CONTINUUM MECHANICS AS A SUPPORT FOR DEFORMATION MONITORING, ANALYSIS, AND INTRPRETATION

*A. Szostak – Chrzanowski**,*
*W. Prószyñski**,*
*W. Gambin****
**Canadian Centre for Geodetic Engineering, University of New Brunswick, Fredericton, N.B., E3B 5A3 Canada; e-mail:* amc@unb.ca
***Department of Engineering Geodesy, Warsaw Technical University, Warsaw, Poland; e-mail: wpr@gik.pw.edu.pl*
****Department of Mechatronics, Warsaw Technical University, Warsaw, Poland; e-mail: wgambin@mchtr.pw.edu.pl*

Abstract: Recent developments in multidisciplinary approach to geodetic and geotechnical monitoring, deformation analysis, and physical interpretation of behaviour of man-made and natural structures call for creation of technological and scientific basis for communication between various sectors of engineering and geosciences. Earlier developed concepts of integrated monitoring and analysis of deformations call for geodetic engineers to become involved in the physical interpretation of structural and ground deformations. The continuum mechanics is proposed as the basis of communication. This presentation gives an overview of geodetic aspects of monitoring and analysis of deformations followed by basic definitions and laws used in continuum mechanics as related to the determination of the load-deformation relationship. The latter is the first step in the physical analysis of deformations.

1. INTRODUCTION

Safety, economy, efficient functioning, environmental protection, and development of mitigation measures in case of man induced or natural disasters require good understanding of deformation process which can be achieved only through proper monitoring and geometrical and physical analysis of deformable bodies. Development of new methods and techniques for monitoring and analysis of deformations and development of methods for the optimal modeling and prediction of deformations is subject of intensive studies.

The FIG Working Group 6.1 on Deformation Measurements and Analysis has always been one of the most vital groups of FIG Commission 6 (Engineering Surveys) and one of the most, if not the most, active international groups dealing with the problems of monitoring and analysis of deformation measurements. Besides the activity of FIG WG6.1, there are a number of other international groups, which are involved in deformation studies. Among the major organizations one should list:

- International Association of Geodesy (IAG) with very active study groups concerning geodynamics, tectonic plate movements, and earth's crust deformations,
- International Society for Photogrammetry and Remote Sensing (ISPRS) within the activity of Commission 5 on Close Range Photogrammetry and Vision Systems,
- International Society for Mine Surveying (ISM) with their very active Commission 4 on Ground Subsidence and Surface Protection in mining areas,
- International Society for Rock Mechanics (ISRM) with their overall interest in rock stability and ground control,

- International Commission on Large Dams (ICOLD),
- International Society of Soil Mechanics and Foundation Engineering, and
- International Association of Hydrological Sciences (IAHS) which organizes international symposia on ground subsidence due to the withdrawal of underground liquids (water, oil, etc.).

Most of the activities and studies of the above groups focus on direct applications to their particular deformation problems. The activity of FIG WG6.1 has been focused on developing new methods and techniques, which could be applied to monitoring and analysis of any type of deformable bodies, using any type of deformation measurements (using geodetic and/or geotechnical/structural instrumentation). Although accuracy and sensitivity criteria for determination of deformation may considerably differ between various applications, the basic principles of the design of the monitoring schemes and analysis of the results remain the same whether one is concerned with the earth crust deformations, slope stability, dam deformations, or displacements of magnets in accelerators of sub-atomic particles. Thus the activity of WG6.1 has always been of an interdisciplinary nature.

Till early 1980s, geodetic deformation models would consider only the geometrical relationship between observations repeated in different epochs of time [1], [2]. In 1988, at the 5-th FIG Symposium the input of specialists in other fields of engineering into the activity of WG6.1 has become very significant. The role of the monitoring surveys, which used to serve only the determination of the change in shape and dimensions of deformable bodies has expanded into verification of material parameters, determination of causative factors, and determination of deformation mechanism [3].

Initially, the main task of the FIG Working Group 6.1 was to organize and to stimulate the international collaboration in the field of deformation measurements by geodetic methods. Since 1986, the tasks have been expanded beyond the use of geodetic methods into other monitoring techniques and into interdisciplinary analysis and interpretation of deformations.This change in the role of geodetic engineers requires from the geodetic community good understanding and adaptation of terminology that has been generally used in other engineering fields and in physics when dealing with deformation processes.

