}^{\mathrm{T}}\mathbf{x} = x_1 + \cos\phi\cos\lambda \cdot x_2 + \cos\phi\sin\lambda \cdot x_3 + \sin\phi \cdot x_4$$

- More complex surface models may be used, including gridded or contour models... ***so-called "hybrid geoid"***

J. Agren, Gravity & Height for National Mapping & Geodetic Surveying, Dublin, Ireland, 2-6 February 2015

Hybrid Geoid Models



- Gravimetric/GGM Geoid systematic misfit with bench marks
- Hybrid Geoid biased to fit local bench marks
- e = h – H - N

D. Roman, Gravity & WHS, Reference Frames in Practice, Rome, Italy, 4-5 May 2012

Ausgeoid09 *Corrector Surface (or Hybrid Geoid)* in Australia

Correction Contours on the GRS80 RE

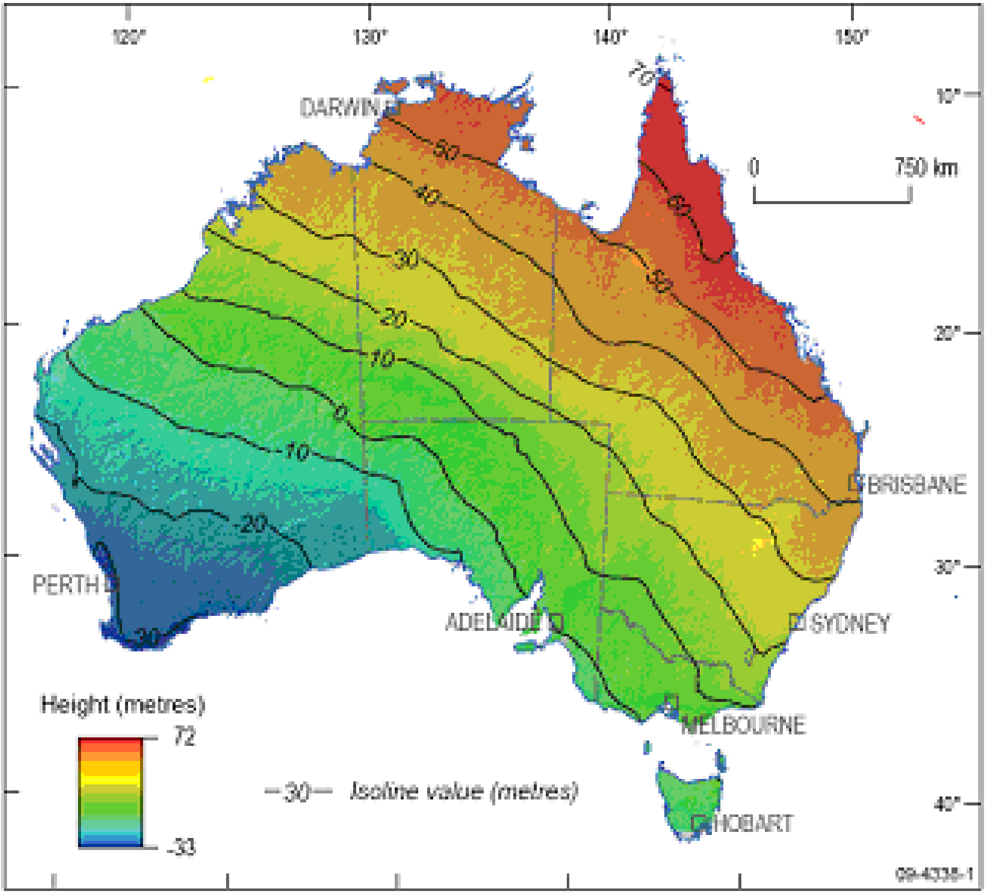
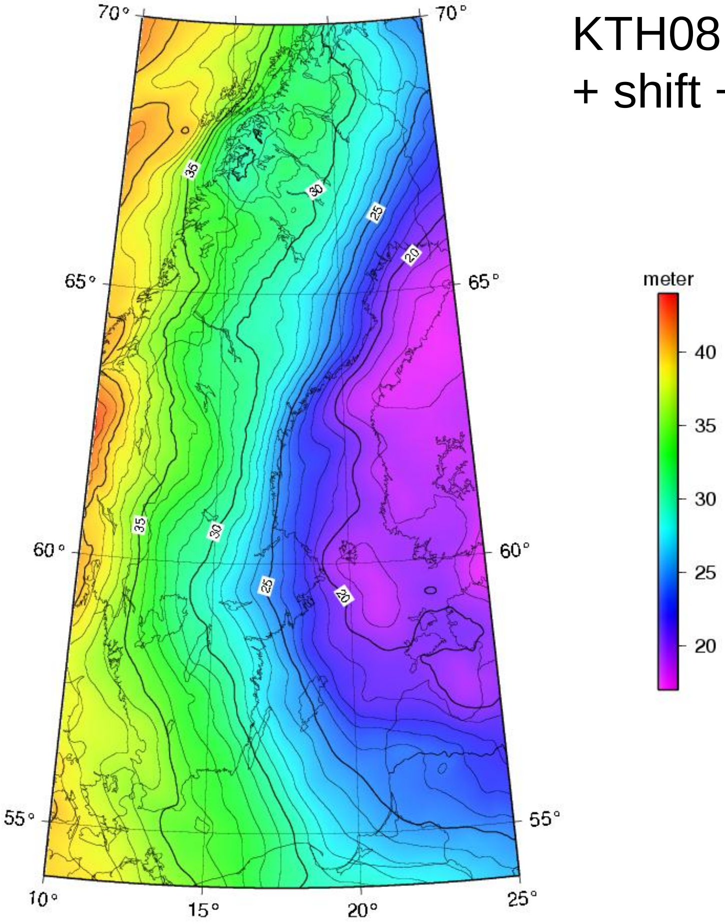