The growing interest among geodetic engineers in the interdisciplinary approach to the integrated analysis and physical interpretation of deformations brought a need for a verification and unification of terminology and formulations used so far in deformation modelling. As a result, in 1992 an *ad hoc* committee (Task Force 6.1.2) was created to look into the terminology and classification of deformation models. Welsch and Heunecke [4] presented an informative report on the proposed terminology, which was based on a system theory, with a focus on data processing. In the system theory, a "black box" is used as a term for the system of unknown structure and parameters. In practice, there is usually some knowledge on the system and on the approximated values of parameters. In such case, the system is referred to as a "grey box". If the system structure and parameters are known, the system is referred to as a "white box" [5]. Although the system theory is very useful in signal (e.g. observed deformation) processing and analysis, one of the main goals of the deformation analysis is determination of the mechanism of deformations. Therefore, the deformable object should be treated as a mechanical system, which undergoes deformation according to the principles and laws of continuum mechanics.

This paper, after reviewing the tasks of geodetic engineers in deformation monitoring and analysis, gives an introduction to basic definitions and laws of continuum mechanics as applied to deformation processes.

2. TASKS OF GEODETIC ENGINEERS IN DEFORMATION ANALYSIS

The interdisciplinary approach to deformation analysis of any type of deformable body includes geometrical analysis and physical interpretation. Geometrical analysis, which is of main interest to geodesists, describes the change in shape and dimensions of the monitored object, as well as its rigid body movements (translations and rotations). The ultimate goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains.

Physical interpretation, which is main interest of structural, rock mechanic, mining, and geophysical specialists, is based on the relationship between the causative factors (loads) and deformations. The relationship can be determined either by statistical method, which analyses the correlation between the observed deformations and loads, or deterministic method based on continuum mechanics which utilizes information on the loads, properties of the material, and physical laws governing the stress-strain relationship. The analysis would follow laws of statics, kinematics, and dynamics. Therefore, in order to have better interdisciplinary cooperation and understanding between geodesists and other engineering specialists the laws and definitions used in deformation analysis should be based on continuum mechanics terminology.

The traditional task of geodetic deformation analysis has been determination of absolute or relative displacements of selected points on the deformable object and eventual determination of the change in shape and dimensions of the object in space and time domains. Geodetic procedures are applied to analyze man-made structures such as dams, bridges, natural structures such as slopes, underground and open pit mining, and natural phenomena such as movements of tectonic plates.

Geodetic monitoring of deformations may have two basic tasks: standard tasks and generalized tasks. The standard task is dealing with deformation models in which the deformations are given in discrete form (determination of displacements of selected points located on an investigated object) [6]. The displacements are given as displacement vectors with given accuracy of their determination The generalized task is dealing with deformation models in which the deformations are given in continuum form.
Interpolation functions are used to transform the discrete data into continuous functions given, for example in [1] [7]. Continuous description of the deformation field is a function of:
1. discrete distribution of data,
2. choice of the interpolation function, and
3. choice of fixed points or frame.

Continuous description of the deformation field is a function of the discrete distribution of data. Therefore it is dependent on the discretization of the geodetic network. The best designed geodetic discretization may not be physically possible or the discreet points may not be located at the location where maximum deformations are expected. The physical analysis is capable to identify the locations of expected maximum deformations. The choice of fixed points or frame is very important in continuous description of the deformation field. The physical analysis may supply information, which of the discrete points or frame may be assumed as fixed. The points assumed to be fixed at the design stage of the structure may not continue to be fixed during life of the structure. The integrated analysis of deformations may lead to identification of time dependent unstable points.

Strain analysis is very widely applied in geometrical analysis. The main applications of strain analysis by geodesists are in:

1. analysis of earth crust movement (e.g. [8]),
2. analysis of structural problems (e.g. concrete dams) (e.g. [9]), and
3. robust analysis of geodetic networks (e.g. [10]).

One should notice that geodesists often introduce their own terminology and create definitions in the strain analysis, which do not follow the definitions established much earlier in continuum mechanics.

3. INTRODUCTION TO BASIC DEFINITIONS IN CONTINUUM MECHANICS

The continuum mechanics, also referred to as phenomenological mechanics of deformable body, is a part of theoretical physics which is based on Newtonian mechanics. On the basis of Newtonian mechanics, the theoretical mechanics formulates the description of a movement of a set of material points and movements of rigid bodies. The continuum mechanics introduces the idea of the deformable body which is treated as a continuum of material particles. The particles are subjected to their own interaction and external action. If the interactions between particles are defined as functions acting on the deformable body, then the material continuum is obtained, which is subjected to deformation movement. This applies to the behaviour of any continuum matter such as solids, fluids, and gases.

A main problem in the methodology of solid mechanics is the way how the governing laws are formulated. The method has phenomenological character, which means that the governing laws of movement are postulated and later experimentally verified. It is assumed that the causes and results are known and the simplest, logical, and interrelated relations are sought.

In continuum mechanics one can list the following basic definitions.

- Physical domain is a three dimensional Euclidian space E^3, where every point has three coordinates (x, y, z). The investigated object is located in Euclidian space.
- Deformable object B is build from a continuum of material particles.
- Material element is a particle with its immediate surrounding. The object may be defined as a rigid or a deformable body.
- Deformable body is an object built from matter forming a continuum.
- Configuration of an object is a mapping function χ of body B in Euclidian space E^3.
- Body movement is treated as a single parameter mapping family χ_t, where t is a time parameter.
- Mass of material element is a positive number. When the volume of an element is approaching zero, then the density of the material particle is obtained.
- Time is an absolute measure of distance between epochs. The time moment is an element of one dimensional Euclidian space I.

Using the above definitions one can construct an "apparatus" which allows for defining movement and deformation of a deformable body. The following must be assumed: a stationary coordinate system called respective (spatial) system $\{x_i\}$ in Euclidian space E^3 and respective configuration χ of object B, which may be called an initial configuration χ_t . The configuration in an actual moment is actual configuration. Set of matter particles in respective configuration forms material space in which material coordinate system $\{X_I\}$ is defined. After the defining the two coordinate systems, the movement of the material object is defined as a changed location in time of each of its particles. The location is given using functions

$$x_i = x_i (X_I,t), \tag{1}$$

or inverse functions

$$X_I = X_I(x_i,t). \tag{2}$$

Derivatives of the functions with respect to time are called material derivatives. They define the velocity fields. If the movement of the deformable body is enforced, then the deformation is defined as a change of distances of body particles. The deformation is expressed as a function gradient $x_i = x_i (X_I,t)$, called deformation gradient F. By connecting F with different coordinate systems, the different deformations measures, used in a spatial or material system, are obtained. Because the functions describing object movement or other functions may be expressed in spatial or material system variables, two descriptions of the movement are distinguished: spatial description and material description. The mechanics of solids is based on material description.

The movement of particles is enforced by external action (acting forces) and internal action (internal stresses). State of stress in a body, similarly as the movement, is defined using described functions. The functions are related to body particles. There are numbers of measures of state of stress and its changes in time. The state of all forces and resulting from them stresses is governed by laws of preservation of: mass, momentum, and energy. Distinguished functions among all functions, which can describe the distribution of forces and stresses in a body, are only those which are allowed from the stand point of physics.

The functions describing body movement and internal state of stress are interconnected by constitutive equations. The constitutive equations are different for different types of materials such as: elastic, viscous, plastic, and their combinations. Modeling of behavior of different materials is governed by the principles of creating the constitutive equations, the dimension analysis, and similarity theory.

4. SAMPLE OF BASIC FORMULATIONS USED IN DEFORMATION ANALYSIS

Deformation analysis based on continuum mechanics of solids is performed using mathematical techniques and physical laws of statics, kinematics, and dynamics when the investigated object is subjected to loading.

Generally, an investigated object may be represented as a mechanical system with following models:
1. rigid body movement (translations and rotations),

2. deformation (change in shape and dimensions) or
3. rigid body movement and its deformation.

The analysis has to follow laws of statics, kinematics, and dynamics. Statics deals with the equilibrium of forces acting on a body at rest or moving with constant velocity. Dynamics deals with bodies in motion. Dynamics is divided into kinematics and kinetics. Kinematics is concerned only with the geometry of motion. It is the study of the positions, angles, velocities, and accelerations of body segments and joints during the motion. Kinetics is concerned with the force analysis of bodies in motion.

The following basic conditions and relations of continuum mechanics must be defined in deformation analysis:

1. conditions of equilibrium (in case of static analysis), or Newton's law of motion in case of dynamic analysis,
2. kinematic relations, and
3. stress-strain relations.