Figure 1. AUSGeoid09 allows GPS users to convert between GPS heights and AHD heights. In southwest Australia, the AHD is up to 33 metres below the ellipsoid and in northwest Australia the AHD is up to 72 metres above the ellipsoid.

SWEN08_RH2000 =
KTH08 + corr. land uplift/permanent tide
+ shift + residual surface (correction surface)



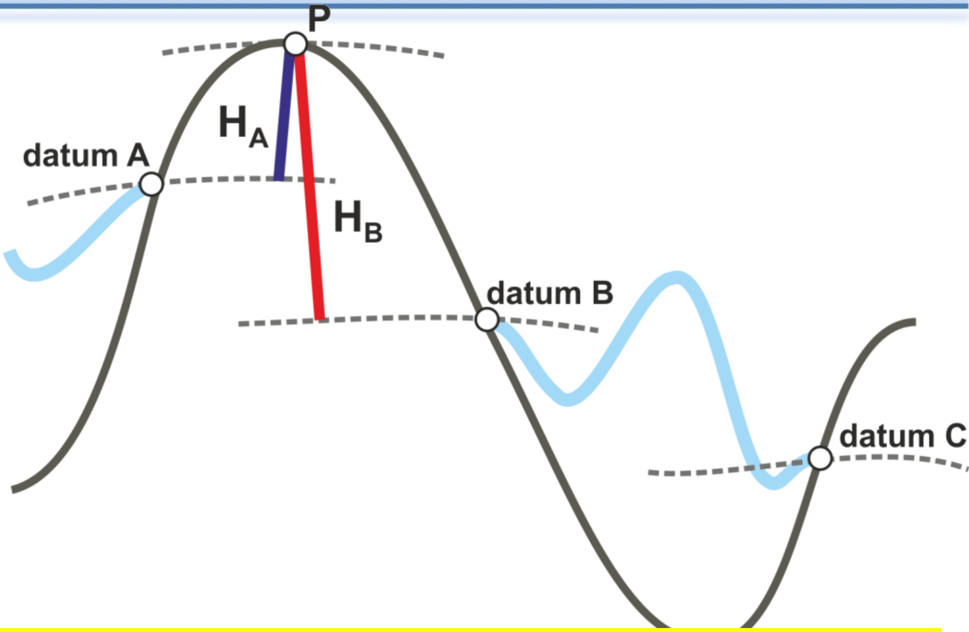
$$N_{correction\,model} = N_{gravimetric} + N_{known\ systematics} + x_{shift} + \delta N_{residual}$$