The conditions of equilibrium are fundamental equations of continuum mechanics. The acting forces on the object must be in equilibrium. The forces acting on the object are divided into external forces acting on boundaries and body forces. An acting force P per unit area A is defined as a stress σ. If σ_{ij} denotes stress tensor and the stress tensor is symmetric, i.e.:

$$\sigma_{ij} = \sigma_{ji} \tag{3}$$

the equation of equilibrium is

$$\sigma_{ij}\ dA_i = dP_j. \tag{4}$$

Eigenvalues of the stress tensor are known as principal stresses. The principal stress directions are the eigenvectors of the stress tensor.

In case of a body in motion, the motion is described by dynamic equations. For small motions, the velocity is a partial derivative of displacement function u and is given as

$$\dot{u}_i = \frac{\partial u_i}{\partial t} \tag{5}$$

Negative product of acceleration and mass density ρ, known as d'Alambert inertia force per unit volume, is added to the acting force P_i. This gives the dynamic equation using Newton's law in the form:

$$\sigma_{ij} dA_i = -dP_i + d(\rho \ddot{u}_i). \tag{6}$$

The kinematic relations are between strain and displacement and relate to the geometry of the motion which leads from undeformed to deformed position. The kinematic relations are independent of acting forces and type of material behaviour (elastic or inelastic).

The strain tensor is noted as ε_{ji} and has nine components in the 3 dimensional space. In case when displacements u are small it means, $u_{i,j} \ll 1$, the strain tensor components are:

$$\varepsilon_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{7}$$

where

$$u_{i,j} = \frac{\partial u_i}{\partial x^j} \tag{8}$$

In matrix notation the strain is given as:

$$\varepsilon_{i,j} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}) & \frac{1}{2}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}) \\ \frac{1}{2}(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}) \\ \frac{1}{2}(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}) & \frac{1}{2}(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}) & \frac{\partial u_3}{\partial x_3} \end{bmatrix} \tag{9}$$

The diagonal components in the above matrix are known as direct strains ε_{11}, ε_{22}, ε_{33}. The off diagonal strain components are known as shear strains. In engineering applications, an engineering shear strain γ is often introduced, which is related to the off diagonal elements of the strain tensor as, e.g: $\gamma_{12} = 2\ \varepsilon_{12}$. This may lead to confusion or even errors in the deformation analysis when the values of the shear strain are not clearly defined whether they refer to γ_{12} or to ε_{12}.

Deformation of a body is fully described if nine components (in 3 dimensional space) of deformation tensor can be determined at any point of the body. The deformation tensor may be decomposed into strain tensor ε and rotation (deviatoric) tensor ω:

$$\frac{\partial u_i}{\partial x_j} = \varepsilon_{ij} + \omega_{ij} \tag{10}$$

where

$$\omega_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}) \tag{11}$$

and ωij is skew symmetric:

$$\omega_{ij} = -\omega_{ji}. \tag{12}$$

The first, second, and third invariants (ε_I, ε_{II}, ε_{III}) of strain tensor ε_{ji} are linear, quadratic, and cubic functions of its components respectively. They are given as:

$$\varepsilon_{I} = trace(\varepsilon) = \varepsilon_{ii} \tag{13}$$

$$\varepsilon_{II} = \varepsilon_{ii}\varepsilon_{jj} - \varepsilon_{ij}\varepsilon_{ji} \tag{14}$$

$$\varepsilon_{III} = \det(\varepsilon)\,. \tag{15}$$

The stress-strain relations are mathematical descriptions of mechanical properties of the material using constitutive matrix. Linear elastic constitutive laws may model almost all materials subjected to sufficiently small loads. For linear elastic material, Hooke's law for a general anisotropic solid is given as:

$$\sigma_{ij} = E_{ijlm}\,\varepsilon_{lm} \tag{16}$$

where E_{ijlm} represents components of the elasticity tensor and i, j, l, m have values 1, 2, 3.

4. CONCLUSIONS

The given basic definitions and formulations of continuum mechanics require a more detailed discussion between geodesists and other professionals in order to develop a common "language" for the interdisciplinary cooperation. The authors propose to establish within the Working Group 6.1 a new Task Force on the "Unification of terminology and procedures in deformation analysis" based on the definitions used in continuum mechanics. The Task Force, consisting of representatives of all disciplines of science and engineering involved in deformation studies, should prepare practical examples in which the unified terminology and procedures would be implemented.

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