$$\hat{H}_{GNSS} = \hat{h}_{GNSS} - N_{correction\ model}$$

J. Agren, Gravity & Height for National Mapping & Geodetic
Surveying, Dublin, Ireland, 2-6 February 2015

Vertical Datum Unification

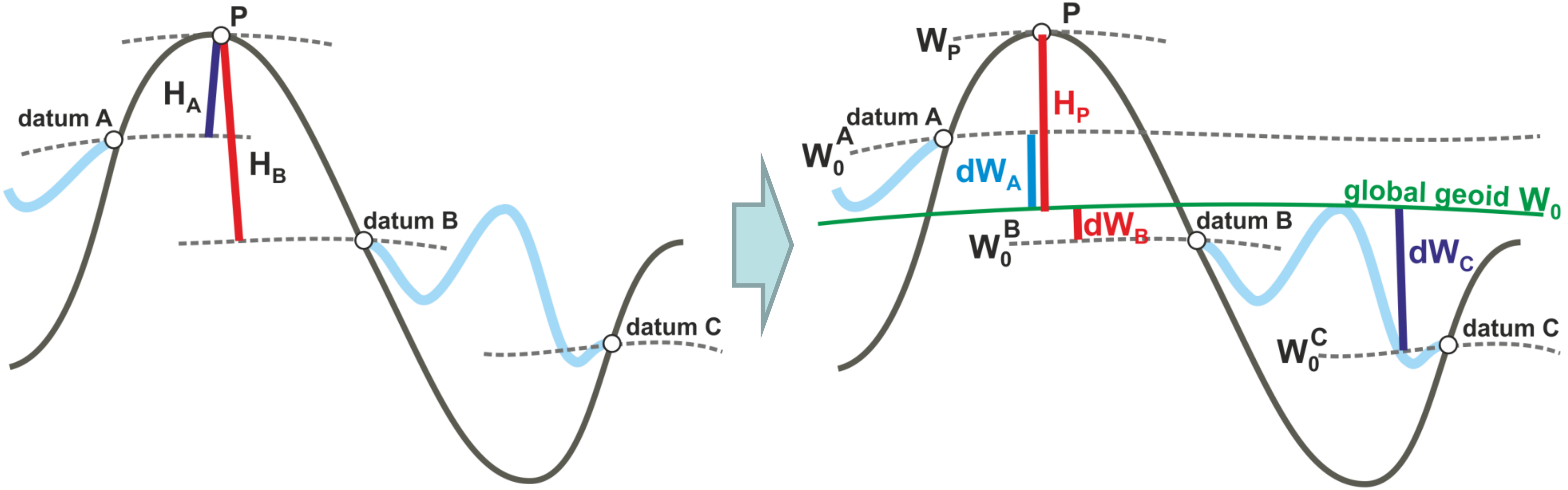
- refer to **different zero levels**
- realise **different types of heights** (normal, orthometric, etc.)
- omit (sea & land) **vertical time variations of displacement**
- do not support precise combination of **h-H-N for GNSS levelling**
- are the basis for **vertical data** produced over **last 150yrs**
- **cannot be replaced** by ellipsoidal heights *(these do not describe flow of water)*



Classical height systems cannot be *discarded*; they should be "modernised" by their integration into an *International Vertical Reference System (IVRS)* or *World Height System (WHS)*

Vertical Datum Unification

Objective: to refer all existing physical heights to one and the same reference level



- Since the primary observables are height differences, the **reference level can be selected arbitrarily**
- The recommended global reference is the **Global Geoid defined by a unique W_o**
- Then necessary to **determine the vertical datum discrepancies** dWi, also called “vertical datum parameters”

Vertical Datum Unification: Methodology

Strategy

The height anomalies ζ can be computed in two ways:

- By comparing geometric heights h with normal heights H^N (derived from levelling + gravity):

$$\zeta_j^{GNSS}(P) = h(P) - H_j^N(P)$$

- By solving the GBVP:

$$\zeta_j^{GBVP}(P) = -\frac{\Delta W_0}{\gamma} + \frac{\delta W_j}{\gamma} + \frac{R}{4\pi\gamma}\iint_\sigma \left(\Delta g_j + G_1^j\right) S(\psi)\, d\sigma + \frac{1}{2\pi\gamma}\iint_\sigma \delta W_j S(\psi)\, d\sigma$$

The comparison of these two estimates allows the formulation of the observation equation for datum unification:

$$h(P) - H_j^N(P) = q\Delta W_0 + e_j \delta W_j + \sum_{\substack{i=1 \\ j \neq i}}^{I} f_i \delta W_i + E(P)$$

$$q := \frac{1}{\gamma} \quad ; \quad e_j := -q + f_j \quad ; \quad f_i := \frac{1}{2\pi\gamma}\iint_{\sigma_j} S(\psi)\, d\sigma$$

$$E(P) := \frac{R}{4\pi\gamma}\iint_\sigma \left(\Delta g_j + G_1^j\right) S(\psi)\, d\sigma = \sum_{j=1}^{I} \frac{R}{4\pi\gamma}\iint_\sigma \left(\Delta g_j + G_1^j\right) S(\psi)\, d\sigma$$

Vertical Datum Unification: Methodology

Observation equations

Putting the known parameters on the left and the unknown parameters on the right, the observation equation for each point P is:

$$\zeta_j^{GNSS}(P) - E(P) = q\Delta W_0 + e_j \delta W_j + \sum_{\substack{i=1 \\ j \neq i}}^{I} f_i \delta W_i$$

For stations connecting two neighbouring datums ($j, j+1$), the observation equation is:

$$\zeta_j^{GNSS}(P) - \zeta_{j+1}^{GNSS}(P) = \left(H_{j+1}^{N}(P) - H_j^{N}(P)\right) = q\left(\delta W_{j+1} - \delta W_j\right) = q\delta W_{j+1,j}$$

with

$$\delta W_j = W_0 - W_0^{j} \quad ; \quad \delta W_{j+1} = W_0 - W_0^{j+1} \quad ; \quad \delta W_{j+1,j} = W_0^{j} - W_0^{j+1}$$

There is an equation observation for each point P and the unknowns (ΔW_0, δW_j) are estimated by means of a least squares adjustment

Details not provided here...
See IAG, Sanchez, Sideris, et al, publications....

International Vertical Reference Frame

Towards a Modern Vertical Reference System

The ITRS/ITRF provides a highly precise geometrical reference frame (consistent at the sub-cm level worldwide)

An *equivalent* physical reference frame is missing, hence need a **unified global vertical reference system**, or its realisation as an **International Vertical Reference Frame**. Main objectives are:

- to provide a reliable frame for consistent analysis and modelling of global phenomena related to the Earth’s gravity field (e.g. sea level variations from local to global scales, redistribution of masses in oceans, continents and the Earth’s interior, etc.)
- to allow the reliable combination of physical and geometric heights in order to explode at a maximum the advantages of satellite geodesy (e.g. combination of GNSS with gravity field models for worldwide unified precise height determination)

Definition & Realisation of a Modern Vertical Reference Frame

Reference for the consistent modelling of geometric and physical parameters, i.e.
$h = H^N + \zeta$ ($\approx H + N$) in a global frame with high accuracy (**$> 10^{-9}$**)

Geometrical Component

Coordinates: Ellipsoidal heights and their change with time

h (t), dh/dt

Definition:

ITRS + **Level ellipsoid ($h_0 = 0$)**

a. (**a**, J_2, ω, GM) **or**

b. (**W_0**, J_2, ω, GM)

Realisation:

1. Related to the **ITRS** (ITRF)
2. Conventional ellipsoid

Conventions:

IERS Conventions

Ellipsoid constants, W_0, U_0 values, reference tide system have to be aligned to the physical conventions.

Physical Component

Coordinates: Potential differences and their change with time

$-\Delta W_P(t) = C_P(t) = W_0 - W_P(t)$; $d\Delta W_0/dt$

Definition:

W_0= *const.* (as a convention)

Realisation:

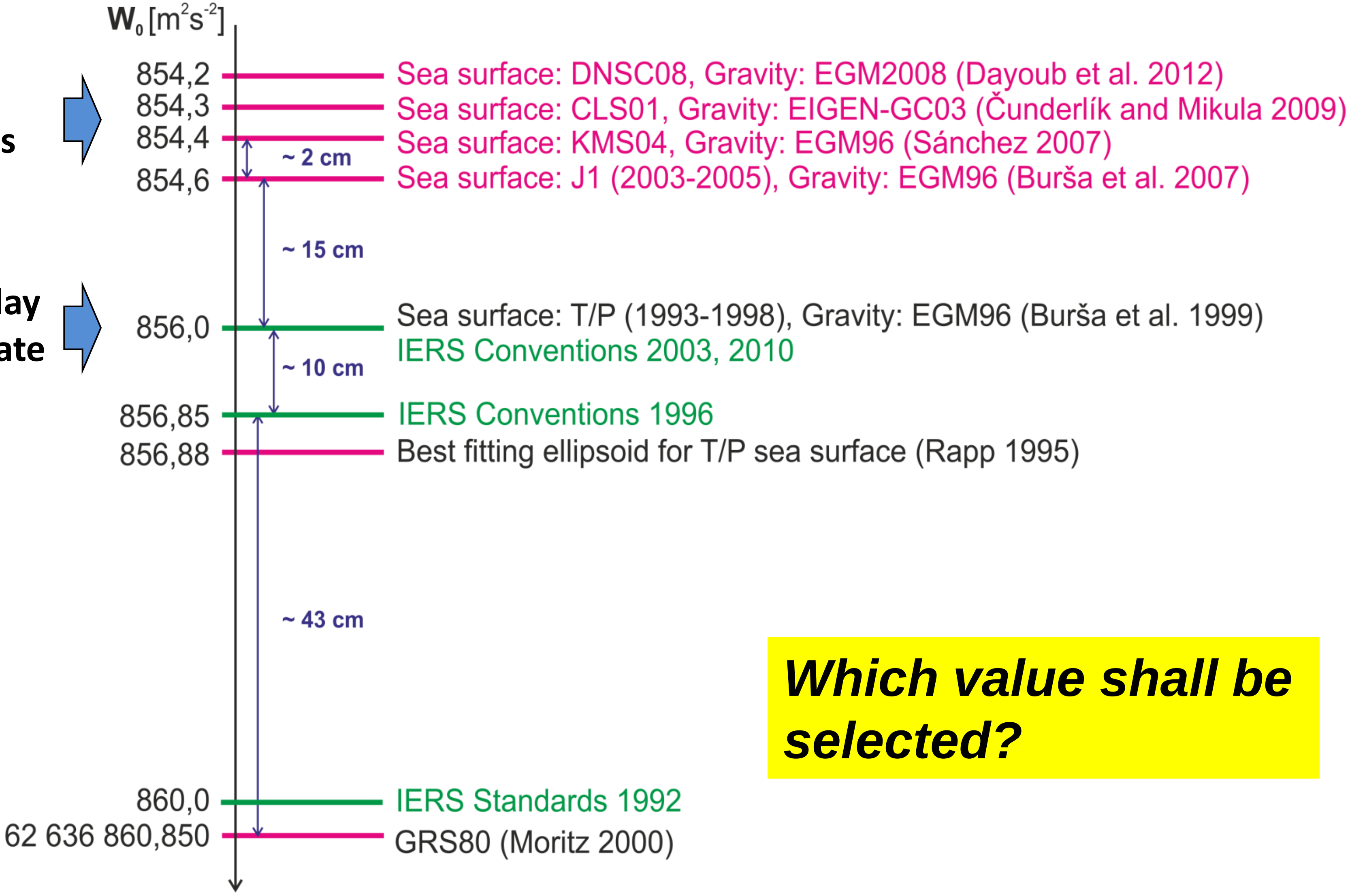
1. Selection of a global **W_0** value
2. Determination of the local reference levels **$W_{0,j}$**
3. Connection of **$W_{0,j}$** with **W_0**
4. Geometrical representation of **W_0** and **$W_{0,j}$** (i.e. geoid comp.)
5. Potential differences into physical heights (**H** or **H^N**)

Zero-tide system

Some Examples of W_0

Recent estimations

Applied today (best estimate 1999)



W_0 [m²s⁻²]

854,2 — Sea surface: DNSC08, Gravity: EGM2008 (Dayoub et al. 2012)

854,3 — Sea surface: CLS01, Gravity: EIGEN-GC03 (Čunderlík and Mikula 2009)

854,4 — Sea surface: KMS04, Gravity: EGM96 (Sánchez 2007)

854,6 — Sea surface: J1 (2003-2005), Gravity: EGM96 (Burša et al. 2007)

856,0 — Sea surface: T/P (1993-1998), Gravity: EGM96 (Burša et al. 1999)
IERS Conventions 2003, 2010

856,85 — IERS Conventions 1996

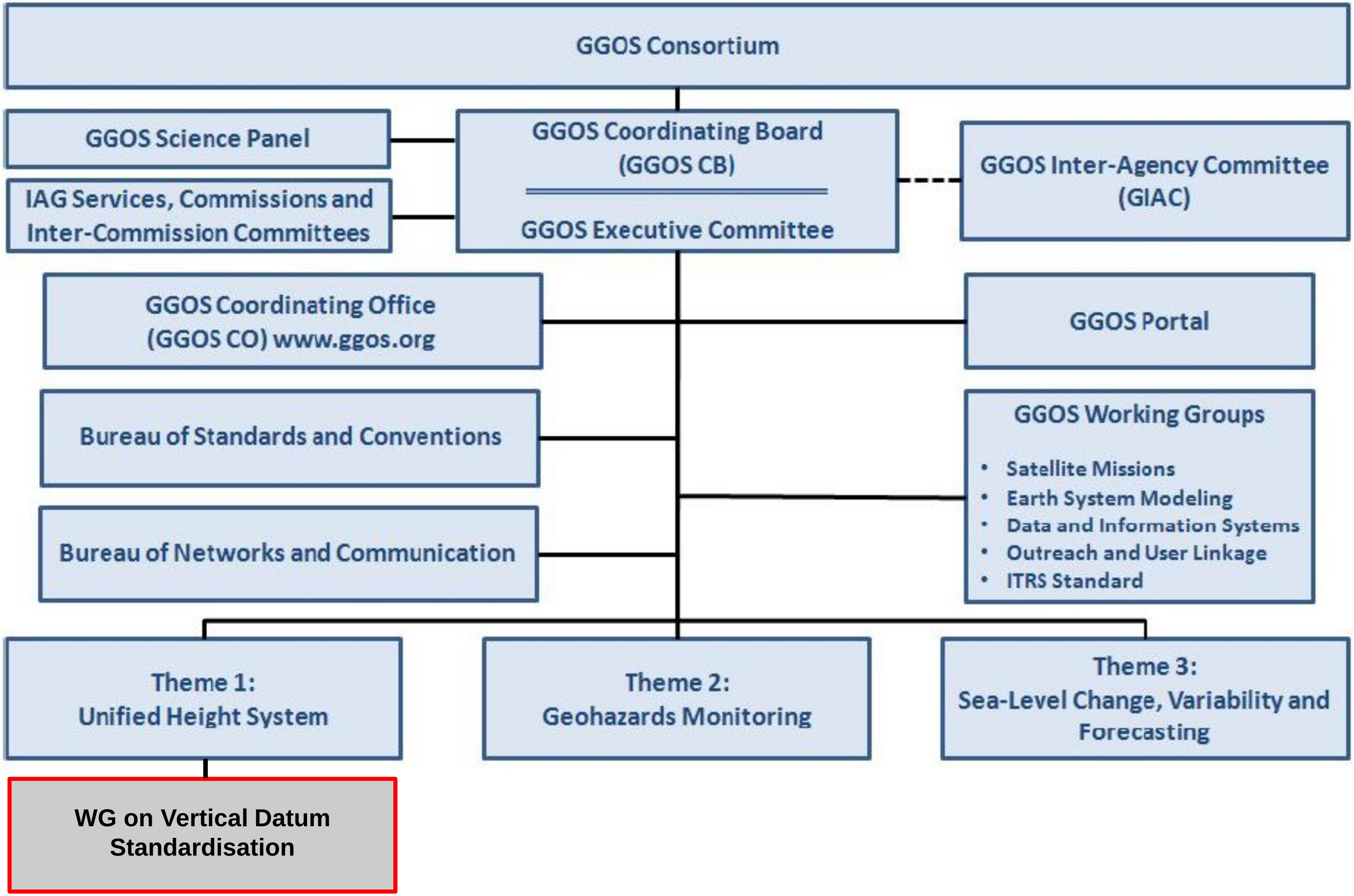
856,88 — Best fitting ellipsoid for T/P sea surface (Rapp 1995)

860,0 — IERS Standards 1992

62 636 860,850 — GRS80 (Moritz 2000)

Which value shall be selected?

A Unified Height System: a GGOS Challenge



GGOS Consortium

GGOS Science Panel

IAG Services, Commissions and Inter-Commission Committees

GGOS Coordinating Board (GGOS CB)

GGOS Executive Committee

GGOS Inter-Agency Committee (GIAC)

GGOS Coordinating Office (GGOS CO) www.ggos.org

GGOS Portal

Bureau of Standards and Conventions

Bureau of Networks and Communication

GGOS Working Groups

- Satellite Missions
- Earth System Modeling
- Data and Information Systems
- Outreach and User Linkage
- ITRS Standard

Theme 1: Unified Height System

Theme 2: Geohazards Monitoring

Theme 3: Sea-Level Change, Variability and Forecasting

WG on Vertical Datum Standardisation

Recommendation on W_0

- The four teams working on the empirical estimation of W_0 have recommended as a **best estimate** the value*

$$W_0 = 62\ 636\ 854{,}0 \pm 0{,}2\ \ \mathrm{m^2s^{-2}}$$

Value used at present: $62\ 636\ 856{,}0 \pm 0{,}5\ \mathrm{m^2s^{-2}}$

(**level difference of about 20cm!**)

- This new W_0 value should be used for:
  - the definition of **the constant L_G** (necessary for the transformation between Time Systems in a relativistic sense)
  - as a **defining parameter for a new reference ellipsoid**
  - as **defining reference level** for the global vertical reference system

* IAG resolution passed in July 2015

Vertical Datum Standardisation in Practice

1) Establishment of a vertical frame including: reference tide gauges, main levelling nodes, ITRF (SIRGAS, EPN, …) stations
2) Connection of the levelling networks between neighbouring countries (or vertical datum regions): $\Delta W_{ij} = C_i - C_j$
3) Computation of T_j (GBVP solution) and comparison with the geometric reference system (γh) and geopotential numbers C_j in three approaches:

Oceanic approach

(DT around gauges)

- **h** from satellite altimetry combined with tide gauge registrations;
- **C_i** = oceanic geopotential numbers ($=\gamma DT$);
- **T_i** from satellite-only GGM.

Coastal approach

(reference tide gauges)

- **h** from GNSS positioning at tide gauge benchmarks;
- **C_i** = 0 (or close to 0 for non-reference tide gauges);
- **T_i** from satellite-only GGM + terrestrial gravity.

Terrestrial approach

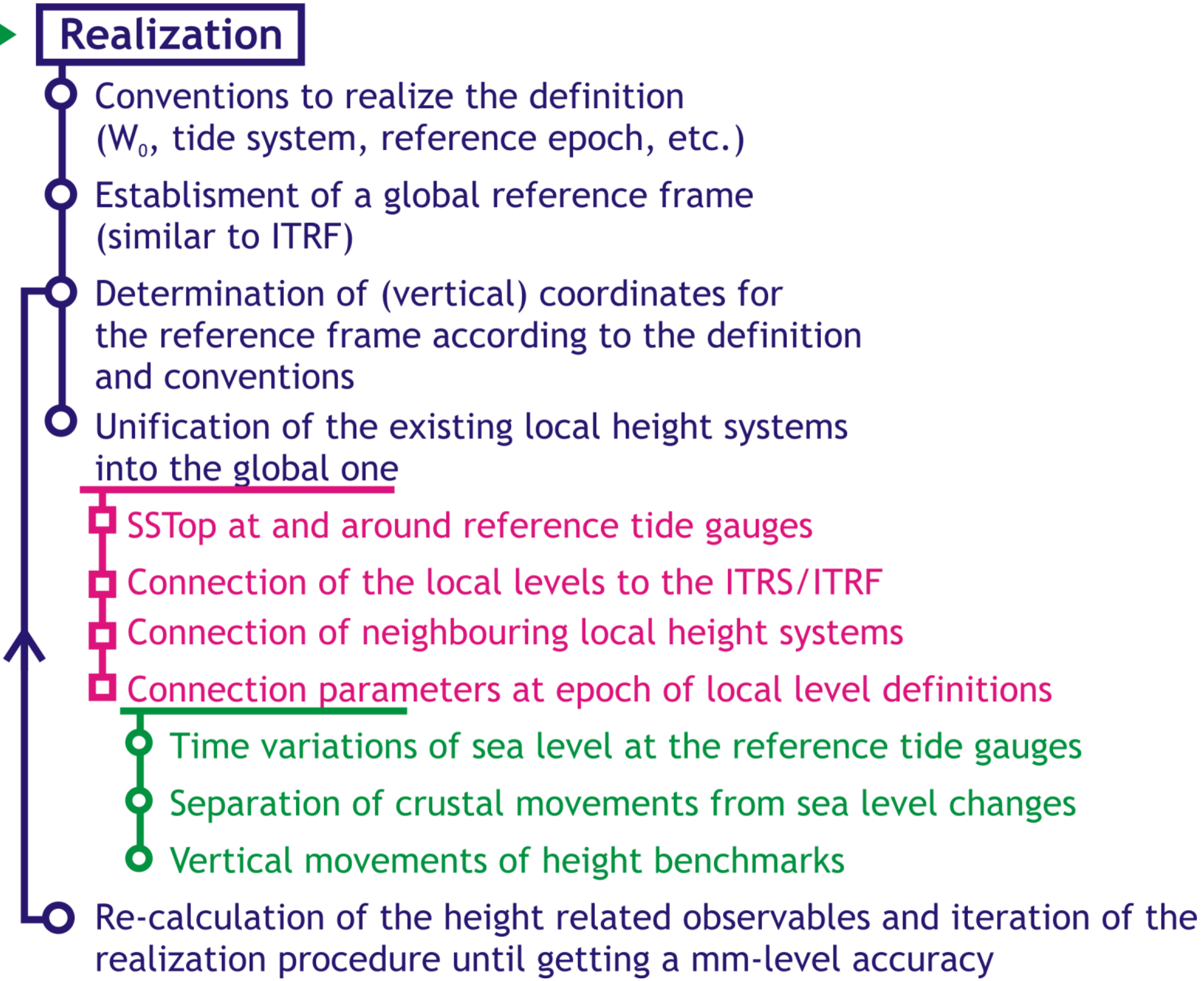
(geometric reference stations)

- **h** from GNSS positioning at ITRF stations and levelling nodes (including points with border connections),
- **C_i** geopotential numbers from levelling,
- **T_i** from satellite-only GGM + terrestrial gravity.

4) Least squares adjustment of (2) and (3)

Definition & Realisation of a Modern VRF: Summary

Definition
type of coordinates, reference surfaces, consistency between geometric and physical heights



Realization

- Conventions to realize the definition (W_0, tide system, reference epoch, etc.)
- Establisment of a global reference frame (similar to ITRF)
- Determination of (vertical) coordinates for the reference frame according to the definition and conventions
- Unification of the existing local height systems into the global one
  - SSTop at and around reference tide gauges
  - Connection of the local levels to the ITRS/ITRF
  - Connection of neighbouring local height systems
  - Connection parameters at epoch of local level definitions
    - Time variations of sea level at the reference tide gauges
    - Separation of crustal movements from sea level changes
    - Vertical movements of height benchmarks
- Re-calculation of the height related observables and iteration of the realization procedure until getting a mm-level accuracy

Details not provided here... See IAG, ESA report, Sanchez, Sideris, et al, publications....

L. Sanchez, 11th Int. School of the Geoid Service: heights and height datum, Loja, Ecuador, 7-11 October 2013

Closing Remarks

1) The availability of **GNSS techniques** motivates the combination of ellipsoidal heights and (quasi-) geoid models to obtain physical heights related, as far as possible, to the local vertical datums
2) **Levelling is expensive, laborious and time-consuming.** In addition, it is difficult in remote and mountainous areas and the inherent systematic errors grow very quickly over large distances
3) On the other hand, h from GNSS can be obtained quickly and inexpensively, and N is usually available from the international geodetic community or from national mapping agencies
4) The relationship $h = H + N$ is widely used for:
   - evaluating or refining global gravity models
   - estimating deformations in the vertical networks
   - determining local reference levels (local W_0 values)
   - vertical datum unification
   - GNSS levelling, etc.

5) In general, the **input data in *h* = *H* + *N* are taken as they are**, from different sources. There are no further considerations concerning issues such as:

- random errors in the heights *h*, *H*, and *N*
- datum inconsistencies inherent among the height coordinate types
- systematic effects and distortions (long-wavelength geoid errors, poorly modelled GNSS errors, over-constrained levelling network adjustments, etc.)
- assumptions/theoretical approximations made in processing observed data (e.g., atmospheric delay in GNSS, neglecting sea surface topography, river discharge corrections at tide gauges, gravity, etc.)
- omission (or approximate use) of gravity height reductions
- instability of reference station monuments over time (geodynamic effects, land uplift/subsidence)

6) There is a growing interest in **modernising vertical datums**, and this includes:
   - Observe high quality surface/airborne gravity data for improved local geoid/quasi-geoid computations
   - Adoption of such improved geoid models as new vertical datum surfaces... *to allow GNSS ellipsoidal heights to be converted to consistent orthometric heights*
   - Defining “hybrid geoids” to link heritage height datums to modernised geoid-based vertical datums... *to allow GNSS ellipsoidal heights to be converted to old datum heights (e.g. AHD71)*
7) **Unification of vertical datums** through the definition of W_o, and using combination of tide gauge heights, standard levelling & GNSS heighting
8) Concern about **time-varying effects on height datums**, e.g. SLR, Geoid height variation, GIA, crustal motion, land subsidence, etc.
9) Definition of an **International Vertical Reference Frame** (IVRF), analogous to the geometry-only ITRF, needs to be realised

Reading & Reference List

Provided by L. Sanchez,

Deutsches Geodätisches Forschungsinstitut (DGFI), Germany

Chair of the IAG/GGOS Working Group on Vertical Datum Standardisation
& SIRGAS Vice-president

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